The Price is Right Again

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Abstract

The Price Is Right (*TPIR*) provides a wealth of material for studying statistics at various levels of mathematical sophistication. The authors have used elements of this show to motivate students from undergraduate probability and statistics courses to graduate level executive management courses. The material consistently generates a high degree of student engagement and lively discussion. This paper describes one classroom activity to help reinforce basic probability and statistics concepts and their potential use in decision making.

1. Introduction

TPIR has been broadcasted in one form or another since its original airing in 1956. It remains a staple for daytime television game show watchers. The show's basic format has contestants compete to win cash and prizes by playing merchandise pricing games. Some of these games are skill based, requiring some pricing knowledge, but many are games of pure probability. Over the years, 105 pricing games have appeared on the show and a subset of 72 games remains in the show's rotation (TPIR Show, 2011). As such, these games and the show's format provide an effective set of learning tools in a wide range of classrooms and a source for statistical review. Butterworth and Coe (2002, 2004) provide an excellent overview of the four main components of the hour long game show and the benefits and challenges of using TPIR in the classroom. They also include a discussion of two pricing games; Plinko and the Money Game. Amy Biesterfeld (2001) provides an example of successfully integrating TPIR into a probability course and covers the Master Key, The Range Game, and Plinko. Wood (1992), Carlton and Mortlock (2003a, 2003b), and Fletcher (2005) all provide additional activities and examples of

using elements of TPIR in the classroom. In addition, McNelis (2008, 2012) discusses the effective use of simulations in teaching statistics in the classroom which includes elements of TPIR and Little (2012) provides a wonderful applet of the Plinko game for collecting data from repeated trials. The show's rich set of elements also lend themselves to statistical analysis. For example, Coe and Butterworth (1995) provide a statistical analysis of determining the optimal stopping point in the Showcase Showdown and their 2002 article, *The Prizes Rite*, provides a combinatory analysis of the Spelling Bee. Grosjean (1998), Tenorio and Cason (2002), Bennett and Hickman (2004), and Hulse (2008) all provide additional statistical analysis of some element of TPIR. This paper adds to this body of knowledge with a look at integrating the show's primetime episodes into a classroom activity.

As a marketing ploy to generate interest in the daytime version of the show, the producers developed the primetime *Million-Dollar Spectacular* (MDS) version of the show in 2003. The MDS changed the show's standard daytime format to provide contestants an opportunity to win a million dollars during the show. During Seasons 31 – 35 (2003 – 2007), the MDS format provided six contestants (and potentially a seventh) the opportunity to spin a wheel for a million dollars, during each show. There were 16 MDS airings under this format, yet no contestants ever won a million dollars. In 2008 (Season 36), CBS changed the MDS format yet again (MDS36) to providing just two contestants the opportunity to win a million dollars.

The intent of the new format was to make the process harder to win while adding more excitement to the show. The current host, Drew Carey, reported that the insurance executives "wanted to put as much pressure on the contestant, making it as difficult as possible so they don't have to give away a million dollars" (Deggans, 2008). However, this new format made CBS' insurance executives nervous, when during three of the first four MDS36 shows they handed out the million dollar top prize. There were no million dollar winners in any of the remaining six MDS36 airings but the initial set of wins raised our curiosity about the new format and the risk for CBS' insurance executives. Did the chance to win a million dollars really become harder and the string of wins simply represents chance or is there another explanation for the string of wins?

The remainder of this paper is divided into three major sections; the classroom activity, the class discussion, and student responses to the activity. The following section describes the classroom activity and is divided to cover both the MDS and MDS36 formats. Each of these sections provides a synopsis of the particular show's format followed by a probability calculation of winning a million dollars.

2. The Classroom Activity

This activity was used in both a one-semester undergraduate probability course, with 16 students and a graduate level statistics for executive decision making course, with an average of 14 students. None of the students in either course were mathematics majors. However, the undergraduate students arrived with three semesters of mathematics that included a semester of modeling with discrete dynamical systems and a semester of calculus. The graduate students were business majors who started the course after a quarter of calculus. We had several goals in mind when developing the classroom activity.

1. Motivate student learning by demonstrating the relevance of their mathematical skills in solving a set of real world related problems.
2. Reinforce student understanding of basic probability concepts including marginal probabilities, conditional probabilities, and independent events.
3. Reinforce student understanding of discrete probability distributions.
4. Serve as a mini-capstone event, pulling together several threads from the course.

To achieve these goals, we assume that the students bring with them a certain set of skills that have been developed previously in the course. In particular, students must be able to:

1. Convert a written depiction of a problem into a probability equation;
2. Understand basic discrete probability distributions;
3. Understand how to construct probability trees and compute marginal, conditional, and joint probabilities;
4. Understand the equation for the binomial probability distribution;
5. Convert probability results into a matrix-vector product;
6. Interpret results and communicate them to a "client".

This activity is designed as an interesting way to review previously taught concepts and to show how basic statistics may help decision making. The activity is usually conducted during a single two hour class period, with a 10 minute break in the middle, but it could be adapted to extend beyond a single period or for outside the classroom work. Experience has shown that providing some level of review may be beneficial since some students struggled to complete the activity in time.

Prior to performing this activity, we recorded an example of spinning the wheel during the Showcase Showdown and the pricing game of Plinko from TPIR. The recording was shown to the students at the being of the exercise to familiarize them with the basic elements of the show's format.

During the class period, the students were broken into groups of three to five students. The intent of the multiple groups was to ensure all students were involved and to provide multiple opportunities for students to communicate their mathematical findings. Each group was provided a short synopsis of both the original MDS and new MDS36 format. The basic question for all groups was to calculate the probability of the insurance executives having to give away a million dollars. After sufficient time passed, we reconvened as a class and several groups were selected to present their solution. Then as a class, we discussed what happened that allowed a million dollar win during three of the first four MDS36 shows.

2.1 Original Format (MDS - Million Dollar Spin) Format Synopsis

During the first 16 MDS shows, contestants were provided the opportunity during the Showcase Showdown to earn a spin for a million dollars. For those individuals not familiar with the format, twice each show three contestants compete to advance to the Showcase round by spinning a large wheel, labeled from five cents to one dollar in five cent increments (20 total spaces). The contestant coming closest to one dollar in no more than two spins, without going

over, advanced to the showcase round. As an addition to the standard showcase showdown in TPIR, MDS adds a bonus spin for a contestant spinning exactly one dollar in one or two attempts. If this bonus spin landed on the one dollar space, then the contestant would win a million dollars. In theory, it was possible for all six contestants to earn a bonus spin and a million dollars.

To make things interesting, if no contestant received a bonus spin during the two Showcase Showdowns, one individual was randomly selected, at the end of the show, for a bonus spin. This format guaranteed at least one spin and a potential of six spins for a million dollars.

Solution

Since each of the six contestants had an opportunity to spin, there were at least six potentially independent opportunities during the show for a contestant to win a million dollars. For the purpose of this activity, we will assume that the outcome of one contestant's spins had no effect on the observed outcome of the other contestants. This implies that each contestant will attempt to win a million dollars and not worry about advancing to the Showcase round. Therefore, each contestant's opportunity to win a million dollars may be considered independent Bernoulli trials. The independence assumption makes a great discussion point at the end of the activity since most contestants are more interested in winning the Showcase Showdown versus the million dollars. This makes the spins non-independent.

Calculating the probability of the insurance executives giving away a million dollars is a relatively straight forward process. Each contestant must first make \$1.00 (in 1 or 2 spins) and then land on \$1.00 during the bonus spin; since the events must occur in order and are assumed independent, the probability can be expressed as

$$P(win\ a\ million) = P(make\ \$1.00) \times P(land\ on\ \$1.00).$$

The probability of making a \$1.00 on one spin or a combination of two spins represents the first probability sub-problem. Considering there are a potential of two spins, the problem is a 2-tuples set of discrete values representing the results of the spin. If the first spin lands on the \$1.00 space, the experiment is over and the contestant receives a bonus spin. However, if the spin lands on one of the other 19 spaces a second spin is required and only one of the 20 potential spaces allows the contestant to make a \$1.00. For example, if the first spin lands on the 10 cent space, the contestant has a 1 in 20 chance of landing on the 90 cent space to earn the bonus spin. The 2-tuple set of spins contains 380 potential outcomes where only 19 sum to \$1.00. Combine this with making \$1.00 on the first spin, provides a 0.0975 (39/400) chance of making \$1.00. The probability of landing on \$1.00 during the bonus spin is clearly a 1 in 20 chance.

Therefore, the probability of a single contestant winning a million dollars is

$$P(win\ a\ million) = \frac{39}{400} \times \frac{1}{20} \cong 0.0049$$

or a 0.49 percent chance.

The construction of a probability tree (Figure 1) will of course provide the same solution. Looking at the probability tree it is clear that only the joint probabilities at point 1 and point 3 will satisfy winning a million dollars. Therefore, the

$$P(win\ a\ million) = (0.05)(0.05) + (0.95)(0.05)(0.05)$$
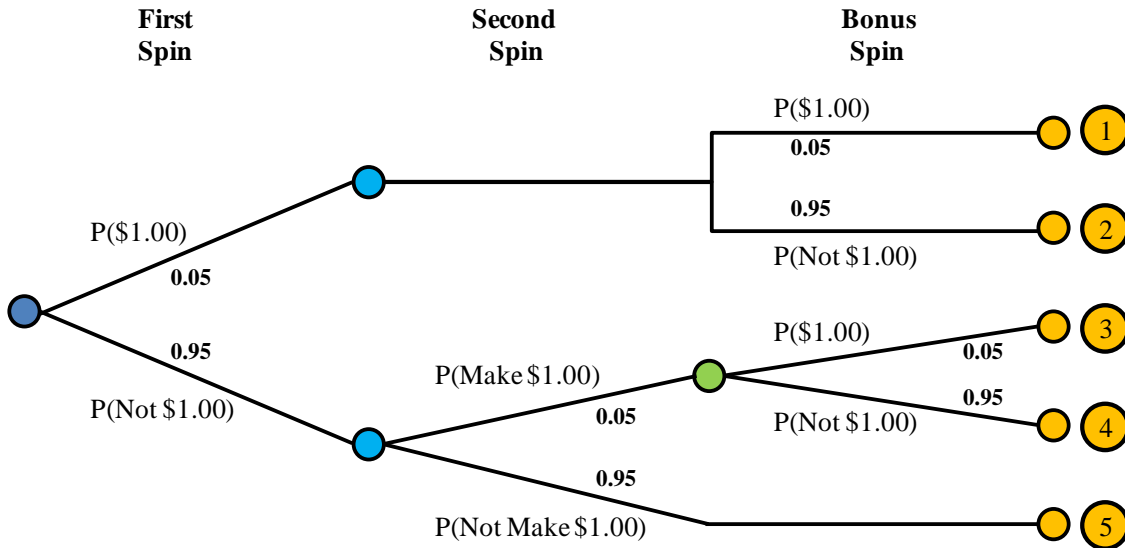$$= 0.0025 + 0.002375 \cong 0.0049.$$

Figure 1. Probability tree for Showcase Showdown in MDS

When we reconvene as a class, this sub-problem solution provides a great discussion point that starts by asking the students if this is a reasonable probability. Students should consider the format and objective of the spin in the Showcase Showdown round and the independence assumption used in the calculation. The contestant's goal is to get into the Showcase round so if their first spin is 95 cents will they really spin again for a chance at a million? A great follow on discussion question is how does the contestant order in spinning change the probabilities? If the first contestant had a sum of 65 cents and the next contestant's first spin was 70 cents, would the contestant spin a second time? It should be noted that the contestant does not have to make the second spin if they are only interested in advancing to the Showcase round. For example, if the first spin was 95 cents, the contestant might choose to keep this score in an attempt to advance to the Showcase round and give up the shot at a million dollars.

Since there are six independent Bernoulli trials to win a million dollars, the overall risk of one success out of six follows the binomial probability distribution. Therefore,

$$P(\ k\ wins\ in\ n\ trials) = \binom{n}{k} p^k q^{n-k}$$

where *n* is the number of trials, *k* is the number of successes or wins, and *q* is the probability of failure. However, the insurance executive's overall risk of giving away at least one million dollars is calculated as

$$P(at\ least\ 1\ winner\ in\ 6\ trials) = 1 - P(0\ winners\ in\ 6\ trials)$$
$$= 1 - \binom{6}{0} 0.0049^0 \times 0.9951^6 = 1 - 0.9711 = 0.0289$$

or a 2.89 percent chance.

It should be noted that some students took the longer route to this solution by calculating the individual probabilities for 1, 2, 3, … 6 wins then summing these individual probabilities.

$$P(at\ least\ 1\ winner\ in\ 6\ trials) = 2.85 \times 10^{-2} + 3.50 \times 10^{-4}$$
$$+2.28 \times 10^{-6} + 8.39 \times 10^{-9} + 1.64 \times 10^{-11} + 1.34 \times 10^{-14}$$

However, if none of the six contestants had an opportunity to win the million dollars, the show randomly selected a seventh contestant to spin the wheel one time for a million dollars. The probability of this contestant winning the million dollars is expressed as

$$P(7th\ contestant\ wins) = P(0\ wins\ in\ 6\ trials) \times P(land\ on\ a\ \$1.00)$$

where
$$P(0\ winners\ in\ 6\ trials) = 1 - P(at\ least\ 1\ winer\ in\ 6\ trials)$$
$$= 1 - 0.0289 = 0.9711$$

and
$$P(land\ on\ a\ \$1.00) = \frac{1}{20}.$$

Therefore, the probability of the seventh contestant winning a million dollars is

$$P(7th\ contestant\ wins) = 0.9711 \times \frac{1}{20} = 0.0486$$

or a 4.86 percent chance.

The overall probability of the insurance executives giving away at least a million dollars during any given MDS airing is 7.75% (2.89% + 4.86%). The construction of a probability tree (Figure 2) provides the same solution. Looking at the probability tree it is clear that only the joint probabilities at point 1 and point 2 will satisfy the condition of the insurance executives giving away at least a million dollars. Therefore, the

$$P(giving\ away\ a\ million) = 0.0289 + (0.9711)(0.05)$$
$$= 0.0289 + 0.0486 \cong 0.0775.$$

This provides another great discussion point of why the show's executives would bring back the seventh contestant. What benefit does the increased payout risk bring to the show?
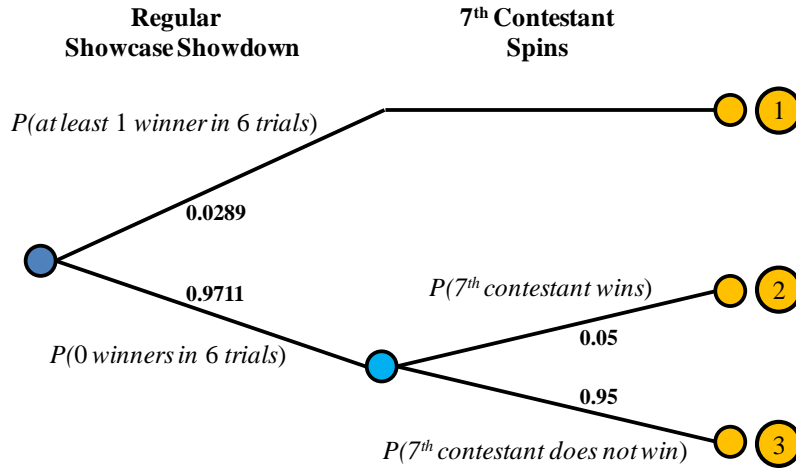
Figure 2. Probability tree for 7th contestant spinning bonus wheel in MDS

2.2 Season 36 Format (MDS36 - Million Dollar Game) Format Synopsis

To make the show a little more interesting and supposedly more difficult, the opportunity to spin for a million dollars was eliminated during Season 36. In its place, each show offered two opportunities to win a million dollars, both of which theoretically could be won by a single contestant. The first opportunity came during the play of what was designated as the Million Dollar Game (MDG), where one of the show's six contestants was selected at random to play the MDG. Executives selected nine games from the show's pool of 72 rotating games to serve as potential Million Dollar Games. These games were selected based on their level of difficulty, and included the Clock Game, Cover Up, One Away, ½ Off, Plinko, Punch a Bunch, Range Game, Safe Crackers, and Switcheroo. Each game offers an opportunity to earn a bonus play that, if successful, would award the million dollars. The second opportunity at a million dollars would come during the Million Dollar Showcase where a single contestant bidding closest and within $1,000 (without going over) of the actual retail price of their showcase would receive a million dollars. This new format reduced the total opportunities for a million dollars during the show from a potential seven to only two. Yet under the MDS36 format, three contestants won the million dollar prize during the first four airings; once with the MDG of Plinko and twice with the Million Dollar Showcase.

2.2.1 Chance during the Million Dollar Game Synopsis

Students were asked to only consider the pricing game of Plinko, since it was the winning MDG. We could have easily asked each group to solve one of the different MDG to see if different games changed the overall probability but wanted to keep it simple. Plinko, which debuted in 1983, is by far the most popular of all pricing games on the show. The game's current version

consists of a pricing part and a part touted by the show as a game of skill. Plinko has a potential top prize of $20,000. In a normal game of Plinko, the contestant receives a single round flat disc, called a Plinko chip, at the start of the game. The contestant then has the opportunity to earn up to four more chips during the pricing part of the game. The contestant is shown four small merchandise items, such as a popcorn maker or a water filter, and displayed next to the item is an incorrect two-digit price. The contestant must decide if the first or second number of the displayed price is correct. The contestant receives one additional chip for each correct guess, providing an opportunity to have up to five chips for the skills portion of the game.

During the skill part of the game, the contestant carries their chips ($1 \le n \le 5$) to the top of the Plinko board (Figure 3) and releases them one at a time in one of nine slots (positions). The disc drops down the board, randomly bouncing off pegs as it falls. The board consists of 102 pegs organized in 12 rows of alternating eight or nine pegs. The chip falls, bouncing left or right 12 times, until it lands at the bottom of the board in one of nine bins. The bins are marked with a monetary value in order, left to right, of $100, $500, $1,000, $0, $20,000, $0, $1,000, $500, $100. The contestant wins the total amount associated with the sum of the bin values their chips occupy. This allows a contestant to win between $0 and $100,000 depending on their number of Plinko chips. (Plinko represents a modification to Galton's 1889 board called a Quincunx (MathsIsFun.com, 2012.))
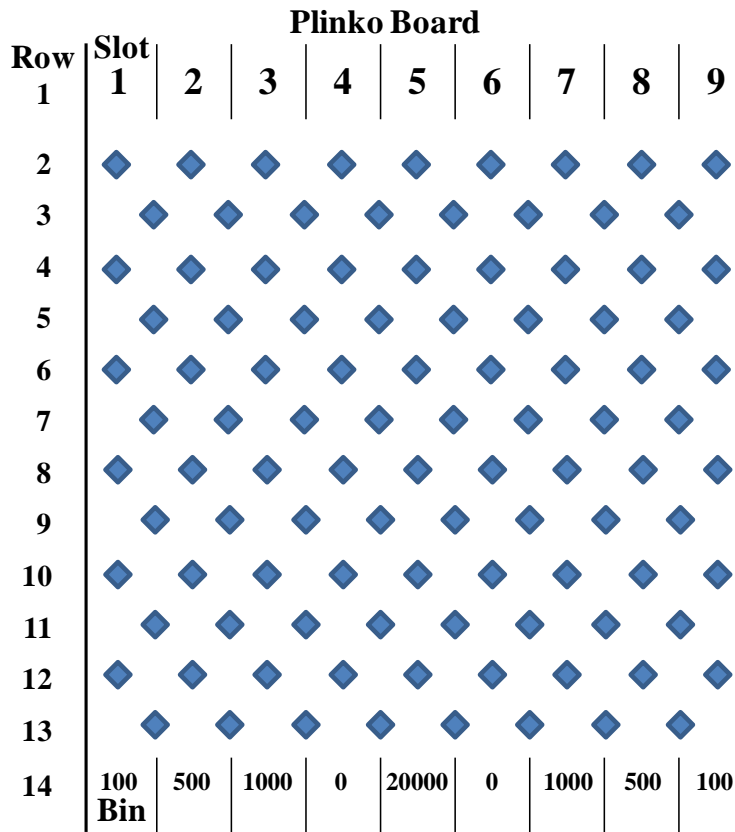

Figure 3. Plinko Board

During a MDG, a contestant earns a special golden chip by getting at least three Plinko chips to land in the center bin during the play of the game. For MDS36, a $1,000,000 bonus is offered if a contestant gets the bonus golden chip in the center ($20,000) bin.

Solution

Calculating the probability of awarding a million dollars requires determining the contestant's probability of getting at least three chips in the center bin earning the bonus golden chip and the probability of getting the bonus golden chip to land in the center bin. For the purpose of this activity, we will assume that the contestant is able to earn all 5 chips. In reality, the contestant only earned all five chips in 30.24 percent (101/334) of the Plinko games played between 2000 and 2011. The drop of each chip may be considered an independent Bernoulli trial, and since the two actions described above must occur in order, the probability can be expressed as a binomial probability. Therefore,

$$P(win\ a\ million) = P(earn\ bonus\ chip) \times P(bonus\ chip\ lands\ in\ center\ bin)$$

Plinko - Landing in Center Bin

The probability of landing in the center bin at the bottom of the board depends on the starting slot of the chip. A review of the first row of nine pegs (Row 2 in Figure 4) shows that there are eight positions a chip can occupy once it bounces off one of the row's pegs. Looking at the board it is clear that a chip dropped in Slot 1 can only fall into position 1, on row 2, because the left wall forces the chip to move to the right (position 1). In similar fashion, a chip dropped in slot 9 moves to position 8. However, a chip dropped in slot 2 has an equal chance of bouncing into either position 1 or position 2. For this activity, let the probability of a chip being in a given position on a row be

$p_r(i) = $ probability that the chip is in position i when it is on row r

and let

$v_r(i) = $ a vector containing the probabilities $p_r(i)$ for each position i on row r, where $r = 1, 2, 3, \ldots, 14$.
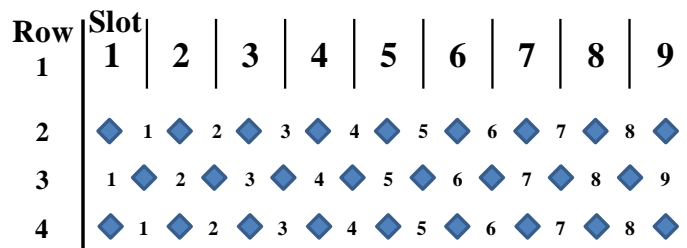
Figure 4. Potential Plinko Chip positions for even and odd rows

Calculating the probabilities for the first row only depends on the slot selected by the contestant. For example, assuming the contestant drops a chip in slot 2 on row 1, then the $p_1(2) = 1$ and all

other probabilities on the row are equal to zero. Therefore, the vector of probabilities for row 1 is

$$v_1(i) = [0,1,0,0,0,0,0,0,0]$$

Calculating the position probabilities for the remaining rows is a little more difficult since they are dependent on the chip's location on the previous row. The probability of a chip being in a given position (i), on an even numbered row (row with eight positions), where $r = 2, 4, 6, 8, 10, 12,$ and 14 is:

$$p_r i = 1 \times p_{r-1}(i) + \frac{1}{2} \times p_{r-1}(i+1), \quad i = 1,$$
$$p_r(i) = \frac{1}{2} \times p_{r-1}(i) + \frac{1}{2} \times p_{r-1}(i+1), \quad i = 2, \dots, 7,$$
$$p_r(i) = \frac{1}{2} p_{r-1}(i) + 1 \times p_{r-1}(i+1), \quad i = 8.$$

Continuing our example where the chip was dropped in slot 2

$$v_1(i) = [0,1,0,0,0,0,0,0,0]$$

$$p_2(1) = 1 \times p_1(1) + \frac{1}{2} \times p_1(2) = \frac{1}{2} \times (0) + \frac{1}{2} \times (1) = 0.50$$
$$p_2(2) = \frac{1}{2} p_1(1) + \frac{1}{2} p_1(2) = \frac{1}{2} \times (1) + \frac{1}{2} \times (0) = 0.50$$

There is no opportunity for the chip to enter positions $3 - 8$. Therefore, the vector of probabilities for row 2 is:

$$v_2(i) = [0.50, 0.50, 0, 0, 0, 0, 0, 0]$$

The third row (Figure 4) consists of eight pegs creating nine potential positions for a chip to occupy once it bounces off one of the row's pegs. A chip in position 1 of the previous even row has an equal probability of moving to either position 1 or 2 of the row. There is only one way for a chip to enter either position 1 or position 9 but a chip may enter all other positions on the row from two directions. Therefore, the probability of a chip being in a given position (i), for an odd row (row with nine positions), where $r = 3, 5, 7, 9, 11,$ and 13 is:

$$p_r(i) = \frac{1}{2} \times p_{r-1}(i), \quad i = 1,$$
$$p_r(i) = \frac{1}{2} \times p_{r-1}(i-1) + \frac{1}{2} \times p_{r-1}(i), \quad i = 2, \dots, 8,$$
$$p_r(i) = \frac{1}{2} p_{r-1}(i-1), \quad i = 9.$$

Continuing from the previous example,

$$v_2(i) = [0.50, 0.50, 0, 0, 0, 0, 0, 0, 0]$$

$$p_3(1) = \frac{1}{2} \times p_2(1) = \frac{1}{2} \times (0.50) = 0.25$$
$$p_3(2) = \frac{1}{2} p_2(1) + \frac{1}{2} p_2(2) = \frac{1}{2} \times (0.50) + \frac{1}{2} \times (0.50) = 0.50$$
$$p_3(3) = \frac{1}{2} p_2(2) + \frac{1}{2} p_2(3) = \frac{1}{2} \times (0.50) + \frac{1}{2} \times (0) = 0.25$$

There is no opportunity for a chip to enter positions $4 - 9$. Therefore, the vector of probabilities for row 3 is:

$$v_3(i) = [0.25, 0.50, 0.25, 0, 0, 0, 0, 0, 0].$$

This process would continue to row 14, which would provide the probability of landing in a given bin if dropped in slot 2. The above equations can be solved analytically with matrix algebra. Since, there is no indication of which slot a contestant might use, we determine which slot provides the best chance to win. Table 1 provides the analytical solution for each of the nine potential starting positions and indicates that the best chance of landing in the center bin is using slot 5 (22.56 percent). This result is intuitively pleasing since most people would guess that starting in the center gives you the best chance of ending in the center. It should be noted that some students struggled with the matrix algebra and needed some additional guidance. A short review the night before might have prevented this problem.

Table 1. Probability of landing in the center ($20,000) bin based on starting (Slot) position

Bin	$100	$500	$1,000	$0	$20,000	$0	$1,000	$500	$100
Slot 1	0.2256	0.3867	0.2417	0.1074	0.0322	0.0059	0.0005	0	0
Slot 2	0.1934	0.3464	0.2471	0.1370	0.0566	0.0164	0.0029	0.0002	0
Slot 3	0.1208	0.2471	0.2417	0.1963	0.1211	0.0537	0.0161	0.0029	0.0002
Slot 4	0.0537	0.1370	0.1963	0.2258	0.1934	0.1208	0.0537	0.0164	0.0029
Slot 5	0.0161	0.0566	0.1211	0.1934	0.2256	0.1934	0.1211	0.0566	0.0161
Slot 6	0.0029	0.0164	0.0537	0.1208	0.1934	0.2258	0.1963	0.1370	0.0537
Slot 7	0.0002	0.0029	0.0161	0.0537	0.1211	0.1963	0.2417	0.2471	0.1208
Slot 8	0	0.0002	0.0029	0.0164	0.0566	0.1370	0.2471	0.3464	0.1934
Slot 9	0	0	0.0005	0.0059	0.0322	0.1074	0.2417	0.3867	0.2256

Plinko – Winning a Million Dollars

For the purpose of this activity, we assume that the contestant will drop their chip in the center slot giving them the best chance of landing in the center bin. Since the observed outcome of each chip is independent, they may be considered repeated independent Bernoulli trials. The probability of earning a bonus chip requires getting three chips out of a potential five chips to land in the center bin and is calculated using binomial probability.

$$P(earn\ bonus\ chip) = P(at\ least\ 3\ in\ 5\ trials)$$
$$= \binom{5}{3} 0.2256^3 \times 0.7744^2 + \binom{5}{4} 0.2256^4 \times 0.7744^1 + \binom{5}{5} 0.2256^5 \times 0.7744^0$$
$$= 0.0689 + 0.0100 + 0.0006 = 0.0795$$

or 7.95 percent.

The probability of the bonus chip landing in the center bin, using slot 5, is simply 0.2256 or 22.56 percent. Therefore, the probability of a single contestant winning a million dollars, using slot 5, is 0.0179 or a 1.79 percent chance.

$$P(win\ a\ million) = 0.0795 \times 0.2256 = 0.0179$$

It is interesting to note that this opportunity to win a million dollars is lower than a single contestant winning under the old format. This also provides another great discussion point. This solution assumed the contestant earned all five chips during the pricing game and that they used slot 5 for all of their drops. Is this a realistic assumption and what is the expected number of discs a contestant might actually win? As discussed above, during the 334 Plinko games played between 2000 and 2011, contestants only earned five chips during 101 games (30.24 percent) and the actual average number of chips was 3.95.

2.2.2 Million Dollar Showcase Synopsis

During the Showcase round, the two contestants who won their respective Showcase Showdown bid on a showcase of merchandise. The contestant who bids closest to the actual retail value of their showcase, without going over, wins and keeps their showcase of prizes. During the MDS36, a single contestant who bids closest and within $1,000 of the actual retail value of their showcase, without going over, wins one million dollars. Calculating the probability of winning a Showcase is difficult. Showcase values typically range between $11,000 and $91,000 and a contestant's bid is likely dependent on pricing knowledge. However, we can look at the historical record to help determine the probability of bidding within $1,000. During seasons 29 – 35 (only seasons with available data), there were a total of 1,214 contestant bids. Out of this set there were only 38 bids within $1,000 of the Showcase value without going over.

Solution

Assuming the historical evidence provides an accurate estimate of the probability of winning, a single contestant has a 0.0313 (38 / 1,214) or 3.13 percent chance of biding within a $1,000 of their Showcase value. It is interesting to note that this event actually occurred in two of the first four MDS36 airings. This provides another great discussion point concerning the validity of using historical data, especially since the collected data was not solely from the MDS or MDS36 shows. The data was from shows where the contestant did not have an opportunity to try for a million dollars. A survey of the class clearly indicated that many of the students would try for a million dollars versus just trying to win the Showcase, especially based on the prizes offered during the Showcase, providing one indication that the data may not be representative.

Combined with the Million Dollar Game, the insurance executive's overall chance of giving away one million dollars under the new MDS36 format is 4.92 (3.13 + 1.79) percent.

3. Class Discussion and Thoughts on the Activity

The solutions indicate that the MDS36 format did provide a reduced risk over the MDS format of giving away a million dollars, 4.92 percent versus 7.75 percent, respectfully. This result is of course based on our set of assumptions:

1. MDS: Independence of spins during the Showcase Showdown;
2. MDS: Contestants spin to win the million dollars during the Showcase Showdown;
3. MDS36: Earning five Plinko chips during the MDG Plinko;
4. MDS36: Dropping all Plinko chips in the 5th slot during the MDG Plinko;
5. MDS36: Historical Showcase data represents accurate estimates of probability.

This conclusion provides another point to continue the discussion of the validity of our claim and the sensitivity of our assumptions on this claim. Assuming for a moment the claim is correct and the new format did reduce the overall risk to CBS' insurance executives, what happened during the first four tapings? This provides a great opportunity to discuss how mathematical probabilities really translate into the real world. The probability of three winners in four shows under the new format is extremely small (4.53×10^{-4}) or about 1 in 2,200, much better than winning the lottery but still an unlikely event.

$$P(\ 3\ winners\ in\ 4\ trials) = \binom{4}{3} 0.0492^3 \times 0.9508^1 = 4.53 \times 10^{-4}$$

However, this provides a clear example of no matter how hard you try sometimes chance just gets in the way. This provides an opportunity to discuss the validity of our assumptions and their impact on the results.

One benefit of the Bernoulli Trials activity was the plethora of extension questions it inspired – many of which were introduced by the students themselves. For example, is it possible to find an "optimal" strategy in the Showcase Showdown? What do we mean by optimal? Are the contestant's spinning of the wheel truly independent if knowledge of other contestant scores influence their decisions? Is it realistic to attempt a guess within $1,000 or just try to beat your opponent in the Showcase? How does a contestant's opinion of their opponent's guess influence their bid amount?

In our experience, this activity continually generates new implementation ideas and additional avenues of exploration for the students. For example, there are several modifications we would consider if we use this activity in the future. We would have each group solve a different Million Dollar Pricing game. This would expand the number of applications and provide a richer set of examples. One additional concept is to issue a follow-on requirement for each group to consider how they would change the show's format to provide more excitement. Then, as a class discuss how these changes impact the overall chance of winning a million dollars. However, we caution to curb your appetite in having students do even more with this activity during the allotted time. For the students to truly enjoy this activity, they need time for classroom

discussions and presentations. Typically, groups complete their work at the classroom chalkboards. During the presentation portion of the class, one or more students will verbally walk the class through their solution process. We have found that this formal presentation is critical to building student mathematical confidence. We encourage you to keep in mind that each addition to this activity should come with an additional allocation of time either before or after the main activity. As a time saver, consider having the students solve the Plinko game the night before the lesson to free class time for other discussions.

We also caution in ensuring the students are ready for the mathematics involved in the activity. The matrix algebra slowed down one group during the class and they needed additional guidance to stay on track to solve the activity. One potential remedy is to offer the activity after a short review of the mathematical concepts. However, it is possible to complete the Plinko portion without an understanding of matrices. One simple alternative is the use of Excel to construct the rows, columns, and probabilities of the Plinko board. An additional method is to manually complete the probability calculations. This requires careful work and time to complete the approximately 110 calculations per slot. If this approach is taken, we suggest limiting the groups to calculating the probabilities for just one slot.

4. Conclusion

Both our own observations and student survey responses suggest that this activity was worth the effort of its inclusion in the course. One striking advantage of activities such as *TPIR* is the opportunity to relate classroom work to actual real-life situations. The format of assigning small groups and allowing students to communicate their own work has been popular with our students. This format reduces some math anxiety and allows the students to determine for themselves which probability tools to use on the problem. Presenting their results allows students to build confidence in their skills and how to translate mathematical results. At the same time, the activity was challenging for ill-prepared students and stressed the importance of coming to class understanding the course material. One consideration to help students prepare for the lesson is to provide a review and potentially have the students solve the Plinko game before the lesson.

The lesson was generally well received by the students. Many students believed it helped reinforce their understanding of the material. Many students appreciated the opportunity of making the connection between their classroom instruction and a real application. In the end, we all learned something and had fun. Who could ask for more from a mathematics class?

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