Assistive Technologies for Second-Year Statistics Students who are Blind

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Abstract

At Wake Forest University, a student who is blind enrolled in a second course in statistics. The course covered simple and multiple regression, model diagnostics, model selection, data visualization, and elementary logistic regression. These topics required that the student both interpret and produce three sets of materials: mathematical writing, computer programming, and visual displays of data. While we did find scattered resources for blind students taking mathematics courses or introductory statistics courses, we found no complete account of teaching statistical modeling to students who are blind. We also discovered some challenges in stitching together multiple partial solutions. This paper outlines our specific approach. We relied heavily on integrating the use of multiple existing technologies. Specifically, this paper will detail the extensive use of screen readers, LaTeX, a modified use of R and the `BrailleR` package, a desktop Braille embosser, and a modified classroom approach.

1. Introduction

1.1 Background

The Americans with Disabilities Act of 1990 (ADA 1990) as amended in the Americans with Disabilities Act Amendment Act (ADAAA) of 2008 and Section 504 of the Rehabilitation Act of 1973 (Section 504 1973) are two pieces of national, civil rights legislation intended to prevent the discrimination of individuals based on their disability. At the postsecondary educational level, colleges and universities are obligated to provide "reasonable accommodations" that would allow students with disabilities the opportunity to participate in the college or university's services and programs, including classroom readings, science labs, etc. Without guidelines and policies provided by the ADA and Section 504 that ensure equal access to educational programs, students with disabilities would face barriers to equal access in postsecondary institutions, and be prohibited from participating fully in college and university life.

According to the ADA, a disability is considered any physical or mental impairment that substantially limits one or more major life activities. Most colleges and universities have a Disability Support Services (DSS) office on campus to help coordinate services and "reasonable accommodations" for students with disabilities. At Wake Forest University, the DSS office is integrated into the larger Learning Assistance Center and Disability Services (LAC), which is a general academic support office that, in addition to coordinating services for students with disabilities, provides academic coaching and peer tutoring to all Wake Forest University students. LAC staff members work closely with university faculty and staff members to facilitate the implementation of accommodations on campus and in classrooms.

To request academic accommodations, students must present recent documentation of their disability to the LAC to determine their eligibility to receive services. Then, the LAC staff, in consultation with other university professionals, (a) review the disability documentation presented by students and (b) determine which accommodations are considered reasonable. Examples of reasonable accommodations could include the following: priority registration; extended time for in-class tests and writing assignments; sign language interpreters; course substitutions; classroom relocation; assistive technology; or a reduced course load. It is important that academic accommodations should not substantially alter or modify essential requirements. Academic accommodations are intended to provide equal access to educational opportunities, not compromise the integrity of course content.

At Wake Forest University, approximately 7% of students have disclosed some type of disability and receive academic accommodations in the classroom. The majority of these

students have a diagnosis of a learning disorder or attention deficit / hyperactivity disorder, and the most frequently provided accommodations for these students are extended time and a low distraction environment for tests and exams. Neither accommodation requires much effort from the faculty member at Wake Forest University. Students with disabilities at Wake Forest University are advised about course selection similarly to students without disabilities. In other words, all students are encouraged to enroll in courses that satisfy Divisional requirements at Wake Forest University, are counted toward their major, and/or satisfy their own personal interests.

At Wake Forest University, a female student presented documentation of a significant visual impairment to the LAC. Her eye condition, congenital microphthalmia, results in reduced visual acuity. In this specific case, the student has significantly reduced visibility in one eye and a prosthetic eye in the other. While she is able detect light, and she can identify high contrasts, she has no usable vision for reading. She used Braille throughout most of her K–12 education and had a moderately high level of knowledge with assistive technology when she entered the university. She enrolled at Wake Forest University with the intention of majoring in mathematical business or a similar field, having previously excelled in her high school AP statistics class.

According to the National Center for Educational Statistics (NCES, Raue and Lewis 2011), 90% of public 4-year and 59% of private 4-year institutions report enrolling 1 or more students with visual impairments. Furthermore, students with visual impairments make up 3% of the total population of students who disclose disabilities among all types of institutions. Therefore, it is likely that an instructor will have a student with a visual impairment at some point in his or her teaching career, particularly instructors who teach frequently at larger institutions. In addition, 87% of public 4-year and 62% of private 4-year institutions provided adaptive equipment and technology for their students with disabilities.

Readers may wonder about the proper use of terms such as *blind* versus *visually impaired*. For the particular student discussed in this paper, the term blind is most appropriate, and is not considered offensive. Indeed, one of the major national advocacy groups calls itself the National Federation of the Blind (https://nfb.org//). The term *visually impaired* encompasses a wide range of visual disabilities and limitations. Many visually impaired students can manage reading ordinary text with minimal accommodations. Given the particular disability of the student at Wake Forest University, the Learning Assistance Center at Wake Forest University suggests that faculty use the term *blind* to properly convey her visual impairment. We have therefore chosen to title this paper "Assistive Technologies for Second-Year Statistics Students who are Blind."

There has been research and discussion on the challenges of teaching the first statistics

course to blind students. Gibson and Darron (1999) discuss the use of physical *manipulatives*—such as modeling clay and raised line drawing kits—and how they can be put to use to convey things like the shape of distributions. Mechan et. al. (1993) similarly discuss these manipulatives, but also share tips and pitfalls on verbalization (i.e. how to dictate mathematical writing in a clear, unambiguous way) and testing strategies. Vita and Kataoka (2014) demonstrate a creative approach to teaching about probability and random walks with a tactile model. Marson et. al. (2013) provide a more recent and comprehensive account of teaching introductory statistics to students who are blind. Each of these papers presents issues and solutions for the introductory course.

Unfortunately, solutions in the first statistics course do not necessarily translate to higher level courses. Consider data visualization, mathematical writing, and software. In the first statistics course, graphical representation of data often involve scatterplots, histograms, boxplots, and so forth. But in a second course, one needs to make residual plots with an eye towards checking for non-linearity, or auto-correlation of residuals. As the number of explanatory variables grows, so too does the number of possible scatterplots and residual plots. The first statistics course can be taught without requiring any computer programming (the author's university avoids all programming in the first course). However, in the second course, one cannot avoid programming to fit logistic regression, use best subsets model selection, and so forth. Finally, the most complicated mathematical writing in a first course may only involve a single equation with few mathematical symbols; in the second course, one computes quantities like Mallow's Cp (shown later as equation 3).

1.2   Common Assistive Technologies for Students who are Blind

Any product or piece of equipment that is used to improve access to course materials for students with disabilities is considered assistive technology. Many postsecondary institutions maintain assistive technology for use by students with disabilities. Providing text in an alternative format is a common accommodation request for students, particularly those with print-related disabilities, which include visual impairments and reading based learning disabilities. With text in alternative format, or alt-text, students can access reading materials (e.g., textbooks, journal articles, workbooks) digitally through the use of assistive technology. Examples of assistive technology include text-to-speech and screen reading software, where a student's personal computer will "read" the information aloud using a digitized voice. At Wake Forest University, when students with disabilities request text in alternative format, they have several options: (a) LAC staff can request digital copies directly from the publisher and then further edit the digital files, using optical character recognition (OCR) software, to be used with the student's preferred assistive technology; (b) students can access digital copies of the text from groups like Bookshare (https://www.bookshare.org/) or

Learning Ally (https://www.learningally.org/), which provide files that are ready to be used with assistive technology; or (c) digital text can be created within the LAC by removing the binding of the book, scanning the individual pages using a high speed scanner, and converting the text to be used with the assistive technology.

Although Braille can provide access to written information for students who are visually impaired or blind, it is generally considered inefficient and cost prohibitive due to the volume of Braille pages that are produced. For example, one textbook in Braille might take up several shelves on a bookcase once printed, so alternative accommodations are preferred. Some blind students use assistive technology known as Braille notetakers. These devices allow students to type notes and read in Braille, and they operate similarly to a laptop computer. However, with Braille readers, Braille dots representing ASCII characters are refreshed at the fingertips of the user, instead of appearing on a computer screen. The student at Wake Forest University used the Braillenote$^{\circledR}$ Braille Reader (Humanware n.d.) and the JAWS screen reader (Freedom Scientific n.d.).

2. Statistics 2: Statistical Models

The course MTH 256 Statistical Models is a 3 credit hour semester long course offered at Wake Forest University. The pre-requisite is effectively "any previous first course in statistics" which may include AP statistics, the introductory statistics course offered in the Mathematics department, or a course from another department, so long as the content is primarily in statistics. We assume no previous background with calculus or computer programming. The course currently uses the textbook *Stat2: Building Models for a World of Data* (Cannon et. al. 2013). In spring 2014 the course covered the following topics: simple and multiple regression, along with diagnostics, testing and model selection (most of chapters 0-4); categorical data and interactions (loosely based on chapters 5 and 6, but taught as a special case of regression); and simple and multiple logistic regression (chapters 9-11). Sighted students use R through the R Commander package (Fox 2005). Our student worked with R in a different manner, which is described in detail in section 3.23.2. Coursework included weekly assignments, and contained a blend of applied data analysis, report writing, and theoretical questions. The course also had three in-class, closed-book exams. For the final project, each student found a suitable dataset, fully analyzed it, and wrote up the findings in a full statistical report. Most reports ranged in the 10–15 page range, including figures. Students could work alone or with a partner on the final project.

In the remainder of this section we demonstrate the visual, computational, and mathematical techniques students need in the second statistics course. The reader is encouraged to consider the challenges that a blind student might face.

2.1 Example: High Peaks Data Set

An example of a typical dataset for this course is the `HighPeaks` data, contained in the `Stat2Data` R package (Lock 2012). These data contain measurements on the 46 "High Peaks" mountains in the Adirondack Park of northern New York State. The High Peaks were originally believed to be all mountains taller than 4000 feet above sea level in the Adirondack Park, though later measurements would confirm that four are slightly below 4000 feet, and one other mountain was missed. Measurements include Elevation (feet), Difficulty of hike (on a scale of 1–7), Ascent (vertical gain in feet from the trailhead to the summit), Length (round trip distance in miles) and Time (average number of hours to complete the round trip hike). We emphasize data visualization prior to model fitting. It is obvious to the students that a longer hike or a higher vertical ascent would make a mountain more difficult. One natural question to ask is if the difficulty is *only* some combination of Length and Ascent, or if it considers other factors. Students might suggest steepness, the presence or absence of a marked trail, and trail conditions as other possible factors. A favorite teaching method of the instructor was to encourage speculation on other possible variables or explanations for observed effects, then ask if it is possible to use the data on hand to answer firmly on the issue. In this case, it is possible. By plotting Length vs. Ascent and using different plotting symbols for Difficulty, a student can identify pairs of mountains in which the hike with both a higher Ascent and longer Distance is classified as easier, disproving the claim that difficulty relates only to Length and Ascent. This is shown in Figure 1, and serves as a nice example of the power of proper displays of data without any model fitting required.

After continuing with additional exploratory data analysis and producing histograms, box-plots, tables, and other visual aids, students next explore what makes a hike take longer. Students learn to fit a simple linear model,

$$\text{Time} = \beta_0 + \beta_1 \cdot \text{Length} + \varepsilon, \quad \varepsilon \sim N(0, \sigma), \tag{1}$$

where β_0 is the intercept, β_1 is the slope, and the errors ε are assumed to be independent and follow a zero-mean normal distribution with some constant standard deviation σ. The fitted model is represented as

$$\widehat{\text{Time}} = \widehat{\beta_0} + \widehat{\beta_1} \cdot \text{Length} = 2.04817 + 0.68427 \cdot \text{Length}. \tag{2}$$
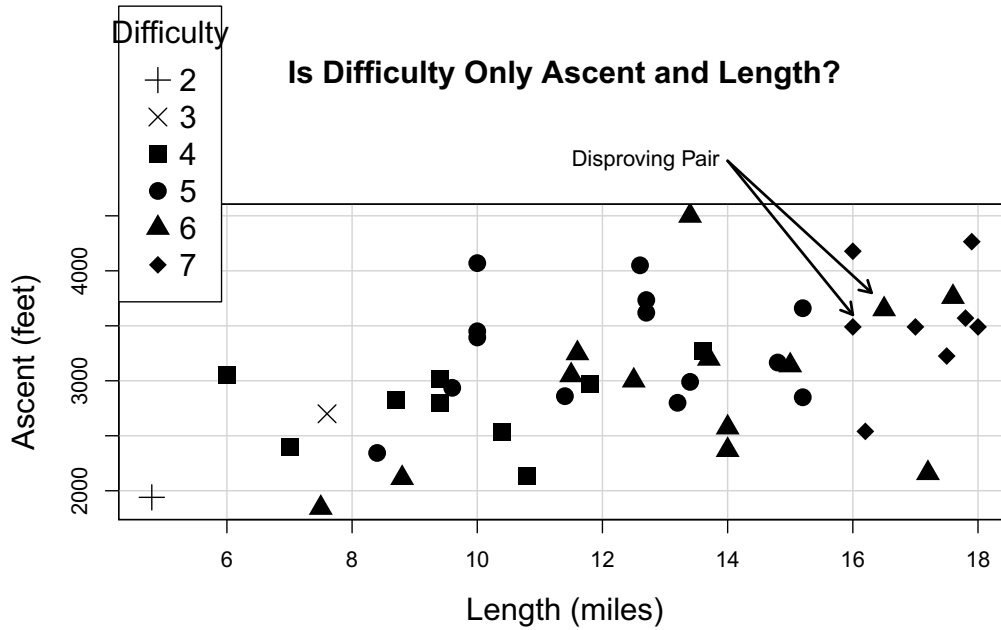
Figure 1.   This scatterplot can be used to disprove the claim that the variable Difficulty is only some combination of the ascent and length of hike. The pair of mountains highlighted shows a case where a mountain with both a larger ascent and longer hike is classified as a less difficult hike.

The R code needed to fit this model and its associated output are both shown below. Students learn to interpret each piece of this output, and scan for the elements they need for specific tests.

```
> library(Stat2Data)
> data(HighPeaks)
> attach(HighPeaks)
> lm1=lm(Time~Length)
> summary(lm1)

Call:
lm(formula = Time ~ Length)

Residuals:
    Min      1Q  Median      3Q     Max
-2.4491 -0.6687 -0.0122  0.5590  4.0034
```
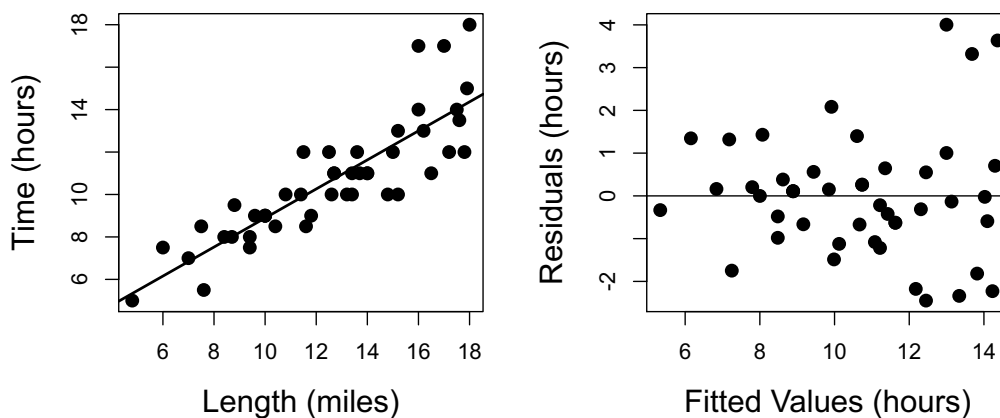
Figure 2. Scatterplot and residual plot for Time (hours) vs. Length (miles). Students will use these figures to assess assumptions of linearity and constant variance.

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.04817    0.80371   2.548   0.0144 *
Length       0.68427    0.06162  11.105 2.39e-14 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.449 on 44 degrees of freedom
Multiple R-squared: 0.737,      Adjusted R-squared: 0.7311
F-statistic: 123.3 on 1 and 44 DF,  p-value: 2.39e-14
```

Figure 2 shows a scatterplot of Time vs. Length, along with a best-fit line and residual plot. Important lessons from this figure include being able to identify signs of nonlinearity, non-constant variance, and possible outliers or influential points. These figures might remain on the projector screen in class for several minutes as students learn to look for important features. Figure 3 shows a QQ plot and histogram of the residuals, both of which are used to assess the assumption of normality of errors. The QQ plot here shows some evidence of non-normality in the tails.

As more complicated models are fit with differing numbers of explanatory variables, stu-
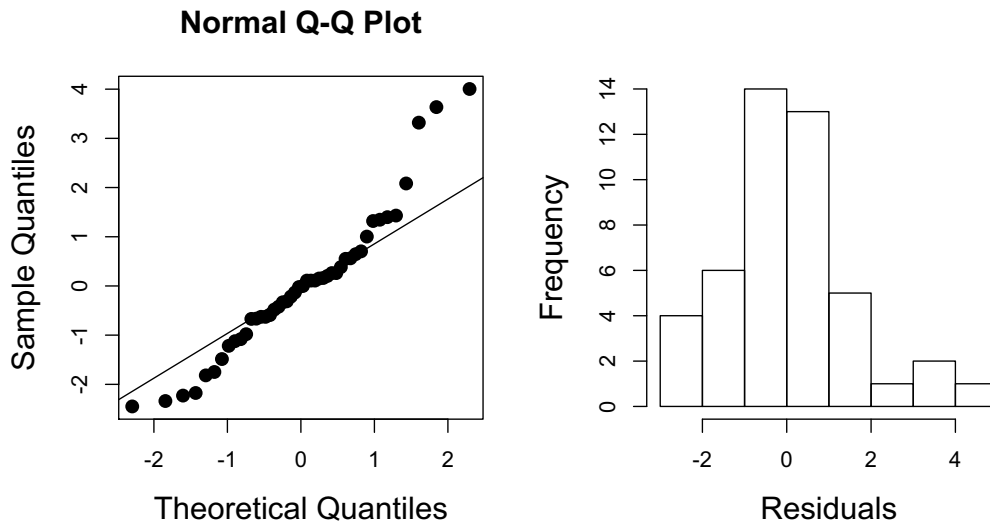
Normal Q-Q Plot

Figure 3.   Quantile comparison and histogram of residuals from Equation 2.

dents turn to model selection criteria such as Mallow's Cp, computed as

$$C_p = \frac{\text{SSE}_m}{\text{SSE}_k/(n-k-1)} + 2(m+1) - n, \tag{3}$$

where *m* is the number of predictors in the model under consideration, *k* is the number of possible predictors in the full model, SSE_m and SSE_k are the sum of squared errors from a model with only *m* and *k* predictors, respectively, and *n* is the number of data points.

This example demonstrates what we define as three sets of materials essential to the course: mathematical writing, computer programming, and visual displays. Equations (1)–(3) show just a bit of the mathematical complexity required; Figures (1)–(3) show examples of images from which we extract information about data or models; and the R commands and output give a flavor for the computer programming needed. For each of these, students must master both interpretation and production. We found it useful to organize these six skills into a table, shown in Table 1. Assisting the student in the course necessarily meant enabling her to master each of these six skills. The next section will discuss the specific technologies and approaches used to help her with each skill.

Table 1. The six required skills a student must master in Statistical Models, labeled *A* through *F*.

	Interpreting	Producing
Mathematical Writing	A	B
Computer Code	C	D
Images and Graphical Displays	E	F

3. Assistive Technologies Employed

3.1 Reading and Writing Mathematics

Grade 2 English Braille involves six dots, placed in two columns of three, which are either raised or flattened. Ignoring the blank space, there are $2^6 - 1 = 63$ unique characters which can be produced with only six dots. With this limitation, a number of conventions have been developed to give this 6-bit system the greatest reach. For example: the numbers 1 through 9 are identical to the letters a through i, and 0 is also j; a special single character dictates whether or not a number or letter is intended; a single character indicates whether a letter is capital or lower case; ligatures such as *th*, *sh*, and *ch* are represented as single characters; and an elaborate system of contractions and short-hand has been developed, with *afn* representing "afternoon", for example. The ASCII symbols 6-bit Braille can replicate are known as Braille ASCII (http://www.dotlessbraille.org/displays.htm).

It should be immediately obvious that it is a struggle to use 6-bit Braille to write higher mathematics. Merely think of all of the symbols from differential calculus, linear algebra, logic, and so forth, and imagine trying to write all of these with a mere 63 characters. Thus, alternative forms of Braille have been developed for special purposes. In mathematics, Nemeth Braille (Nemeth 1972) was first developed in 1952 by Abraham Nemeth to better represent mathematical equations and writing. Physicist John Gardner and Mathematician Norberto Salinas developed Gardner-Salinas Braille (Jackson 2002), an 8-bit Braille which extended Grade 2 English Braille to better capture symbols commonly used in mathematics, such as operators, Greek letters, and various punctuation marks.

An alternative is to use LaTeX for writing mathematical and statistical material. LaTeX is a markup language used in the statistics community for typesetting statistical writing. All mathematical writing can be reduced to ASCII characters used to represent various mathematical symbols. For those less familiar with LaTeX, this manuscript was written in LaTeX. Equations (1)–(3) in this paper were typed as:

```
Time = \beta_0 + \beta_1 \cdot Length + \epsilon,
\hspace{4mm} \epsilon \sim N(0, \sigma)
```

```
\widehat{Time} = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot Length
= 2.04817 + 0.68427 \cdot Length


C_p = \frac{SSE_m}{SSE_k/(n-k-1)} + 2(m+1) - n
```

The crucial value of LaTeX for the blind student is that it can represent all *information* of a mathematical expression purely in terms of ASCII characters, which can themselves be easily read by either screen readers or Braille displays. Since the student would very likely continue to take mathematics courses, we decided the time investment to learn LaTeX was worthwhile. Most mathematics faculty at Wake Forest University are already fluent in LaTeX, and can provide materials relatively easily. Due to the mathematical symbols and equations required, many textbooks in mathematics and statistics are often first written in LaTeX, and we suspected it might be easier to get accessible materials from publishers. Finally, since the student could type on a regular keyboard, typing LaTeX source code would be a viable means of her writing her own mathematics and statistics. LaTeX is simply a convention for typing mathematical symbols using only ASCII characters, it is not a software program itself. Various proprietary compilers and editors have been developed, but these are not expensive. For example, the widely used WinEdt (http://www.winedt.com/) has a single educational license for $60, and 5 educational licenses can be purchased for only $200.

The student found it easiest to open LaTeX files both on her laptop and also in her Braillenote refreshable display. She would listen to the code being read by JAWS on the laptop, while feeling the Braille version of these equations on her Braillenote. It wasn't until after the course ended that the student discovered the possibility of syncing her screen reader JAWS and the Braillenote so that she could simultaneously hear and feel the LaTeX code. She described that screen readers are designed to read either word-by-word, or character-by-character. A document written in LaTeX is best read by combining these two approaches. For example, paragraphs of text are read word by word; equations are read as a combination of words and characters. Syncing (through bluetooth) the screen reader and Braillenote would help a student learn LaTeX.

When it came time for her to type mathematics for assignments or exams, the student would simply type the code into a .txt file on her laptop. As it was not possible for the student to compile the code to see if it was all correct, very minor coding errors such as missing dollar signs, unclosed braces, missing slashes, or an incorrect use of the % symbol would need to be quickly corrected before her work could be graded. Over time, the faculty member stopped spending time to correct minor LaTeX mistakes and simply graded the LaTeX source code without compiling. This focus on the LaTeX source code (and not the compiled text)

is an important consideration when teaching blind students. Consider the formatting of the "four steps" of modeling, as written in the notation common in the Cannon et. al. (2013) book:

1. **Choose** the model $Y = \beta_0 + \beta_1 \cdot X + \varepsilon, \quad \varepsilon \sim N(0, \sigma)$.

2. **Fit** the model to obtain $\widehat{Y} = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot X$.

3. **Assess** the model.

4. **Use** the model to predict at $X = 100$.

The precise code used to write the above steps is:

```
\begin{enumerate}
\item \textbf{Choose} the model $Y = \beta_0 + \beta_1 \cdot X + \epsilon,
    \hspace{4mm} \epsilon \sim N(0, \sigma).$
\item \textbf{Fit} the model to obtain $\widehat{Y} = \widehat{\beta}_0
    + \widehat{\beta}_1 \cdot X$.
\item \textbf{Assess} the model.
\item \textbf{Use} the model to predict at $X = 100$.
\end{enumerate}
```

The compiled, formatted writing looks quite elegant. The `enumerate` environment kept track of counting for us and we never needed to write out step numbers, the `widehat` looks a bit prettier than the smaller `hat`, the `hspace` added a bit of blank space in line (1) between the ε and its distribution, and so forth. However, the blind student reading the source code directly would find some of this unnecessarily distracting. A more efficient means of conveying the same mathematical information in LaTeX would be

```
1. Choose the model $Y = \beta_0 + \beta_1 \cdot X + \epsilon,
    \epsilon \sim N(0, \sigma).$
2. Fit the model to obtain $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 \cdot X$.
3. Assess the model.
4. Use the model to predict at $X = 100$.
```

In particular, the changes are: the `enumerate` environment has been scrapped in favor of writing "1," "2," . . . instead; all instances of `widehat` have been replaced by the simpler `hat`; the command `hspace` was removed; bold was removed, and the double slash used to indicate a line break was also removed. If this simpler code is compiled, the result looks

quite a bit worse to a sighted user, as shown below:

1. Choose the model $Y = \beta_0 + \beta_1 \cdot X + \varepsilon, \varepsilon \sim N(0, \sigma)$. 2. Fit the model to obtain $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 \cdot X$. 3. Assess the model. 4. Use the model to predict at $X = 100$.

However, the scaled back source code shown above is clearly preferable for someone reading the code directly. Thus, items A and B on Table 1 were handled through the use of LaTeX, and a quick summary of these accommodations is shown in Table 2. Next, we move on to items C and D.

Table 2. A summary of the major accommodations made for mathematical reading and writing, and remaining challenges

Challenge	Accommodation and Assistive Technology	Remaining Challenges
Obtaining an accessible textbook	We obtained a high quality, searchable pdf file from the publisher, and a graduate student copied this text into a text editor and re-typed equations in LaTeX.	Some organization of multiple files was required. Images were stored separately.
Course notes	The instructor would re-write some notes in "minimal" LaTeX and send to the student in advance.	Additional merging of files was required if the student wished to add notes during class time.
In-class notes	The instructor practiced a clear, verbal description of all equations and images displayed.	A more timid student who is blind might not wish to repeatedly ask for clarification.
Writing mathematics	The student typed all work in LaTeX on a regular keyboard.	Submitted LaTeX files often did not compile (minor errors, missing $ symbols, etc.) so the instructor either fixed and compiled, or graded the LaTeX source directly.

3.2 Computer Programming

In "Statistical Software from a Blind Person's Perspective," Godfrey (2013) discusses in great detail how blind users interact with different statistical software programs. The paper's subtitle says "R is the best, but we can make it better," a statement we agree with in full. R can deal exclusively in text for data input, analysis, and output; it requires no graphical interface; help pages are written as text files; and functions and commands can be modified to suit the needs of particular users. In these respects, R offers the blind user many advantages. However, two essential modifications for the blind user are described

by Godfrey in clear detail. These are 1. how to run R in a terminal window, and 2. how to utilize the package `BrailleR` (Godfrey 2012) to get full compatibility with the screen reader.

We developed this approach using R version 2.15.1. Most students using `R` early in their statistics careers will run the program inside a window, with several nested windows inside. One is the R console window which shows directly typed commands and R output. Another shows the script, and additional windows show images. These nested windows interfere with the ordinary use of the screen reader, and thus navigating was not possible for the blind user. To our knowledge, there is no easy way to toggle between these nested windows in a manner that allows the screen reader to work properly. Following Godfrey (2012), we instead set up R to run in the terminal window. By running R in the terminal window, it ensures that the screen reader will stay focused on the window with both input and output. By spending a bit of time creating a desktop shortcut and modifying the PATH environment variable, it is possible for the student who is blind to open a working R session with minimal effort, and be totally independent with computer input and output. The specific steps to achieve this for the Windows 7 operating system are as follows:

1. On the desktop, create a new shortcut to the Command Line terminal `cmd.exe`. This is the link the blind user will open and simply type the letter "R" and hit enter to open up a session. However, this requires that one modify the PATH environment variable.

2. Modify the Path variable:

   (a) Go to *Control Panel*, *System*, *Advanced System Settings*, *Environment Variables*

   (b) Find Path under System Variables, and click Edit

   (c) Add the directory containing R.exe to the Path variable. On the author's computer with Windows 7, the directory was

   ```
   C:\Program Files\R\R-2.15.1\bin\i386;
   ```

3. Open cmd.exe on the desktop, and simply type R and press enter. R should open.

The end result of this is that one can find the cmd.exe icon on the desktop, open it, and simply type the letter R to run R in the terminal window. Packages can be installed in the terminal by typing `install.packages("BrailleR")`, for example. After waiting a few seconds, the blind user can simply press the ENTER key which will select the default "0-Cloud" as the mirror for the installation; only in rare cases would a package be

unavailable from this mirror. Unless there is an error of some sort, the rest will happen automatically. Once a package has been installed correctly, on subsequent sessions the user can simply type `library(BrailleR)` to load it. The working directory is identified by typing `getwd()`.

The second essential modification is to create a file which captures all user keystrokes along with R output which can be easily read with a screen reader. Here we will describe what happens in a trivial example of using R, in which the user only wants to add 1 and 2:

```
> 1+2
[1] 3
```

Once the user has typed "1+2" the screen reader recited this entry once, and then recites the output line "[1] 3" exactly once. The screen reader will read the bracketed [1], which is unfortunate but not too bothersome after a while. However, suppose that the user needed to hear things again... there is no way to get the screen reader to repeat what has been run. Pressing the up arrow will bring back previously typed commands which are read by the screen reader, but there is no simple way to have the reader re-read the R output in response to those commands. Hence, it is useful to have a file which captures all input commands and output results into a log, and the user can repeatedly refer to this log. This file can be created by typing

```
library(BrailleR)
txtStart(file="output.txt").
```

This creates a file called "output.txt' inside the working directory, and this file will capture all input and output commands. This file is best opened in a web browser such as Internet Explorer (we used version 9.0). By pressing Alt-Tab, the user can toggle between the terminal running `R` and the file `output.txt`. When in Internet Explorer, the user can press F5 and the browser will refresh the file, showing all typed commands and R output in response. This file works quite well with screen readers, allowing the student to listen line by line or character by character. This file also serves as a useful learning tool for programming. Indeed, sighted students in the class expressed an interest in this file as it serves as a log of all successful (and unsuccessful!) commands typed, archiving all output.

The student commented that the `BraillerR` package was one of the most important assistive technologies in the course. The `VI()` command (described in the next section) allowed her to interpret all histograms and assess normality independently; the alternative would

Table 3. A summary of the major accommodations made for computer programming and interpretation, and remaining challenges

Challenge	Accommodation and Assistive Technology	Remaining Challenges
Getting R to work with the screen reader	We set up R to run in the terminal window by modifying the `path` variable.	This change would need to be made to every computer the student uses.
Losing the cursor in the terminal window in Windows 7	Press the `Alt` key.	None.
Recording submitted code and output	The student used the `BrailleR` package and the `txtStart()` command to make a log. This file was opened in an internet browser which allowed for refreshing the file.	The log file records everything, so it can become quite long with accumulated errors.
Interpreting R output	The student used the `VI()` command when applicable, but otherwise simply practiced listening for relevant output.	Tabulated output (from the `summary()` of a linear model, for example) must be listened to row by row instead of column by column.

have been to print all histograms as tactile images, which is more time consuming and requires an assistant. The `txtStart()` command essentially made learning R programming accessible. Most students refine programming skills through trial and error, and the log file saving all commands and output allowed her to do this as well.

Two final comments are needed. First, there is a bizarre tendency for the cursor to get "lost" when toggling between the terminal window and Internet Explorer. This is a well-documented issue in which the keyboard becomes unresponsive when toggling back to the terminal window from a different window. Fortunately, quite by accident Godfrey and Erhardt (2014) discovered a solution; pressing the `Alt` key after toggling back to the terminal window remedies the lost cursor. Second, we did not consider alternative statistical software programs and so cannot comment on how well they would have performed for our needs. Our university teaches exclusively in R, and given the available packages to make R more accessible to blind students, we simply chose this option.

Table 3 shows a summary of these accommodations for computer programming.

3.3 Producing and Interpreting Images

The question we asked ourselves for each image in class was this: How does a visual representation function for students who are sighted, and what would serve as a comparable function for a student who is blind? Sometimes, an image is not at all needed; it is only a convenient way for sighted people to store and present the information. Within this spirit, the `BrailleR` package (Godfrey 2012) contains a useful command `VI()` which can wrap other commands to make the R output more accessible to the blind user. Consider the histogram shown on the right panel of Figure 3. If the R command `hist` is placed inside the wrapper function `VI()`, the output is:

```
>VI(hist(lm1$resid))
This is a histogram, with lm1$resid  marked on the x-axis, unless you explicitly
used the xlab argument.
There are a total of 46 elements for this variable.
It has 8 bins with equal widths, starting at -3 and ending at 5.
The mids and counts for the bins are...
mid = -2.5  count = 4
mid = -1.5  count = 6
mid = -0.5  count = 14
mid = 0.5  count = 13
mid = 1.5  count = 5
mid = 2.5  count = 1
mid = 3.5  count = 2
mid = 4.5  count = 1
```

Thus, the command has extracted the needed information from the histogram, and described it in a format which is accessible to the blind user. The student made extensive use of this command when checking for normality of variables or residuals. A histogram may not be as valuable for detecting non-normality as a quantile quantile plot such as the one shown in Figure 3, but it is often enough. Information lost in the translation from a QQ plot to a histogram include things like: the values of the theoretical quantiles at which the departure from normality begins (seen as where the points depart from the line on a QQ plot); a sense of whether a few points are merely outliers, or truly indicative of a non-normal tail; and so forth. Given that histograms could be made fully accessible, whereas something like a QQ plot poses much more of a challenge, we felt it reasonable to allow the student to solely use histograms to assess normality of errors for homework and projects. If the `VI()` command is used with a function for which it has no special modification, it will simply "print" the object, as shown here:

```
> VI(boxplot(Elevation))
```

```
No VI method has yet been written for this type of object
so it has been printed for you in its entirety.
$stats
        [,1]
[1,] 3820.0
[2,] 4100.0
[3,] 4380.5
[4,] 4627.0
[5,] 5344.0
attr(,"class")
...
```

This is still quite useful, once the user has learned the structure of these R objects. For the boxplot, the five numbers are always the lower whisker, first quartile, median, third quartile, and upper whisker.

For images which cannot be easily converted or described in text, we purchased a Desktop Braille Embosser from Viewplus (http://www.viewplus.com/products/braille-printers/desktop-braille-printers/). An embosser prints dots in high durability paper. It can be used to print straight text which is simply Braille, or images which are known as *tactile images* in the blind community. At the time of purchase the embosser cost $4,995, and was by far the greatest expense associated with accommodating the student. Our justification was that the student was a first year student at Wake Forest University, interested in majoring in a quantitative field, and likely to take numerous additional courses with images, graphs, and other visual information.

Figure 4 shows an example of a tactile image (it is the same scatterplot as in Figure 2). Darker dots are taller, and lighter dots are shorter, so the student can feel around 5 different "shades." All images in our course were first created in R. The only modifications at this stage are to ensure that any plotting symbols or lines are heavy enough to print well. Plotting symbols should be solid (pch=15 produces solid squares, which work well with the Braille Embosser) and large (cex=1.4 works well). Lines should have a line width of lwd=2 at least. Colors, different plotting symbols, different line types (dashed, dotted, solid) and figure legends did not print well on the Embosser and we made no use of them. Returning to Figure 1, it is not possible to convert this to a tactile image on a regularly sized sheet of paper. This message would need to be conveyed in some other fashion.

Once an image has been produced, it is copied as a bitmap. The Tiger Software Suite (http://www.viewplus.com/products/software/braille-translator/) that comes with the Braille Em-
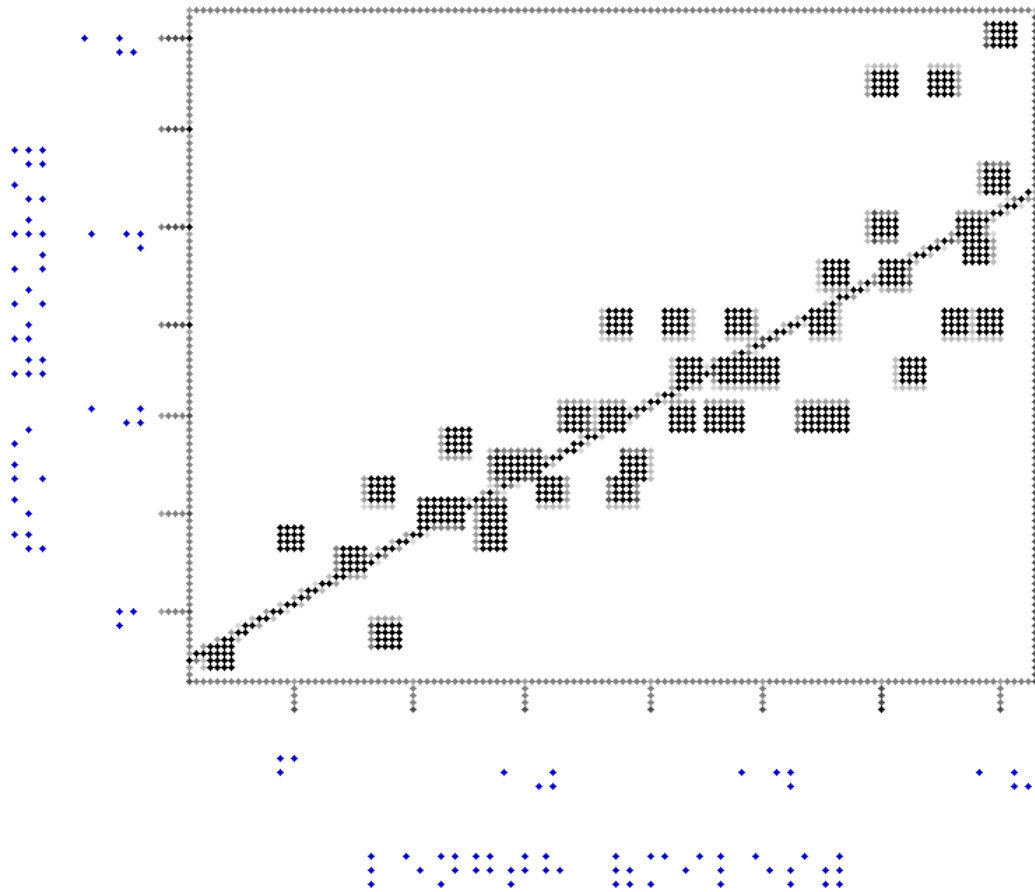
Figure 4. Scatterplot of Time vs. Length of the 46 High Peaks when pasted into Tiger Designer. Plotting symbols should be dark, large, but overlapping to a minimal degree. The line must be wide enough to be felt. It is also important that the entire plotting area be boxed, so the student can understand the range of possible points. Braille labels take up much more space than text (think of writing in size 19 font), so space is at a premium. Only every other tick mark on the axes was labeled, and the axis labels appear much larger.

bosser had a program called Tiger Designer, which facilitated the conversion of an image. The simplest way we found to use it was to paste the bitmap from R, adjust the contrast as needed, and retype over all axis labels using Braille. The software allows one to type in English, but Braille is displayed. These were then printed on Braille paper, three hole punched, and stored in a three ring binder. Over the course of the semester, the number of images grew quite rapidly so organization became more of a challenge. We found it useful to write information explaining each image to a sighted person (titles, page numbers, etc.) on the back of each, to remedy the inevitable—when the binder rings fail to close, and all pages fall to the floor.

We should also mention another more affordable solution known as a Picture In a Flash, or PIAF (Humanware n.d.). This prints an image using special ink on paper, and the sheet is then heated which causes the ink to blister, and physically deform the paper. We originally thought a PIAF would work best as it can print a straight line, but found that the Braille embosser offers greater flexibility, and longer lasting images.

What has been described above only deals with item E on Table 1. It may come as a surprise to learn that Braille embossers are not really designed to be operated independently by the blind user. So, images in lectures or notes could be prepared by the faculty member in advance and provided to the student, but the greatest challenge with images were those situations where the student needed to produce an image. Think of the process of choosing an appropriate model for data. Sighted students might make several dozen images of scatterplots, residual plots, and so forth when deciding how to choose a best model for a given dataset. Each image might be viewed for only a few seconds. The student could not use the Braille embosser independently, and thus could not produce these images herself. For homework assignments and projects where producing and interpreting intermediate images was essential to the task at hand, the student either worked directly with the faculty member or with a graduate student trained in making tactile images.

Table 4 shows a summary of these accommodations for images.

Table 4.   A summary of the major accommodations made for images, and remaining challenges

Challenge	Accommodation and Assistive Technology	Remaining Challenges
Histograms (for residuals)	The `VI()` command in the `BrailleR` package converts these to a more accessible format.	There are minor issues (bin sizes) that are overcome with practice.
Images which can be easily described	The instructor developed a convention for describing those images used in class for 30 seconds or less.	One-on-one meetings with the student were needed to confirm that this convention was effective.
Images which cannot be easily described	We printed tactile images on a Braille embosser, labeled them in both Braille and English, and stored them in binders.	The number of images posed some storage and access issues as the semester progressed. Also, essential images for class needed to be prepared in advance.

4. Other Changes Required

The textbook for the course was not available in any accessible format. We are deeply grateful to W. H. Freeman for providing us with a very high quality, searchable PDF version of the entire Stat2 textbook, though obtaining this was not a simple matter. When we contacted W. H. Freeman asking for a digital version of the text, we were initially told they could only provide a free hardcopy of the textbook or free access to an e-copy. We pressed, insisting that neither were accessible to the student. W. H. Freeman eventually agreed to send a high quality, searchable PDF version of the text, but their reluctance is quite understandable. One could imagine such a file being posted online, which would greatly impact book sales. However, trust must be maintained between all those involved in education for the blind.

The searchable function of this file was essential. What we mean by searchable is that the PDF is not merely an image of text, but the text itself can be highlighted and copied. This was necessary to convert the PDF into an accessible format. In principle, the program In-ftyReader (http://www.inftyreader.org/) can be used to convert PDF back into LaTeX source code, and InftyReader has been successfully utilized to enhance accessibility in mathematics for the blind (Yamaguchi et. al. 2008). However, in our particular course we found that the formatting of the textbook PDF file prevented InftyReader from working well. We therefore handled the conversion ourselves. For all text, we were able to highlight and copy the text directly, and paste it into an ordinary text file. Equations were re-typed in LaTeX source code. Tables were re-typed using the `tabular` environment in LaTeX. In place of images, we would write "(Image 1.1 goes here: Scatterplot of Time vs. Length for the 46 Adirondack High Peaks)" into the LaTeX file, then re-make the image ourselves using the Braille embosser. A graduate student in Mathematics at Wake Forest University worked on this chapter-by-chapter and would send the completed files and tactile images throughout the semester. The faculty member wrote out all classroom notes in LaTeX in advance and e-mailed them to the student. The student was able to take modest notes during class using her Braillenote, though she primarily just listened during class.

During class, all notes written on the white board were simply dictated by the faculty member when written. For example, when writing the equation for Mallow's Cp (shown earlier as Equation (3)), the faculty member would say "Cp equals SSEm divided by the quantity SSEk over n-minus-k-minus-1, and then to this entire quantity is added 2-times-m-plus-one, minus n." On rare occasions, the student would ask for a brief clarification ("is SSEk over n-minus-k-minus-1 all in the denominator?"), but this happened very rarely, possibly because all class notes were provided in advance so the student had it written in LaTeX correctly already. Sighted students did not seem bothered by this dictation, and indeed a few seemed to prefer it since it helped remove confusion with symbols, notation,

and faculty penmanship.

Images that were essential to a lecture—involving, say, at least 30 seconds of class time—were all printed by the faculty member in advance using the Braille embosser. When the relevant image was displayed on the projector, the faculty member would simply direct the student to the correct tactile image being discussed. We found that many images in Stat2 are displayed and discussed for very little time. For these, we would simply describe them, or ask sighted students to describe them. This turned out to have unexpected educational benefits for the sighted students as well. For example, the residual plot shown on the right in Figure 2 might be displayed, and the professor would ask "are the assumptions of simple linear regression met here?" One sighted student might say "No, the variance is not constant" to be quickly rebutted by another sighted student saying that "yes, it's close enough." The faculty member would ask each to elaborate. This begins by stating what type of plot is being shown, and what the two axes represent. The first student might describe a general funnel shape extending to the right, indicating the residuals were larger for larger fitted values; the second might simply point out that it's only perhaps 3 data points in the upper right corner giving this illusion, and we shouldn't over-interpret outliers. In any event, these verbal descriptions were originally used to benefit the student who is blind, but in fact they helped sighted students see different features of these images. They also forced all students to spend a moment describing an image before they jumped into drawing conclusions about it—surely this is a habit we could all stand to benefit from.

The faculty member learned to avoid the use of ambiguous pronouns such as *this*, *these*, *it*, and so forth, and instead refer to all objects by their proper names. At first, the faculty member also deliberately put his hands in his pockets to avoid the temptation to gesture or point during class discussions. Hand motions and gesturing are of course a positive teaching technique for sighted students, but the danger is that they can be used to substitute for verbal descriptions, and therefore the blind student could not follow the lecture. Over the course of the semester, these negative habits abated.

A final piece of the accommodations was a standing, weekly one-on-one meeting between the faculty member and the student. These meetings allowed both to clear up any errors, misunderstandings, or issues that arose in the previous week, and set clear goals for the upcoming week. Sometimes these meetings would last only 30 minutes, and deal with a few homework questions. Other times these meetings used substantial time for working with R, teaching LaTeXcommands for new material, revisiting class notes, or just generally providing support and advice to the student in all her academic pursuits. An honest and open relationship—in which the student feels free to express any and all frustrations or questions about the course—is absolutely essential to success in the classroom.

5. Discussion

Despite postponing her first statistics course until the spring semester, allowing the university more time to determine how to best provide accessible math and statistics coursework, the staff at Wake Forest University were still overwhelmed with the number of potential and novel solutions. It had been much easier to provide accessible material to courses in the Humanities and even some of the Sciences, where the coursework was more text-rich or the text in alternative format already existed. Our efforts at researching best practices provided some helpful starting points, but we were never certain that our proposed approach using varied assistive technologies would be appropriate.

Providing the technology and the alternative course material for this statistics course required the efforts of several faculty, staff, and students. In addition to the time and energy both the faculty member and the LAC staff member spent researching and proposing alternative methods for presenting the course information, many hours were spent by the mathematics graduate student: (a) retyping all of the equations into LaTeX; (b) printing all needed images using the embosser; and (c) collating all of the classroom examples to provide a course packet of examples for the student. Furthermore, the graduate student and faculty member were required to become familiar with Braille and with the assistive technology essential to providing the alternative text and using the Braille embosser. It would have been valuable to have a full-time staff member at the university who is dedicated to making course material accessible and who has the knowledge and skills to provide ongoing support for assistive technology. However, we did not have such a staff member, and were extremely grateful to our graduate student for filling this need.

Even though the student will likely benefit in her academic future and her anticipated vocation from having learned LaTeX, she faced a steep learning curve and had to become fluent in a short period of time. Her incoming knowledge of assistive technology was extremely helpful, and it is important to note that knowledge and experience will vary among students, depending on the degree to which they used assistive technology in their primary and secondary education. Here we were fortunate that the student was well versed in screen readers, Braille notetakers, and the use of a regular keyboard for typing. This allowed us to focus heavily on learning LaTeX and R right away. The student recommends that others who are blind and wish to study statistics should learn LaTeX as soon as possible, preferably even before the second statistics course. This would allow them to focus more on learning the material.

Due to the nature of how we provided access to this course, the student did not actually register for the same course number as the other students enrolled in the course. Instead, the faculty member, in consultation with the department, chose to follow an independent study

model. In addition to attending the class meetings and completing the class requirements for the original course, the student and faculty member supplemented instruction through individual meetings. At the end of the semester, she met (if not exceeded) the same learning objectives as the other students enrolled in the actual course, and she is prepared to advance through her major at Wake Forest University. However, the amount of time the faculty member spent researching and meeting individually with the student is more accurately reflected as an independent study.

The faculty member who taught this course is currently the only one in the department with the knowledge and experience to provide an accessible experience for courses that rely heavily on the visual representation of data. Ideally, his knowledge and experience can be easily transferable to other mathematics, statistics, or business courses, because this student intends to complete a major in one of these areas. We have found that the upfront and ongoing costs of providing these accommodations are likely to be highly variable from university to university. Much of the cost estimates would depend on the particular student, background of the instructor, depth and resources of the Learning Assistance Center at the university, background of student assistants, and so forth.

Providing an accessible course in this fashion highlighted the importance of collaboration between faculty and student services on college campuses. Faculty members should be encouraged to consult and problem solve with other campus resources, particularly the Disability Support Services offices. The responsibility of providing accessible material is shared among the faculty, student, and staff members at the institution.

It has already been noted that teaching a statistics course to a student who is blind can enhance not only the instructors pedagogical approach, but the student-instructor relationship, too. The individual attention required at this level of instruction provided helpful guidance and feedback for both the instructor and the student. It seems unlikely that a traditional classroom environment, even with a high level of participation and engagement, could achieve the same level of understanding and learning. We recommend to mathematics and statistics faculty that they find ways to work with their administrators to protect the weekly one-on-one hours which are surely necessary.

We should mention that in situations where the student is completing graded work with an assistant, there is a danger that the "assistance" will grow beyond what it should be (Marson et. al. (2013) call this *countertransference*). This can be difficult to avoid, but must be avoided if the student is to have a fair and equitable learning environment. In addition, research has shown that students feel resentment towards teachers who give too much assistance (Landau et. al. 2003). For graded exams, the Learning Assistance Center administered all graded exams for the student, overseen by a trained specialist rather

than the faculty member who would grade the exam. The student was asked to sign an exam contract with the Learning Assistance Center. This contract reminds the student of the Honor Code and the list of impermissible resources. Homework problems posed less of a concern, because all students in the class are permitted to discuss problems with one another so long as each student fully writes and hands in his or her own solutions. Many of the conversations between the blind student and her assistant would mirror those among sighted students, and thus posed no real problem. As for interactions with the faculty member regarding grading (discussing how partial credit was assigned, for example) the faculty member simply had to listen carefully and use professional judgement.

Another point that warrants discussion is the relationship between what we did in this second course and what experience the student had in the first course. Returning to the table of six challenges (shown in Table 1), we consider them for a hypothetical student who is blind. For items A and C, we assume no previous background in either case. We assume only that the student has a means of reading LATEX and computer programming output (a screen reader, refreshable Braille display, etc). However, prior exposure to LATEX in the intro course would have been valuable. Item A further requires some basic familiarity with algebra and commonly used equations in statistics (mean, standard deviation, standardizing a random variable, manipulating square root, square, logarithm, etc.). For items B and D, we do assume that the student is able to use a regular keyboard for typing LATEX and computer code. However, there is nothing from the introductory course that would be needed in this regard. For item E, we assume that in the student's first course they have learned the basics of a scatterplot, fitting a straight line to a scatterplot, histograms, the normal distribution, etc. It is extremely difficult for a student who is blind to master item F unassisted, so we make no assumptions on the background obtained in the introductory course.

The student at Wake Forest University had done well in the introductory AP statistics course, and seemed to have roughly the same background skills and knowledge as any other sighted student taking a second statistics course. She felt that her preparation was excellent coming into the second course. Since she was planning to major in a quantitative business field, the investment of time in LATEX was worthwhile. We suspect that there may be situations in which a student who isn't planning to take a large number of math and statistics courses may hesitate to make such a time investment learning LATEX; however we still recommend its use. The student also remarked that the greatest remaining challenge was time. Reading LATEX source code character by character, getting R code to work properly with the `txtStart()`, and working with an assistant on homework and projects all take time. The student also would sometimes first write her answers in Braille (using a shorthand convention that she understood), and then translate this into LaTeX for submission. This additional step adds time. Instructors must be aware of the inherent additional

time demands placed on the student who is blind.

We firmly hope that this description of our experience will help promote the discipline of statistics to the visually impaired and blind, and that other faculty will find a framework to build upon. There is no doubt that additional technologies and approaches can be incorporated to enhance the learning experience. Indeed, one of the greatest sources of satisfaction for us at Wake Forest University is that the student has registered for the next courses in sequence, and is proceeding with her chosen major. We would gladly discuss our work in further detail to any interested person, and hope to continue to be of service to the community of blind and visually impaired students, and their universities.

6. Acknowledgments

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