Key Results of Interaction Models With Centering

David Afshartous
Vanderbilt University

Richard A. Preston
University of Miami

Abstract

We consider the effect on estimation of simultaneous variable centering and interaction effects in linear regression. We technically define, review, and amplify many of the statistical issues for interaction models with centering in order to create a useful and compact reference for teachers, students, and applied researchers. In addition, we investigate a sequence of models that have an interaction effect and/or variable centering and derive expressions for the change in the regression coefficients between models from both an intuitive and mathematical perspective. We demonstrate how these topics may be employed to motivate discussion of other important areas, e.g., misspecification bias, multicollinearity, design of experiments, and regression surfaces. This paper presents a number of results also given elsewhere but in a form that gives a unified view of the topic. The examples cited are from the area of medical statistics.

1. Introduction

We consider the case of simultaneous variable centering and interaction effects in linear regression. The goal is to create a useful reference for teachers and students of statistics, as well as applied researchers. Thus, we technically define, review, and amplify many of the statistical issues for interaction models with centering and provide a comprehensive summary and discussion of key points. While many of the points we raise have been made elsewhere, they are somewhat scattered across a voluminous literature. The examples cited are from the area of medical statistics.

By the term variable centering we mean subtracting either the mean value or a meaningful constant from an independent variable. It is well-known that variable centering can often increase the interpretability of regression coefficients as well as reduce multicollinearity between lower and higher-order predictor variables.

To discuss characteristics of interaction effects, we consider a model with two predictors and their cross-product term. For ease of illustration we assume that continuous predictors are linearly related to the dependent variable.[1] Interaction effects arise when the effects of predictor variables are not additive, i.e., the effect of one predictor variable depends on the value of another variable. For example, consider Figure 1 in the context of a potassium challenge experiment[2] where Y represents urinary potassium excretion, X_1 represents serum potassium level, and X_2 represents glomerular filtration rate (GFR). As GFR is a measure of kidney function one might expect that the slope of the response Y against serum potassium level X_1 would increase for higher GFR levels X_2. This is often referred to as a reinforcement or synergistic interaction, whereas an offsetting or interference interaction effect occurs when the slope of the response decreases for higher GFR levels X_2. Moreover, centering of GFR and serum potassium could enhance the interpretability of the regression coefficients given that it is not meaningful to consider a subject with a zero value for GFR or serum potassium.

When adding centering to a model that includes an interaction term, the magnitude and standard error of certain estimated coefficients change. Indeed, as researchers often sift through several different models, many of which yield the same fitted values merely under different parameterizations, the potential for confusion is high. In this paper we attempt to provide a compact guide to help reduce such confusion. In Section 2, we provide separate overviews of variable centering and interaction effects. In Section 3, we consider simultaneous centering and interaction effects via a sequence of models. We derive expressions for the change in the regression coefficients for the new models from both an intuitive and

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[1] For a discussion of relaxing the linearity assumption see Harrell (2001), p.16

[2] Potassium challenge experiments involve the administration of a potassium load to experimental subjects in order to investigate the physiology of potassium handling.
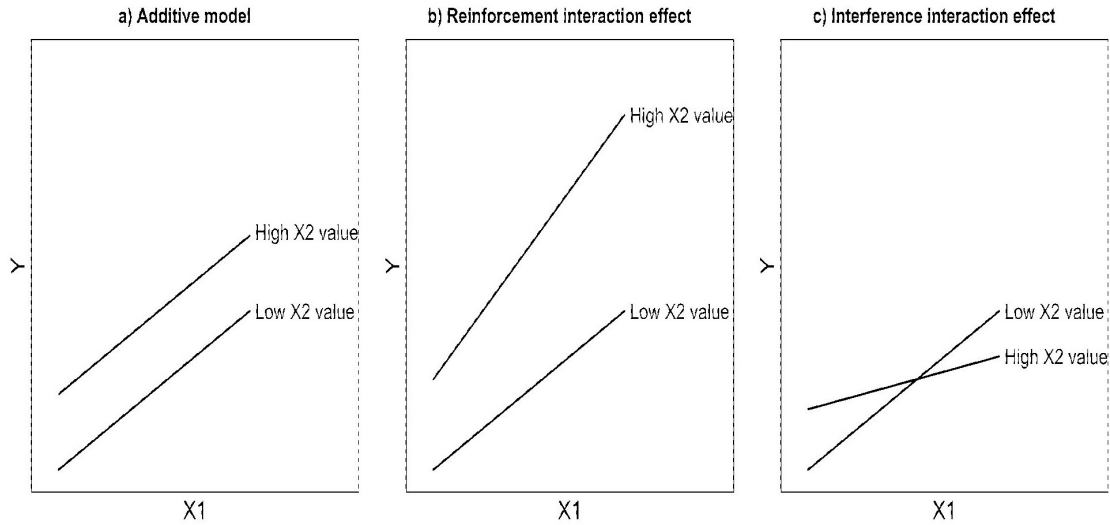
Figure 1.   Illustration of reinforcement and interference interaction effects. In the additive model (a), the relationship between Y and X_1 does not depend on the value of X_2. In a reinforcement interaction effect (b), the slope between Y and X_1 increases for higher X_2 values, while in an interference interaction effect (c) the slope between Y and X_1 decreases for higher X_2 values.

mathematical perspective. In Section 4, we provide a list of key points to guide both teaching and applied work with interaction models and centering.[3] We conclude with a brief summary in Section 5.

2. Variable Centering and Interaction Effects

2.1   Variable Centering

Motivations for employing variable centering include enhanced interpretability of coefficients and reduced numerical instability for estimation associated with multicollinearity. Consider the standard bivariate linear regression model where scalars X_i and Y_i represent the predictor and response variables, respectively, for the ith observation, and scalar ε_i represents the corresponding random error term where the standard assumption is that $\varepsilon_i \sim N(0, \sigma^2)$. Omitting the subscript without loss of generality, the "true" population model is[4]:

$$Y = \alpha + \beta X + \varepsilon, \tag{1}$$

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[3]For assessments of the methodology to detect interaction effects in certain fields (that also attempt to identify key points) see Carte and Russell (2003); Champoux and Peters (1987).

[4]Throughout the paper only scalar notation is employed. Greek letters are employed for population parameters while the corresponding English lower-case letter represents the corresponding estimator, e.g., (α, β) versus (a, b).

yielding $E(Y) = \alpha + \beta X$ for known predictor variable X. However, it is often more appealing to consider the centered population model:

$$
\begin{aligned}
Y &= \alpha^* + \beta^*(X - k) + \varepsilon & (2) \\
&= \underbrace{(\alpha^* - \beta^* k)} + \beta^* X + \varepsilon, & (3)
\end{aligned}
$$

where one may consider this as the regression of Y on the transformed predictor variable $X^* = X - k$. For instance, consider $k = \mu_X = E(X)$, the population mean of X.[5] Although this change of location of the predictor variable shifts the 0 point to μ_X, other changes of location to another meaningful value k are possible as well. Since $E(Y) = \alpha^* + \beta^*(X - \mu_X)$, the new intercept α^* represents the expected value of Y when $X = \mu_X$, i.e., the expected value of Y for the average predictor value. If the X variable is a physiological variable such as weight or blood pressure, the centered model provides a much more meaningful intercept. Since both population models must yield the same expected values for the same given X values, it follows that $\alpha^* = \alpha + \beta \mu_X$ and $\beta^* = \beta$. For instance, $E(Y|X = \mu_X) = \alpha + \beta \mu_X = \alpha^*$ and $E(Y|X = 0) = \alpha = \alpha^* - \beta^* \mu_X$, from which both results follow. Since correlation properties between variables do not change under linear transformations, the fact that the estimated slope should not change is also intuitive. It also follows that centering (or any linear transformation) does not alter the coefficient of determination R^2 (Arnold and Evans 1979; Allison 1977).

In practice, the population parameters are unknown and must be estimated via sampled data $(X_i, Y_i), i = 1, \ldots, n$, yielding the analogous equations for the estimated regression coefficients, e.g., $a = a^* - b\overline{X}$ and $b^* = b$. Note that centering predictors by their sample mean also has the beneficial effect of making the estimate of the intercept independent of the estimate of the slope.[6]

In multiple regression, variable centering is often touted as a potential solution to reduce numerical instability associated with multicollinearity, and a common cause of multicollinearity is a model with interaction term $X_1 X_2$ or other higher-order terms such as X^2 or X^3. For the case of two predictor variables X_1 and X_2, when X_1 and X_2 are uncorrelated in the sample data the estimated regression coefficient b_1 is the same regardless of whether X_2 is included in the model or not (similarly for b_2 and X_1). This may be seen from the following algebraic expression for b_1 in the standard multiple regression model with two

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[5]Asterisks are employed to denote corresponding parameters and estimators in a transformed model versus the original model, e.g., α^* is the intercept in the centered model while α is the intercept in the original model.

[6]This result no longer holds if one centers via k where k is not the sample mean.

predictor variables (Kutner et al. 2004):

$$b_1 = \frac{\frac{\sum(X_{i1}-\overline{X}_1)(Y_i-\overline{Y})}{\sum(X_{i1}-\overline{X}_1)^2} - \left(\frac{\sum(Y_i-\overline{Y})^2}{\sum(X_{i1}-\overline{X}_1)^2}\right)^{1/2} r_{Y2}r_{12}}{1 - r_{12}^2}, \tag{4}$$

where r_{Y2} represents the sample correlation coefficient between Y and X_2 and r_{12} represents the sample correlation coefficient between X_1 and X_2.[7] However, if the predictor variables are (perfectly) uncorrelated we have $r_{12} = 0$ and it immediately follows that

$$b_1 = \frac{\sum(X_{i1}-\overline{X}_1)(Y_i-\overline{Y})}{\sum(X_{i1}-\overline{X}_1)^2}, \tag{5}$$

which by definition is the estimated slope in the bivariate regression of Y on X_1 alone. Note that predictors are often correlated, except for designed experiments where the experimenter may choose the levels of the predictor variables.

When predictor variables are perfectly correlated infinitely many estimated coefficients provide the same predicted values and fit to the data. Perfect correlation, however, is not as troublesome as near perfect correlation. Under perfect correlation, the simple solution is to remove one of the variables since doing so does not remove any information. On the other hand, if $|\mathrm{Cor}(X_1,X_2)| < 1$, removing one of the variables entails a loss of information. Practically, the estimated coefficient b_1 changes depending on whether the predictor variable X_2 is included in the model or not. This change may be quantified and is commonly referred to as specification bias. Specifically, if $\mathrm{Cov}(X_1,X_2) = \sigma_{12}$ and one estimates a model without X_2 when the model should include X_2, one may show that the resulting estimate for the regression coefficient of X_1 has $E(b_1) = \beta_1 + \beta_2 \frac{\sigma_{12}}{\sigma_1^2}$, i.e., the expected bias in b_1 is thus $\beta_2 \frac{\sigma_{12}}{\sigma_1^2}$ (Goldberger 1964). Even if both variables are included, inference becomes more difficult in the presence of inflated standard errors, i.e., estimation uncertainty, where a small change to the data can result in a large change to the estimated coefficients. The more advanced reader may find further details regarding multicollinearity in Christensen (2002).

2.2 Interaction Effects

Consider multiple regression with two predictor variables. An interaction effect may be modeled by including the product term $X_1 \times X_2$ as an additional variable in the regression, known as a two-way interaction term. If there are k predictor variables in the multiple regression, there are $\frac{k!}{2!(k-2)!}$ potential two-way interactions, and analogously for three-

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[7]Note that there exists the distinction between the population correlation and the sample correlation, and correlated in sample does not necessarily imply correlated in population, and vice versa.

way and higher-order interactions. For a simple model with two-way interactions only, the population model is:

$$
\begin{aligned}
Y &= \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \varepsilon & (6) \\
&= \alpha + \underbrace{(\beta_1 + \beta_3 X_2)} X_1 + \beta_2 X_2 + \varepsilon & (7) \\
&= \alpha + \beta_1 X_1 + \underbrace{(\beta_2 + \beta_3 X_1)} X_2 + \varepsilon. & (8)
\end{aligned}
$$

The re-arrangement of terms in Equations 6–8 demonstrates the meaning of an interaction effect, i.e., the slope associated with X_1 is no longer simply a constant β_1, but rather $(\beta_1 + \beta_3 X_2)$, which clearly depends on the value of X_2, and similarly the slope associated with X_2 is now $(\beta_2 + \beta_3 X_1)$. The coefficient β_1 now represents the effect of X_1 on Y *when* $X_2 = 0$, whereas β_1 in a model without interaction represents the effect of X_1 on Y for *all* levels of X_2. The effect of X_1 on Y for non-zero values of X_2 is affected by the magnitude and sign of β_3, e.g., if $\beta_3 < 0$, the effect of X_1 on Y is less for higher values of X_2 and greater for smaller values of X_2 (interference or offsetting interaction, Figure 1), and vice versa for $\beta_3 > 0$ (synergistic or reinforcing interaction, Figure 1).

For instance, for $X_2 = 0, 1, 2$, we have three different lines for the effect of X_1 on Y:

$$
\begin{aligned}
E(Y|X_2 = 0) &= \alpha + \beta_1 X_1 \\
E(Y|X_2 = 1) &= (\alpha + \beta_2) + (\beta_1 + \beta_3) X_1 \\
E(Y|X_2 = 2) &= (\alpha + 2\beta_2) + (\beta_1 + 2\beta_3) X_1,
\end{aligned}
$$

and the bivariate relationship between Y and X_1 depends on X_2. Note that β_3 in isolation lacks information about the relative strength of the interaction. For instance, β_1 may be so large that even for a seemingly large β_3 there is not a substantial impact over the range of X_2 values considered.

Interaction effects are sometimes called joint effects, where the focus (instead of the conditional focus above) is more on how the two variables interact when accounting for the variance in Y over and above the contributions of the individual additive effects. Indeed, the interaction term does not assess the combined effect, e.g., a positive interaction coefficient $\beta_3 > 0$ only provides slope change information: higher values of X_2 correspond to a greater slope between Y and X_1. On the other hand, $\beta_3 > 0$ provides no information whatsoever regarding whether Y achieves its highest values for the highest values of X_1 and X_2 (Hartmann and Moers 1999). For example, in Figure 2a and Figure 2b the sign and magnitude of the interaction coefficient β_3 is the same. However, for the range of X_1 shown in Figure 2a, Y is higher when both predictors are high, while in Figure 2b we have Y higher when X_1 is
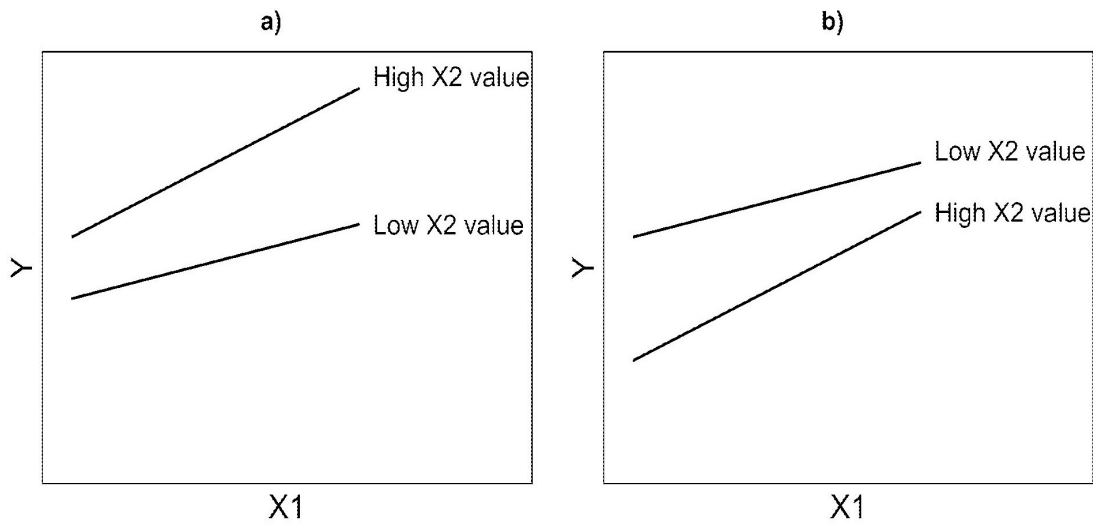
high and X_2 is low.[8]

Figure 2. Interaction coefficient does not provide information with respect to where dependent variable is higher. In both a) and b), the sign and magnitude of the interaction is the same. In a), Y is higher when both predictors are high, while in b) Y is higher when $X1$ is high and $X2$ is low.

Another problem common with including interaction terms in a regression model is that such terms are often highly correlated with the corresponding lower order terms. That is, $X_1 X_2$ is often highly correlated with both X_1 and X_2, similar to how in polynomial regression the higher-order terms like X^2 are highly correlated with X. It is often stated that the standard error of the lower order terms increases when the higher-order terms are added to a model with only the lower order terms, while the same is not the case in reverse, i.e., when the lower order terms are added to a regression with only a higher-order term the standard error of the higher-order term is unaffected (Aiken and West 1991).

Saunders (1955, 1956) was apparently the first to propose methodology to detect interactions for the continuous case and introduced the terms "moderator" and "moderated multiple regression" (MMR) to describe interaction effects.[9] The usual test for interaction in MMR is via the statistical significance of the semipartial coefficient of determination, i.e., the increment in the coefficient of determination between the simple additive model and the full interaction model (ΔR^2). This test is equivalent to the standard t-test for the interaction term as indeed the F-statistic for ΔR^2 equals the square of the t-statistic for the interaction

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[8]The figure was generated via the equation $Y = \alpha + \beta_2 X_2 + (\beta_1 + \beta_3 X_2)X_1$, with $\alpha = 1, \beta_1 = 0.1, \beta_2 = 0.5,$ $\beta_3 = 0.3$, and X_2 values fixed at either $X_1 = 1$ or $X_1 = 2$ for different ranges of X_1 values.

[9]Baron and Kenny (1986) later distinguished mediator effects from moderator/interaction effects; a mediator effect is often defined as a relationship such that an independent variable X_1 causes a mediating variable X_2 which then in turn causes the dependent variable Y; see Bauer et al. (2006); MacKinnon and Luecken (2008); Kraemer et al. (2008) for a full discussion.

term (Cohen 1978; Cohen et al. 2003). Some authors have critiqued this approach to assess the strength of interaction via incremental change in R^2 and some have offered alternative measures (Champoux and Peters 1987). Perhaps a more viable approach is to avoid focusing on a single measure and employ graphical techniques and predictive comparisons (see Section 4).

3. Simultaneous Variable Centering and Interaction Effects

Although the inclusion of an interaction term in a regression model by definition creates multicollinearity and its ensuing potential problems, this may be improved via centering. Specifically, if X_1 and X_2 are not centered, they are highly correlated with the product $X_1 X_2$ by definition. However, if one assumes bivariate normality between X_1 and X_2, then each of the centered variables $X_1 - \mu_1$ and $X_2 - \mu_2$ has a zero covariance and thus zero correlation with $(X_1 - \mu_1)(X_2 - \mu_2)$ (Aiken and West 1991). It follows that if the variables are centered, any correlation with the corresponding interaction term is solely due to nonnormality of the variables.[10] This follows from a result for symmetric variables X_1 and X_2 (Aiken and West 1991, p.180):

$$\text{Cov}(X_1 X_2, X_1) = \sigma_1^2 \mu_2 + \text{Cov}(X_1, X_2)\mu_1, \qquad (1)$$

where $\text{Var}(X_1) = \sigma_1^2$. Thus, we immediately see that if both variables are centered, i.e., have mean zero, then the correlation between $X_1 - \mu_1$ and $(X_1 - \mu_1)(X_2 - \mu_2)$ is zero. The correlation that exists between X_1 and $X_1 X_2$ when they have non-zero mean is often referred to as nonessential multicollinearity or nonessential ill-conditioning: it is due to the scaling and disappears with a rescaling; on the other hand, the part of the correlation that is due to skew is referred to as essential multicollinearity and cannot be removed via re-scaling (Marquardt 1980). In any event, for the case where we have only nonessential multicollinearity within an interaction model, via centering we know that failure to include the interaction term does not bias the estimated lower order coefficients (since the multicollinearity has been removed).

However, when centering variables in a model that includes an interaction term, students are often puzzled by the fact that the estimated coefficients for the lower order terms differ from those in the corresponding uncentered model. For instance, if the predictor X_2 was centered in Equation (6), β_1 would represent the effect of X_1 on Y when $X_2 = \mu_2$ instead of when $X_2 = 0$. As this represents a different conceptual entity altogether, one should expect an accompanying change in the estimated coefficients after such a transformation. Indeed, prior to making such transformation, it helps if the student knows which estimated coefficients change and which do not, thereby reducing the chance of misinterpretation. It

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[10]Of course in practice we form $X_1 - \overline{X}_1$ instead of $(X_1 - \mu_1)$, but the same results still follow.

is instructive to emphasize that standard errors of prediction, standard errors of fit, and the F statistic do *not* change for equivalent models. To assess the aforementioned changes in the estimated regression coefficients, consider the following sequence of models:

$$\textbf{M1:} \quad Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \varepsilon$$

$$\textbf{M2:} \quad \begin{aligned} Y &= \alpha^* + \beta_1^* X_1 + \beta_2^*(X_2 - \mu_2) + \beta_3^* X_1(X_2 - \mu_2) + \varepsilon \\ &= \underbrace{\alpha^* - \beta_2^* \mu_2} + \underbrace{(\beta_1^* - \beta_3^* \mu_2)} X_1 + \underbrace{\beta_2^*} X_2 + \underbrace{\beta_3^*} X_1 X_2 + \varepsilon \end{aligned}$$

$$\textbf{M3:} \quad \begin{aligned} Y &= \alpha^{**} + \beta_1^{**}(X_1 - \mu_1) + \beta_2^{**}(X_2 - \mu_2) + \beta_3^{**}(X_1 - \mu_1)(X_2 - \mu_2) + \varepsilon \\ &= \underbrace{\alpha^{**} - \beta_1^{**} \mu_1 - \beta_2^{**} \mu_2 + \beta_3^{**} \mu_1 \mu_2} + \underbrace{(\beta_1^{**} - \beta_3^{**} \mu_2)} X_1 + \underbrace{(\beta_2^{**} - \beta_3^{**} \mu_1)} X_2 \\ &\quad + \underbrace{\beta_3^{**}} X_1 X_2 + \varepsilon \end{aligned}$$

For a given set of predictor variables, the expected value of Y is the same under all models since they merely have changes of location for the same set of predictor variables. Consider the expected value of Y when $X_2 = \mu_2$ for Model 1 and Model 2:

$$\begin{aligned} \textbf{Model 1:} \quad E(Y|X_2 = \mu_2) &= \alpha + \beta_1 X_1 + \beta_2 \mu_2 + \beta_3 X_1 \mu_2 \\ &= (\alpha + \beta_2 \mu_2) + (\beta_1 + \beta_3 \mu_2) X_1 \\ \textbf{Model 2:} \quad E(Y|X_2 = \mu_2) &= \alpha^* + \beta_1^* X_1. \end{aligned}$$

Thus, equating the respective intercept and slope terms we immediately see that $\alpha^* = \alpha + \beta_2 \mu_2$ and $\beta_1^* = \beta_1 + \beta_3 \mu_2$. Since the coefficient β_2^* in Model 2 still has the same meaning, i.e., the effect of X_2 on Y when $X_1 = 0$, we should have $\beta_2^* = \beta_2$. This may be shown algebraically by conditioning on different values of both predictor variables, e.g.,

$$\begin{aligned} \textbf{Model 1:} \quad E(Y|X_1 = 2, X_2 = 0) &= \alpha + 2\beta_1 \\ \textbf{Model 2:} \quad E(Y|X_1 = 2, X_2 = 0) &= \alpha^* + 2\beta_1^* - \beta_2^* \mu_2 - 2\beta_3^* \mu_2. \end{aligned}$$

Equating the expected values and substituting the initial results for α^* and β_1^*, it readily follows that $\beta_2^* = \beta_2$ and $\beta_3^* = \beta_3$.

Regarding Model 3, the meaning of β_1^{**} is exactly the same as that of β_1^*, so once again we have $\beta_1^{**} = \beta_1^* = \beta_1 + \beta_3 \mu_2$. Similarly, due to symmetry we have $\beta_2^{**} = \beta_2^* = \beta_2 + \beta_3 \mu_1$. Equating expected values when $X_1 = \mu_1$ and $X_2 = \mu_2$ for Model 1 and Model 3, it immediately follows that $\alpha^{**} = \alpha + \beta_1 \mu_1 + \beta_2 \mu_2 + \beta_3 \mu_1 \mu_2$. These results are summarized in Table 1:

Table 1.　Summary of expected coefficients for each model with respect to M1

	Intercept	X_1 Slope	X_2 Slope	Interaction Slope
M1 (no centering)	α	β_1	β_2	β_3
M2 (X_2 centered at μ_2)	$\alpha + \beta_2\mu_2$	$\beta_1 + \beta_3\mu_2$	β_2	β_3
M3 (both centered at μ_1, μ_2)	$\alpha + \beta_1\mu_1 + \beta_2\mu_2 + \beta_3\mu_1\mu_2$	$\beta_1 + \beta_3\mu_2$	$\beta_2 + \beta_3\mu_1$	β_3

The reverse direction, i.e., solving for the coefficients of Model 1 in terms of the centered Models 2–3, is much simpler as one can directly see the results for factorization of the centered models accordingly. For example, $\alpha = \alpha^{**} - \beta_1^{**}\mu_1 - \beta_2^{**}\mu_2 + \beta_3^{**}\mu_1\mu_2$ when comparing Model 1 to Model 3. The results are summarized in Table 2:

Table 2.　Summary of expected coefficients for M1 with respect to M2 and M3

	Intercept	X_1 Slope	X_2 Slope	Interaction Slope
M2 (X_2 centered)	$\alpha^* - \beta_2^*\mu_2$	$\beta_1^* - \beta_3^*\mu_2$	β_2^*	β_3^*
M3 (both centered)	$\alpha^{**} - \beta_1^{**}\mu_1 - \beta_2^{**}\mu_2 + \beta_3^{**}\mu_1\mu_2$	$\beta_1^{**} - \beta_3^{**}\mu_2$	$\beta_2^{**} - \beta_3^{**}\mu_1$	β_3^{**}

The expressions above lead directly to calculations of corresponding variances and standard errors for the estimated coefficients in the various models, either solving for the variance of a coefficient in a centered model in terms of variances and covariances of coefficients in the uncentered model, or vice versa. For example, since $b_1^* = b_1 + b_3\overline{X}_2$, we have

$$\mathrm{Var}(b_1^*) = \mathrm{Var}(b_1) + \overline{X}_2^2\mathrm{Var}(b_3) + 2\overline{X}_2\mathrm{Cov}(b_1, b_3). \tag{2}$$

Note that the covariance term in the expression above is likely to be negative, and thus potentially yield a standard error that is lower than that from the uncentered model. This follows from the result that predictor variables that are positively (negatively) correlated have estimated coefficients that are negatively (positively) correlated. Similarly, although the standard error for b_1 in the uncentered interaction model is likely to be higher than that in the simple additive model, there may exist values of X_2 where the standard error for a particular conditional effect $b_1 + b_3X_2$ may be less than the standard error of b_1 in the simple additive model. Moreover, one may show that the standard error of a conditional effect for X_1 in the uncentered interaction model is minimized when $X_2 = \mathrm{Cov}(b_1, b_3)/\mathrm{Var}(b_3)$ (Friedrich 1982). Since the covariance term is only zero if X_1 is independent of X_1X_2 (Kmenta 1971) and this rarely occurs, this explains why the standard error of b_1 in the uncentered model is usually higher than that in centered models.[11]

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[11]The minimal conditional standard error in the interaction model equals the corresponding standard error in the additive model multiplied by the ratio of the standard error of estimate from the interaction model to that from the additive model (Friedrich 1982); and since this ratio is by definition less than one, there exists

For an illustration of these models in practice, we consider kidney function data from 33 male subjects where the predictor variables are serum creatinine concentration (SEC, mean = 1.25 mg/dL), age (mean = 56 yrs), and weight (mean = 72 kg) and the dependent variable is creatinine clearance (CCL, mean = 85.2 ml/min) (Kutner et al. 2004).[12] The results for employing only serum creatinine concentration (SEC) and age as predictor variables when fitting Models 1 through 3 are shown in Table 3, with the simple additive model included for reference.

Table 3.   Estimated regression coefficients for kidney function data

	Intercept	SEC Slope	AGE Slope	SEC*AGE Interaction
M0 (additive)	176.24 (10.23)	−43.41 (7.12)	−0.66 (0.18)	− −
M1 (no centering)	232.65 (37.38)	−93.28 (32.59)	−1.57 (0.61)	0.77 (0.49)
M2 (AGE centered at mean)	145.03 (9.77)	−50.09 (8.15)	−1.57 (0.61)	0.77 (0.49)
M3 (both centered at mean)	82.42 (3.25)	−50.09 (8.15)	−0.60 (0.18)	0.77 (0.49)

In the uncentered model, the large standard errors follow from the high correlations between SEC and AGE with their product term: 0.9 and 0.8, respectively. The corresponding correlations in the centered model are 0.5 and 0.07. Moreover, the results illustrate the aforementioned behavior in the estimates and standard errors, and the mappings between the coefficients can be verified via Table 1. We note that the standard errors for the coefficients in Model 1 are the same regardless of whether they are obtained directly from Model 1 or indirectly via a linear combination of terms from Model 3, e.g., $b_2 = b_2^{**} - b_3^{**}\overline{SEC}$. Recall the lower-order coefficients in Model 1 represent conditional effects of a variable when the other variable is equal to zero. Tate (1984) remarks that for the "central mean effects", i.e., the conditional effects when the other variable is equal to its mean, the effects are estimated just as precisely regardless of whether the centered or uncentered model is used, and this is indeed verified for these data. Further insight is obtained by plotting the conditional regression line of creatinine clearance (CCL) versus SEC for several AGE values as in Figure 3. Taking this one step further, Figure 4 is a plot of the actual CCL-SEC conditional slope versus the continuum of AGE values along with corresponding confidence bands, where one may attempt to assess significance via non-overlap with the horizontal zero line. For these data the range of significance essentially corresponds to the range of the AGE values, and this significance should not be confused with the lack of significance of the interaction term.[13] Both plots are instructive: The former is a direct illustration of

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a conditional standard error in the interaction model that is less than the standard error in the simple additive model.

[12]Creatinine is a break-down product of creatine phosphate in muscle and is usually produced at a fairly constant rate by the body. Creatinine is mainly filtered out of the blood by the kidneys and creatinine levels in blood and urine may be used to predict creatinine clearance (CCl) which reflects renal function.

[13]The confidence bands in Figure 4 are drawn without consideration to multiple testing and are thus valid

the relationship between CCL and SEC and contains information on the range of the SEC, values but is limited to pre-selected discrete AGE values. The latter focuses on the conditional slope itself and its potential statistical significance across the range of all AGE values and also indicates where the conditional slope is estimated most precisely. See Section 4 (Point 2) for further discussion of regions of statistical significance.[14]
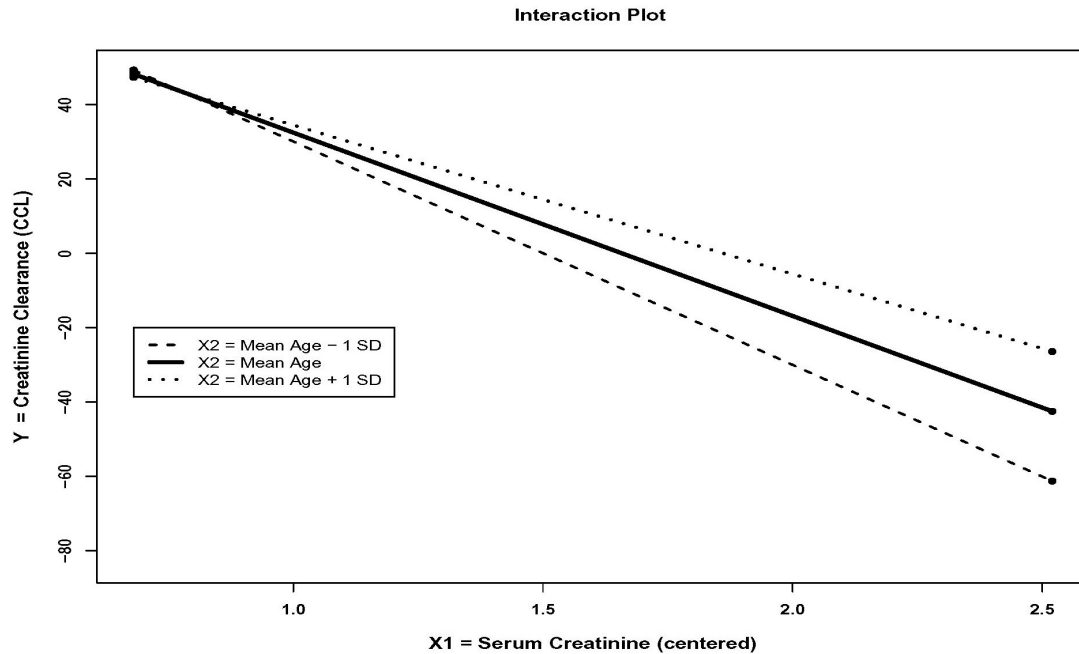

Figure 3.   Interaction plot for Model 3.

4. Key Aspects and Recommendations for Applied Practice

In general, it is recommended to consider centering continuous predictor variables in applied practice to enhance interpretability of coefficients and reduce numerical instability. Although some authors advocate standardized transformations for regression predictors (i.e., subtracting the mean and dividing by one standard deviation), this topic is often controversial (for an annotated bibliography see Firth 1998). For interaction models with centering, one should be careful when presenting results as the potential for mis-interpretation

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for a single test only. These would have to be modified via a suitable multiple testing method such that inferences would be valid for a specific number of values for the moderator X_2 and a given familywise error (the probability of at least one false rejection); see Bauer and Curran (2005) for details.

[14]Figure 3 and Figure 4 were generated via java applets that generate corresponding R code (R-Project 2008); See Bauer and Curran (2005) for details.
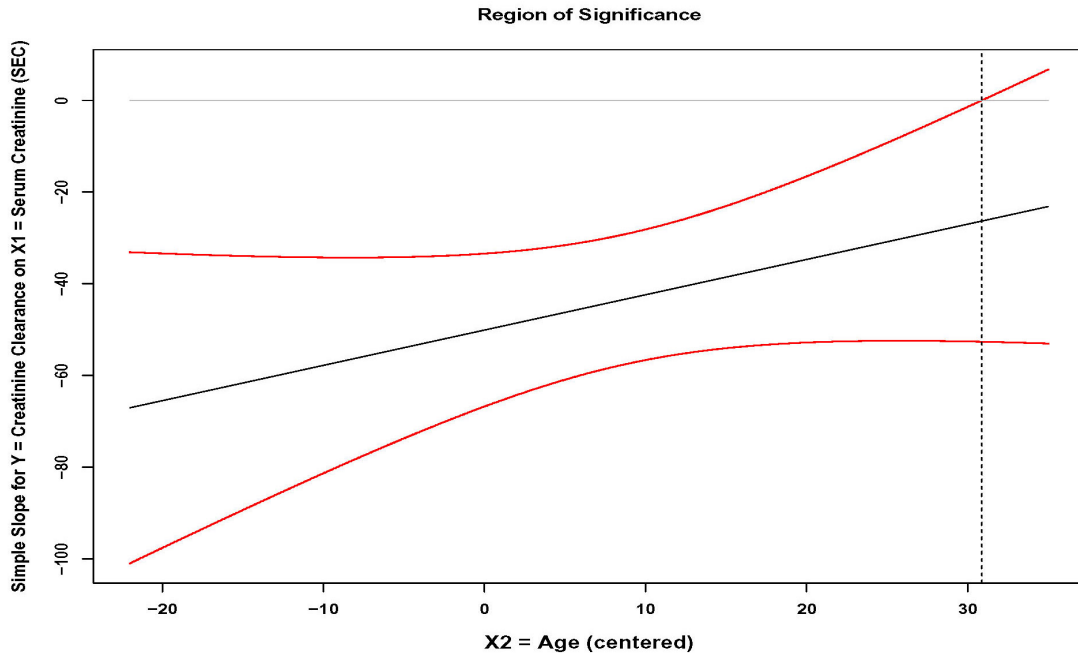
Region of Significance

Figure 4. Plot of conditional slope of Y (creatinine clearance) on X_1 (serum creatinine) versus X_2 (centered weight) for Model 3. The region of significance corresponds to the X_2 values for which the confidence bands do not overlap with zero. Checking whether the confidence bands overlap zero at a particular X_2 value assesses the Y versus X_1 relationship for that particular X_2 value.

is high. We provide the following key points and recommendations:

Remark 1. There are no "main effects"

It is fairly common for the lower-order coefficient estimates in an interaction model to be referred to as "main effects", e.g., the estimate b_1 for the model in Equation 6. Such language increases the chances that b_1 is not understood to be a conditional effect, i.e., the effect of X_1 only when $X_2 = 0$. Indeed, all effects in an interaction model are conditional, and with continuous variables there exists an infinite number of conditional effects and there is no reason why one of them should be called the main effect. Moreover, once one uses the term "main effect", one is inherently driven towards assessing its particular statistical significance and not the range of significance (see below). The same arguments apply for the centered models, as b_1^* and b_2^* still represent conditional effects, albeit conditioned upon different values. If one does not call them main effects, one option is to call them "linear effects" or "conditional main effects", thereby emphasizing their inherently

conditional nature.

Remark 2.   There is a range of significance

If one realizes that there exists a continuum of conditional effects instead of a single "main effect", one could determine the subset within this continuum that the variable is statistically significant. Specifically, since the effect of X_1 on Y can be expressed as a function of X_2, i.e., $b_1 + b_3 X_2$, not only does the effect change at each point, so does the standard error (see Equation 2), t-value, and hence p-value. Thus, one may determine the values of X_2 for which the effect of X_1 on Y is statistically significant, and similarly for the effect of X_2 on Y. A useful exercise is to graph $b_1 + b_3 X_2$ versus X_2 with individual or joint confidence bands and examine for which values of X_2 the confidence bands do not overlap with zero (see Tate 1984; Bauer and Curran 2005 for details). Such a plot not only illustrates the range of significance but also provides information regarding the values of the moderator (X_2) for which we have the most confidence (via the width of the confidence interval) in estimating the conditional slope (see Figure 3).

It is instructive to consider the above suggestion with respect to the so-called Johnson-Neyman method. Specifically, while analysis of covariance (ANCOVA) is commonly employed to assess differences between groups when controlling for a continuous covariate, the Johnson–Neyman method determines the range of values for the continuous control variable(s) for which the conclusion of significant mean differences holds (Johnson and Fay 1950; Johnson and Neyman 1936; Abelson 1953). For example, when controlling for two continuous covariates the region of significance would be a two-dimensional region. Unlike the usual case for interaction models, however, these control variables are of secondary interest and only serve to properly adjust the mean comparison between the groups via essentially matching on known characteristics. Moreover, if there is no interaction between such characteristics and group membership then the region of significance for the mean difference does not depend upon such characteristics and hence the Johnson–Neyman technique is unnecessary. Note that the focus of the Johnson–Neyman technique is on group mean differences as opposed to conditional slopes. However, if X_1 is a dichotomous variable defining group membership and X_2 is continuous, then the region of significance for the conditional slope $b_1 + b_3 X_2$ (of the X_1 variable) is equivalent to the region of significance for the mean difference between the groups. This is because the regression slope b_1 defines a conditional mean difference between the groups given the coding of X_1. For the general case where $X_2, \ldots, X_n$ are continuous, and we only have two-way interactions, it follows that the $(n-1)$-dimensional region of significance of the conditional slope for X_1 (which depends on $X_2, \ldots, X_n$) is equivalent to the $(n-1)$-dimensional region of significance for the mean difference between the groups. For complex models the correspondence between the region of significance for the mean difference and that of a conditional slope

may not be as clear.

Remark 3. Different models do not induce multicollinearity and "change" coefficient estimates

The change in the estimated regression coefficients of an additive model upon introducing an interaction term is commonly attributed to the multicollinearity introduced by adding X_1X_2 to the additive model. This stems from a misunderstanding of the difference between the reason for the changing coefficients and the ramifications of multicollinearity. Multicollinearity is reflected in large standard errors in estimated regression coefficients, and this essentially means that the coefficients are not very stable, i.e., small changes to the data can result in large changes in the estimated coefficients (sample to sample variation). When we go from the additive model to the interaction model, not only do the estimated coefficients correspond to different parameters with different meaning, we are using the same data, so regardless this "change" in the estimated coefficients has nothing to do with multicollinearity.

Remark 4. Multicollinearity does not result in low power in an interaction model

The potential for high probability of Type II error (and thus lower power) in testing for the existence of interaction effects is often recognized (Zedeck 1971; Morris et al. 1986; Chronbach 1987; Champoux and Peters 1987; Cortina 1993; McClelland and Judd 1993). Such potential low power, however, is often attributed to multicollinearity, and some authors have offered solutions to this "problem" (Morris et al. 1986; Cortina 1993). However, although reducing multicollinearity has benefits as discussed previously, those benefits do not include increased power when testing for an interaction effect via the standard method of assessing the statistical significance of the change in the coefficient of determination ΔR^2 between the simple additive model and the interaction model (this also corresponds to the t-test for the interaction coefficient in the full model). Indeed, while a linear transformation like centering can reduce multicollinearity, the test for interaction in the transformed model is identical to the test for interaction in the interaction model without centering, i.e., the test is invariant with respect to a linear transformation. Thus, multicollinearity does not reduce the power of the standard test (Cohen 1978; Arnold and Evans 1979; Chronbach 1987; Dunlap and Kemery 1987). On the other hand, the joint distribution of the predictor variables does indeed affect the power to detect interactions and this explains how tests for interactions in field studies often exhibit substantially less power than those in optimally designed experiments (McClelland and Judd 1993; Schepanski 1983; Stone-Romero et al. 1994).[15] Other valid concerns for low power in detecting interaction effects relate to measurement error in individual predictor variables that is compounded when forming X_1X_2 to

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[15]It should be emphasized that the comments above apply to the case where the interaction model represents the true model; otherwise, see Point 4.5.

produce lower reliability of the product term (Dunlap and Kemery 1988; MacCullum and Marr 1995; Busmeyer and Jones 1983).

Remark 5.   Multicollinearity does matter for curvilinear terms: Type I error (spurious interactions) and Type II error (lower power)

Several authors have considered the issue of the relationship between interaction and quadratic effects (i.e., higher powers X_i^2) (MacCullum and Marr 1995; Busmeyer and Jones 1983; Lubinski and Humphreys 1990; Shepperd 1991). Initial focus was on spurious (false positive) interactions since the likelihood of Type I error is indeed increased when the true model contains these higher-order terms and only the cross-product interaction term is included in the estimated model (Busmeyer and Jones 1983; Lubinski and Humphreys 1990; Shepperd 1991). In addition, there exist scenarios where this omission produces an estimated interaction effect in the wrong direction when such interaction effects do indeed exist, i.e., indicating a synergistic/reinforcing interaction when it should be an off-setting/interference interaction, and vice versa (similar misleading results may occur for the quadratic terms when they do indeed exist and the cross-product term is omitted, i.e., indicating a concave relation when it should be convex, and vice versa) (Ganzach 1997).

Spurious and misleading estimates are not the only danger, however, as the probability of Type II error for the interaction effect may increase when quadratic terms are added when the true model does *not* include quadratic terms, and also when quadratic terms are omitted when the true model *does* include quadratic terms (Ganzach 1998). These results are dependent upon the degree of multicollinearity, where the main message is that the increase in Type II error is only substantial when multicollinearity is very high (i.e., above 0.7) (Ganzach 1997, 1998). Thus, while reducing multicollinearity is not relevant to interaction detection as discussed above in Point 4.4 (where the true model is the interaction model and the interaction model is estimated), it is relevant to the present discussion of curvilinear terms and the assessment of corresponding mis-specified models. As in practice one does not know if the "true" model contains quadratic terms, whether or not to include them depends on the relative costs of Type I and Type II error. As recommended by Ganzach (1998), if multicollinearity is not very high, one strategy would be to include quadratic terms: this provides protection against Type I and Type II error associated with the interaction when the true model contains quadratic terms, and if the true model does not contain quadratic terms the resulting increase in Type II error (and loss of power) is relatively small.

Remark 6.   Correlation interactions are not the same as regression interactions

In addition to interaction effects in linear regression, it is common to observe similar discussions for correlation coefficients where for example the correlation between two vari-

ables is stratified per levels of a third variable. For instance, it is common in medical statistics for this third variable to be a continuous variable made discrete by partitioning into "high" and "low" groups. Many authors have cautioned against the arbitrariness and loss of information of such an approach (Cohen 1978; Cohen et al. 2003; Altman 1991). Thus, it is important to recognize the different types of information conveyed by correlation coefficients and regression coefficients to avoid confusion. While the square of the correlation coefficient r_{XY}^2 indicates the percentage of Y variance explained by X variance, the corresponding regression coefficient indicates the expected change in Y for a unit change in X ($r_{XY} = b_{YX}$ only when $\sigma_Y = \sigma_X$). While it has been suggested that testing for interaction in linear regression yields information regarding correlation interactions (Stone and Hollenbeck 1984), this has been clearly shown to be incorrect as the regression interaction tells us nothing regarding whether or not correlations differ across a moderator variable (Arnold 1982, 1984). Indeed, Arnold (1982) contains applied examples where an interaction exists for the regression but not the correlation, and vice versa. An excellent theoretical example therein to underline this point employs a rectangle where Y = area, X_1 = width, and X_2 = length: for each level of length, the correlation between area and width equals one and thus does not change, whereas the regression coefficient (the change in area for a one unit change in width) depends on the length; hence the interaction exists for the regression coefficient but not for the correlation. Another useful tool to clarify the distinction between an interaction for a correlation versus that for a regression coefficient is Figure 5 from Hartmann and Moers (1999), where correlation and regression are referred to as strength and form, respectively. As shown in the figure, a strength interaction involves a change with respect to the tightness around the same directional relationship, whereas a form interaction involves a change in the direction of the relationship. For example, the relationship between income and education might have the same form but different strength in males versus females, or the relationship might have the same strength but different form.

Remark 7.   Standard errors have not "increased"

In addition to the common admonitions about changing regression coefficients, students are often frustrated by the "increased" standard errors in the interaction model as compared to the simple additive model. However, the standard error has not really *increased*, i.e., the b_1^{**} in Model 3 is estimating a different parameter than the b_1 in Model 1, so it is just as erroneous to state that the standard error has increased as it is to state that the estimated coefficient has changed. To be sure, the standard errors of b_1^* and b_2^* may indeed be large, but this only means that this one particular data set does not allow the precise estimation of these particular parameters, and that small changes to this particular data set can result in larger changes to these estimated parameters. Centering may be used to estimate coefficients with lower standard errors, but once again it must be noted that these estimates apply to different population parameters. One should be just as careful with re-
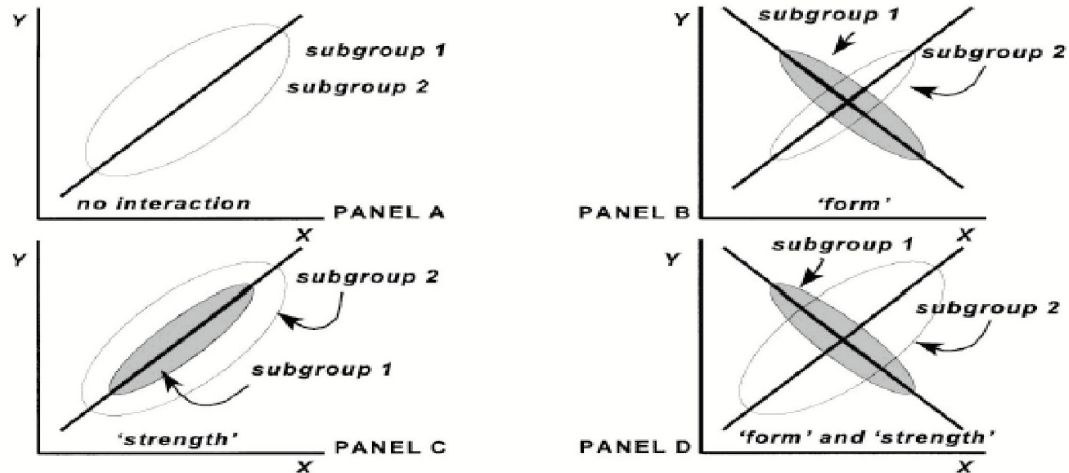
Figure 5. Illustration of the difference between strength (correlation) and form (regression), reproduced from Hartmann and Moers (1999) (Figure 3) with permission of Elsevier.

spect to statements about comparing standard errors as one is with respect to statements about comparing estimates from different models.

Remark 8. No need to estimate centered model results from uncentered model

Related to the previous points, one often finds in the literature the suggestion that the coefficients of the uncentered model can be better estimated from the coefficients of the centered model via linear combinations as shown by the horizontal brackets in Models 2–3, e.g., see Smith and Sasaki (1979). The motivation offered is that the centered model has less multicollinearity and thus lower standard errors, and thus the estimates of the original uncentered model are estimated with reduced numerical error. Several points are worth noting here. First, regarding the point estimates of the coefficients in the uncentered model, they are exactly the same regardless of whether they are estimated via the uncentered model or via the linear combinations of coefficients estimated from the centered model. Second, as noted above, one must be careful by saying that the coefficients of the centered model have lower standard error, since the coefficients pertain to different population parameters and thus the standard errors should not be directly compared in terms of one being lower than the other. Third, it is somewhat pointless to calculate the coefficients of the original model in such a manner in the first place. The concepts represented by the coefficients b_1 and b_2 of the original model, viz., conditional effects of X_1 and X_2 when $X_2 = 0$ and $X_1 = 0$, respectively, also exist in the centered model. There is no need to go back to the original model in practice and there is nothing to be gained from a practical standpoint. The mappings of the coefficient estimates between the different types of models are instructive

from a theoretical standpoint, but certainly not as a practical guide for doing statistics.

Remark 9. The simplest moderator variable is an indicator or dummy variable

A point that is often lost amidst the many points about interaction models is that the concept of an interaction effect directly applies to the case of dummy or indicator variables. For example, if we are trying to predict urinary potassium excretion and X_1 is a continuous variable such as serum potassium level and X_2 is a dummy variable for diabetes, adding the product term X_1X_2 assesses whether the effect of serum potassium level depends on diabetic status. If this is the case, the conditional nature of X_1 is anchored to only two values, the value for diabetic versus non-diabetic, whereas if X_2 were a continuous variable there would exist a continuum of conditional effects as discussed earlier. Furthermore, the distinction can be made that the dummy variable case represents a scenario where variable centering should not be used since the mean value of the dummy variable has no practical meaning. After teaching dummy variables in a regression setting, the student has already been exposed to interaction effects, so the case of interaction effects for continuous variables should not be introduced as a completely new topic. This is a problem with statistics education in general, as the student is often frustrated by the impression that there are countless topics, when indeed there really are only a few big concepts that have small variations for different cases.

Remark 10. An interaction model represents a curved surface

When introducing multiple regression, most standard statistics texts make the point that while bivariate regression is a model of a line in two dimensions, multiple regression with two predictor variables represents a model of a plane in three dimensions (and a hyperplane in $p+1$ dimensions when there are p predictor variables). When an interaction effect is present in a model, the plane is no longer flat, but rather curved or warped and the degree of this distortion from flatness is quantified via b_3. Moreover, the shape of the surface does not change when centering is introduced, it is merely shifted in scale (for illustrations see Cohen et al. 2003, p. 259). This can be used to emphasize that the interaction coefficient does not change from the uncentered to the centered model, and that the interaction coefficient quantifies the degree of warping in the regression surface. Moreover, centering does not change the predicted values.

Remark 11. The average effect is worth considering

When one centers both predictor variables and includes the corresponding interaction term, the coefficients for the lower-order terms have a new meaning as well, viz., as the *average* effect of the given predictor variable on Y across the range of the other predictor variables. For example, this can be shown manually by regressing Y on X_1 separately for each value of X_2. Assuming there exists an equal number of observations at each X_2 level, then averag-

ing the resulting coefficients yields the estimated coefficient of X_1 in the centered multiple regression; otherwise, the same result is obtained by weighting the average via the number of observations at each level (Cohen et al. 2003, p.261). A natural extension to this method is Gelman and Pardoe (2007), where regression coefficients are augmented with a new measure, *average predictive comparisons*, obtained as the mean value of the predictive comparison over some specified distribution of the inputs and parameters. Note that predictive comparisons (i.e., the expected change in the response variable for a given change in a predictor variable assuming all other predictors are held constant) directly correspond to regression coefficients for additive models.

5. Summary

We have revisited simultaneous variable centering and interaction effects in linear regression. Our compilation represents a reference that covers a wide range of issues and highlights common mistakes and misunderstandings in the literature. While many of these points have been made elsewhere, they are somewhat scattered across a voluminous literature and are not easily extracted into a useful format. Although the focus has been on linear regression, similar issues could be considered for more elaborate models such as mixed-effects or generalized linear models.

REFERENCES

Abelson, R. (1953). A note on the Neyman-Johnson technique. *Psychometrika*, 18(3):213–218.

Aiken, L. and West, S. (1991). *Multiple Regression: Testing and interpreting interactions*. Sage, Thousand Oaks.

Allison, P. (1977). Testing for interaction in multiple regression. *American Journal of Sociology*, 83:144–153.

Altman, D. (1991). *Practical Statistics for Medical Research*. Chapman & Hall, London.

Arnold, H. (1982). Moderator variables: A clarification of conceptual, analytic, and psychometric issues. *Organizational Behavior and Human Performance*, 29:143–174.

Arnold, H. (1984). Testing moderator variable hypotheses: A reply to Stone and Hollenbeck. *Organizational Behavior and Human Performance*, 34:214–224.

Arnold, H. and Evans, M. (1979). Testing multiplicative models does not require ratio scales. *Organizational Behavior and Human Performance*, 24:41–59.

Baron, R. and Kenny, D. (1986). The moderator-mediator variable distinction in social psychological research: Conceptual, strategic and statistical considerations. *Journal of Personality and Social Psychology*, 51(1):1173–1182.

Bauer, D. and Curran, P. (2005). Probing interactions in fixed and multilevel regression: Inferential and graphical techniques. *Multivariate Behavioral Research*, 40:373–400.

Bauer, D., Preacher, K., and Gil, K. (2006). Conceptualizing and testing random indirect effects and moderated mediation in multilevel models: New procedures and recommendations. *Psychological Methods*, 11:142–163.

Busmeyer, J. and Jones, L. (1983). Analysis of multiplicative combination rules when the causal variables are measured with error. *Psychological Bulletin*, 93(3):549–562.

Carte, T. and Russell, C. (2003). In pursuit of moderation: Nine common errors and their solutions. *MIS Quarterly*, 27(3):479–501.

Champoux, J. and Peters, W. (1987). Form, effect size and power in moderated regression analysis. *Journal of Occupational Psychology*, 60:243–255.

Christensen, R. (2002). *Plane Answers to Complex Questions: The Theory of Linear Models*. Springer, New York, third edition.

Chronbach, L. (1987). Statistical tests for moderator variables: Flaws in analyses reported. *Psychological Bulletin*, 102(3):414–417.

Cohen, J. (1978). Partialed products *are* interactions; partialed powers *are* curve components. *Psychological Bulletin*, 85:858–866.

Cohen, J., Cohen, P., West, S., and Aiken, L. (2003). *Applied Multiple Regression/correlation analysis for the behavioral sciences*. Lawrence Erlbaum Associates, Hillsdale, NJ, third edition.

Cortina, J. (1993). Interaction, nonlinearity, and multicollinearity. *Journal of Management*, 19(4):915–922.

Dunlap, W. and Kemery, E. (1987). Failure to detect moderator effects: Is multicollinearity the problem? *Psychological Bulletin*, 102(3):418–420.

Dunlap, W. and Kemery, E. (1988). Effects of predictor intercorrelations and reliabilities on moderated multiple regression. *Organizational Behavior and Human Decision Processes*, 41:248–258.

Firth, D. (1998). Annotated bibliography: Relative importance of explanatory variables. http://www.nuff.ox.ac.uk/sociology/alcd/relimp.pdf, Oxford: Nuffield College.

Friedrich, R. (1982). In defense of multiplicative terms in multiple regression equations. *American Journal of Political Science*, 26(4):797–833.

Ganzach, Y. (1997). Misleading interaction and curvilinear terms. *Psychological Methods*, 2(3):235–247.

Ganzach, Y. (1998). Nonlinearity, multicollinearity and the probability of type II error in detecting interaction. *Journal of Management*, 24(5):615–622.

Gelman, A. and Pardoe, I. (2007). Average predictive comparisons for models with nonlinearity, interactions, and variance components. *Sociological Methodology*, 37(1):23–51.

Goldberger, A. (1964). *Econometric Theory*. Wiley, New York.

Harrell, F. E. (2001). *Regression Modeling Strategies*. Springer, New York.

Hartmann, F. and Moers, F. (1999). Testing contingency hypotheses in budgetary research: an evaluation of the use of moderated regression analysis. *Accounting, Organizations and Society*, 24:291–315.

Johnson, P. and Fay, L. (1950). The Johnson-Neyman technique, its theory and application. *Psychometrika*, 15:349–367.

Johnson, P. and Neyman, J. (1936). Tests of certain linear hypotheses and their applications to some educational problems. *Statistical Memoirs*, 1:57–93.

Kmenta, J. (1971). *Elements of Econometrics*. Macmillan, New York.

Kraemer, H., Kiernan, M., Essex, M., and Kupfer, D. (2008). How and why criteria defining moderators and mediators differ between the Baron & Kenny and MacAuthor approaches. *Health Psychology*, 27(2):S101–S108.

Kutner, M., Nachtsheim, C., and Neter, J. (2004). *Applied Linear Regression Models*. McGraw Hill, New York, fourth edition.

Lubinski, D. and Humphreys, L. (1990). Assessing spurious "moderator effects": Illustrated substantively with the hypothesized ("synergistic") relation between spatial and mathematical ability. *Psychological Bulletin*, 107:385–393.

MacCullum, R. and Marr, C. (1995). Distinguishing between moderator and quadratic effects in multiple regression. *Psychological Bulletin*, 118:405–421.

MacKinnon, D. and Luecken, L. (2008). How and for whom? Mediation and moderation health psychology. *Health Psychology*, 27(2):S99–S100.

Marquardt, D. (1980). You should standardize the predictor variables in your regression models. *Journal of the American Statistical Association*, 75:74–103.

McClelland, G. and Judd, C. (1993). Statistical difficulties of detecting interaction and moderator effects. *Psychological Bulletin*, 114(2):376–390.

Morris, J., Sherman, J., and Mansfield, E. (1986). Failures to detect moderating effects with ordinary least squares-moderated multiple regression: Some reasons and a remedy. *Psychological Bulletin*, 99:282–288.

R-Project (2008). R - *Version 2.7.1*. The R Project, http://www.r-project.org/.

Saunders, D. (1955). The "moderator variable" as a useful tool in prediction. *Proceedings of the 1954 Invitational Conference on Testing Problems*, pages 54–58.

Saunders, D. (1956). Moderator variables in prediction. *Educational and Psychological Measurement*, 16:209–222.

Schepanski, A. (1983). The predictive ability criterion in experimental judgement research in accounting. *Decision Sciences*, 14:503–512.

Shepperd, J. (1991). Cautions in assessing spurious "moderator effects". *Psychological Bulletin*, 110:315–317.

Smith, K. and Sasaki, M. (1979). Decreasing multicollinearity: A method for models with multiplicative functions. *Sociological Methods and Research*, 8(1):35–56.

Stone, E. and Hollenbeck, J. (1984). Some issues associated with the use of moderated regression. *Psychological Bulletin*, 34:195–213.

Stone-Romero, E., Alliger, G., and Aguinis, H. (1994). Type II error problems in the use of moderated multiple regression for the detection of moderating effects for dichotomous variables. *Journal of Management*, 20(1):168–178.

Tate, R. (1984). Limitations of centering for interactive models. *Sociological Methods and Research*, 13(2):251–271.

Zedeck, S. (1971). Problems with the use of "moderator" variables. *Psychological Bulletin*, 76:295–310.

David Afshartous, Ph.D.
Department of Biostatistics
Vanderbilt University Medical Center
Nashville, TN 37203
Email for correspondence: d.afshartous@vanderbilt.edu

Richard A. Preston, M.D., M.S.P.H., M.B.A.
Division of Clinical Pharmacology
Department of Medicine
Miller School of Medicine
University of Miami
Miami, FL 33136

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