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Teaching Bits: "Random Thoughts on Teaching"

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Rejoinder to "Let's Just Eliminate the Variance"

In the letter "Keep Teaching the Variance" the author makes some excellent counterpoints regarding my "Random Thoughts on Teaching" column in the November 2009 issue. The title of that column was "Let's Just Eliminate the Variance." I very much appreciate the author's letter, and I'm happy to have the opportunity to respond.

The author's main point is that variance should not be excluded from what I'll refer to as "Chapter 1" of a typical introductory statistics course. Among the author's arguments for including variance in "Chapter 1" are 1) because variance itself can help students understand standard deviation; and 2) because its early introduction provides for better understanding of SSE (in linear models) and ANOVA.

With regard to the need for a good concept of variance to understand sums of squares, I am entirely in agreement with the author. In truth, the main focus of my column (although not explicitly stated) was on my one-term course, which ends before ANOVA is introduced. However, even if ANOVA were included in my course, I would still disagree with the author's rationale for including variance in Chapter 1 – my reasoning follows.

In my opinion the main objective of Chapter 1 is describing data in terms of shape, center and spread using graphs and descriptive statistics. The sample variance, as a descriptive statistic, is not helpful to understanding spread because variance is in square units and hence cannot be

interpreted. I think Chapter 1 should focus on statistics that are commonly used as well as interpretable in the context of the data, with standard deviation being one of those statistics.

Including variance as a measure of spread is somewhat similar to including the mode as a measure of center. Twenty years ago every statistics textbook included the mode, and some still do. But the majority of today's books and courses leave it out. The mode doesn't often represent our modern day interpretation of center: a balancing point in terms of numerical values or location within a data set. I argue that the same thing has happened to our use of the variance in Chapter 1. Since it's not interpretable, it's being included less and less often. Now whether it should still be mentioned at all, even though it isn't as meaningful as standard deviation, is another topic for debate; my opinion is that if it's not core to understanding and interpreting spread, we should leave it out.

However regarding the units issue, it is only fair to point out that the way measures of spread are presented and used in Chapter 1 of a typical textbook and course is also a problem. Most books that include variance don't emphasize the fact that it is in square units; however, they also don't emphasize that standard deviation is in original units. Indeed, the concept of units often disappears altogether when it comes to presenting and discussing measures of spread. This is a shame, because we spend a great deal of energy telling students that a measure of center is not enough to describe a data set, yet we often fail to include and/or emphasize units in the interpretation of spread.

The concept of spread is a very difficult one for introductory level students to grasp, especially in Chapter 1. (I am not certain that the word 'spread' is even a good word to use. I have better success using words like 'diversity' or 'concentration' around the mean.) We certainly need to concentrate on ways for students to understand variability - this is one of the main elements of the GAISE guidelines (Garfield, et al, 2005). However, variability as a concept and variance as a statistic are not one and the same.

That said, I don't think the issue is whether or not we think variance is important; the main point of discussion between the author and I is its placement in the course. David Moore once told me when it comes to teaching statistics, don't go off on a side road if you can help it, and don't cover a topic you aren't going to use later unless it's a really big deal. As a result, my teaching mantra centers on the use of "big ideas and common threads" throughout my courses. I try not to let anything get in the way of the main ideas, and I try not to take side trips or leave time gaps that are so long that students lose the connection you want them to make later. This approach has been helpful to me over the years.

However, if one must include the variance in Chapter 1, I think standard deviation should be covered first as the main attraction, followed by a quick mention of variance, alluding to its major significance later in the course. If you think about it, the notation for variance is "s-squared", so one can argue that it makes good mathematical as well as statistical sense to present "s" first, and then talk about "s-squared". Perhaps that's why early statisticians used "s" for standard deviation rather than for variance. I don't know if David would agree with me on this one, but I don't think sample variance as a descriptive statistic in Chapter 1 is one of the big

ideas, and including it in Chapter 1 and not again until ANOVA and/or regression (if your course reaches point) does not follow the "common threads" approach. The time gap is just too long.

There is also an issue of added understanding when you think about placement of variance in Chapter 1 vs. later on (e.g. the chapter on regression and SSE). The author points out that "the student who has not yet learned about the variance is at a disadvantage in understanding the notion of a least-squares best fit line." I disagree. If students understand the concept of spread (and standard deviation in particular) well enough and they have used it in other situations such as the normal distribution, and the 68-95-99.7 rule (aka Empirical Rule) I believe they can understand the idea of the sum of squared differences around a mean, or from a given model. At the point in the course where SSE and/or ANOVA comes into the picture, discussing sums of squares rather than the square root of sums of squares is quite relevant and takes only a small jump in statistical reasoning. Students are more established in their knowledge at this point in the course so they can likely handle the ideas of variance and sums of squares presented in conjunction with each other better than one might think. This would be an interesting topic for statistical researchers to explore.

Regarding ANOVA, the partitioning of the sums of squares has a very geometric and visual connotation, where areas of actual squares can be used to make the point in a natural way, based directly on square units. In this setting, whether a student has already covered sample variance in Chapter 1 is less of an issue in my opinion.

The debate over the "whether and when" to teach variance will no doubt continue; but in a broader sense the author's letter reemphasizes the importance of periodically revisiting and rethinking the concepts we introduce, and when and how we introduce them. In our efforts to improve statistics teaching by including real data, hands-on activities, new technologies and more effective pedagogical elements, we must remember that in the end, our goal is for students to take away a set of big ideas that help them understand, reason with, and solve problems using statistics in their careers and everyday lives.

Our list of the big ideas we want students to take away from our courses will change over the years (indeed, the theme of the upcoming United States Conference on Teaching Statistics – USCOTS 2011 – is "The Next Big Thing.") However the process by which those big ideas are determined and the approach we take to incorporating them into our courses and textbooks need not change if we follow W. Edward Deming's 14 Points of Management (1986). Among the many of Deming's 14 points that can be directly applied to teaching, perhaps the most important ones are the following:

1. Create constancy of purpose and continual improvement – long term planning must replace short term reaction. (For example, we need to continue examining and modifying our list of the big ideas as time goes on, and let those big ideas drive which day-to-day concepts are included in our courses and textbooks, not the other way around).

2. Build quality into the product and process in the first place. (The GAISE guidelines (Garfield, et al, 2005) provide a template for us to do exactly that.)

3. Improve constantly and forever to reduce variation in all aspects e.g. planning, production, and service. (Aim to create courses of consistently high quality by researching and applying best practices and always looking for ways to improve; discussions such as the one sparked here are an important part of this process.)

I again thank the author for submitting a response to my November, 2009 column, and I appreciate the opportunity to write a rejoinder to it and discuss some general thoughts that arose from it.

As a final note, this is the last entry in the "Random Thoughts on Teaching" column. I've thoroughly enjoyed the opportunity to contribute to JSE through this column, and I'm glad it has generated discussion amongst our readers. I look forward to watching JSE continue to grow and change, and I look forward to reading more papers published by fellow statistics educators who like to push the window, stir the pot, and challenge others' ideas. These efforts help us continue to examine not only the way we teach statistics, but what we teach, and why we teach it.

References

Deming, W. Edwards (1986). Out of the Crisis. MIT Press.

Garfield, J., et al (2005). *Guidelines for Assessment and Instruction in Statistics Education (GAISE)*. The American Statistical Association.
