The Effects of Data and Graph Type on Concepts and Visualizations of Variability

Linda L. Cooper
Felice S. Shore
Towson University

Abstract

Recognizing and interpreting variability in data lies at the heart of statistical reasoning. Since graphical displays should facilitate communication about data, statistical literacy should include an understanding of how variability in data can be gleaned from a graph. This paper identifies several types of graphs that students typically encounter—histograms, distribution bar graphs, and value bar charts. These graphs all share the superficial similarity of employing bars, and yet the methods to perceive variability in the data differ dramatically. We provide comparisons within each graph type for the purpose of developing insight into what variability *means* and how it is evident within the data's associated graph. We introduce graphical aids to visualize variability for histograms and value bar charts, which could easily be tied to numerical estimates of variability.

1. Introduction

The Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report (PreK-12) states that in order to develop the ability to think statistically, students must learn that "statistical problem solving and decision-making depend on understanding, explaining, and quantifying the variability in the data" (Franklin et al., 2007, p. 6). Being able to recognize and understand the ways that variability is manifested in different types of graphs is part of this ability to think statistically. Unfortunately, this area of statistical reasoning may not receive

adequate emphasis during the course of students' exposure to graphical representations. Often the focus of pre-college graph instruction has been on construction of various graphs and identification of overall trend, shape and outliers, without developing a robust understanding of the interplay between statistical concepts (e.g. data type, center and spread) and key graphical features (e.g. meaning of axes) that differ among graph types. As Friel et al., (2001) explain, "g*raph sense* develops gradually as a result of one's creating graphs and using already designed graphs in a variety of problem contexts that require making sense of data" (p. 145). We contend that the questions about graphs that students are asked may be limited, resulting in the failure to bring attention to statistics' key concept of variability. Indeed, in their research on undergraduate students' misconceptions about sampling distributions presented by histograms, Chance, et al., (2004) attributed much of the difficulty students have to their "not fully understand[ing] the meanings of distribution and variability" (p.312). Interestingly, another study on students' conceptual understanding after a first course in statistics by delMas et al., (2007) reported a high success rate on items asking for comparisons of trend and center between distributions represented in histograms; yet, the same study identifies students' inability to recognize critical differences between histograms and other graph types that use bars. Consistent with that finding, Shore and Cooper (2009) reported difficulties that 75 in-service teachers (grades 4-12) enrolled in a graduate program in mathematics education had with identifying different types of graphs. Twenty-eight percent failed to identify a histogram as a histogram, preferring to call it a bar graph, while fifty-one percent referred to all non-histogram graphs that use bars [and will be discussed later in this paper] as simply bar graphs. We maintain that being able to differentiate the underlying structures of different graph types, including identifying the type of data and on which axis it is plotted, is a necessary step to being able to perceive variability graphically.

Pre-college students quickly encounter measures of variability when introduced to the concept of variability in data. We suggest that prior to being introduced to methods of *quantifying variability*, students need experiences in which they simply *compare* the variability of data sets for the purpose of developing an understanding of and insight into what variability *means* for different types of data. *Visual* comparisons of the magnitude of variability encourage this discussion of what makes a data set more or less variable, which necessitates a discussion of how variability is manifested in the particular graph type. In general, variability is perceived differently depending on graph type. Our discussion in this article focuses on *visual comparisons* of variability in three common graph types—value bar charts, histograms, and bar graphs [for categorical data]. We chose these three graphs specifically because 1) they share the superficial similarity of using bars, and yet the role of the bars in representing data differs across graph type; 2) they present examples of qualitative and quantitative data; 3) they are examples of both distribution and non-distribution graphs; and consequently 4) the methods by which variability for each is judged differs. Embedded within our discussion of visual comparisons of variability, we present basic misconceptions found in the literature that hinder students' abilities to interpret and compare some types of graphs.

In each case we have framed the concept of variability in relation to the center of the data as it is perceived in the graph. We contend that in order to strengthen both graph sense and understanding of variability, teachers should explicitly and routinely connect graphical representations, and their underlying structures, to discussions of center and variability. To facilitate such discussion, we suggest how visual conceptions of center and variability can be

more concretely illustrated through the use of visual aids such as those that we introduce, which then also lead to connections to computational measures. We hope that the ideas in this paper gain recognition and utility among classroom teachers at the pre-college level promoting discussion of data, center, and variability whenever different graph types are presented.

2. Comparisons of Variability in Graphs

Before we can discuss visual comparisons of variability in graphical representations, we first need to delineate the underlying structures of various graphs. It is precisely these structures which dictate how we interpret variability. For any given data set there is usually more than one type of graph that can be used to help the reader visually summarize the data. The type of graph is in part determined by the type of data (qualitative or quantitative). For example, bar graphs and pie charts are used to represent the distribution of qualitative (nominal and ordinal) data; histograms, time plots, stem-and-leaf plots, dot (line) plots, box plots, and value bar charts are choices for quantitative (interval and ratio) data. In the following sections we discuss how variability is expressed in the value bar chart and histogram, both used to represent quantitative data, and the bar graph, used to represent qualitative data.

2.1 Value Bar Charts

The value bar chart is often encountered by students as early as kindergarten. It provides an illustration of numerical data in which the height of a bar corresponds to the magnitude of its corresponding data value. The relevance of the ordering of the bars depends on whether the values along the horizontal axis are nominal or ordinal. Only recently has more attention been given to distinguishing these graphs from traditional distribution bar graphs, even though their structure merits an entirely different interpretation of center and spread, as we shall see. The middle school curriculum Connected Mathematics 2 (Lappan et al., 2006) refers to these graphs as "*value* bar graphs," distinguishing them from "*frequency* bar graphs." In other places, such as the National Council of Teachers of Mathematics' Navigations Series, these graphs are called "case-value plots" (Bright et al., 2007). We choose to refer to them as "value bar *charts*" to further differentiate them from distribution bar graphs, while at the same time emphasizing their pictorial nature. In popular media and even in research papers, value bar charts are often used to compare summary data, such as means or proportions, for several groups.

The example we provide in Figure 1 is typical of how value bar charts appear in elementary and middle school curricula. The graphs illustrate the floor plans of two builder companies, Windsor Homes and Vincent Builders. Both builders offer five different floor plans of various areas. The heights of the bars *are* the data, the area in square feet of each floor plan. The mean can be interpreted as the square footage resulting if total area was spread evenly among the floor plans. The mean square footage, the height at which all bars would be level, is 2,340 square feet for both builders.
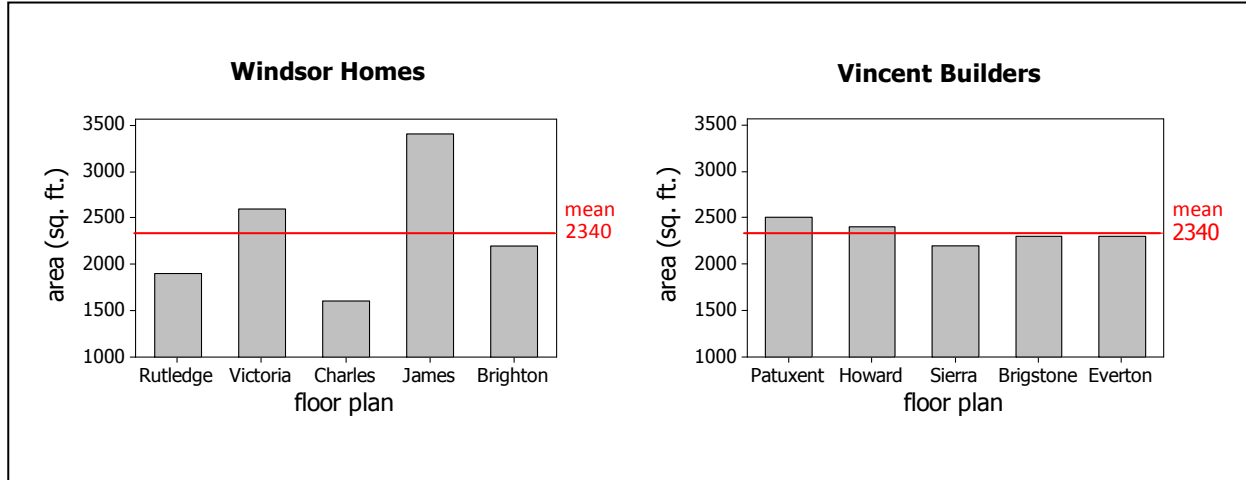
Figure 1. Value bar charts for two home builders; floor plans of Windsor Homes are more variable than those of Vincent Builder.

The comparison of variability of floor plans for the two builders is made by looking at the magnitude of the differences between the heights of the bars and the mean line. The larger these differences, the more variation in the data from the mean. The square footage for the five floor plans from Vincent Builders, depicted as bars, all hover close to the mean line, whereas the square footage, heights of the bars, for Windsor builders' floor plans deviate greatly from the mean line. Thus, Windsor Homes offers floor plans of more variable area than does Vincent Builders.

In Figure 2 we provide a second comparative example, that of time plots, that introduces a visual aid to compare the variability of two data sets. A time plot that uses bars may be considered a special case of the value bar chart, in the sense that center and spread are perceived in the very same manner in the graph. We present an example of time plots because they are ubiquitous in the mass media, and in some cases, the naturally occurring shape of the trend may be misinterpreted as a shape of a distribution, e.g. Beijing's "bell-shaped" rainfall trend. Consider the graphs of Beijing and Toronto, both of which depict the change of rainfall over time. The height of each bar, plotted by month along the horizontal axis, depicts quantitative data, in this case monthly rainfall. Again, variability is judged by deviations from the mean monthly rainfall, the superimposed horizontal line. In our visual aid the vertical deviation lines indicate the differences between each data value and the mean value. The greater the deviations, the more variable the data. The arrow tips shown in the graph for Beijing merely serve as a reminder that deviation refers to distance *from* the mean. Arrow heads are not shown on the bars in Toronto simply because the deviation lengths were too small to accommodate them. As there is little deviation of monthly rainfall from the mean in the Toronto graph as compared to that of Beijing, one can conclude that there is less variability in Toronto's monthly rainfall as compared to Beijing.
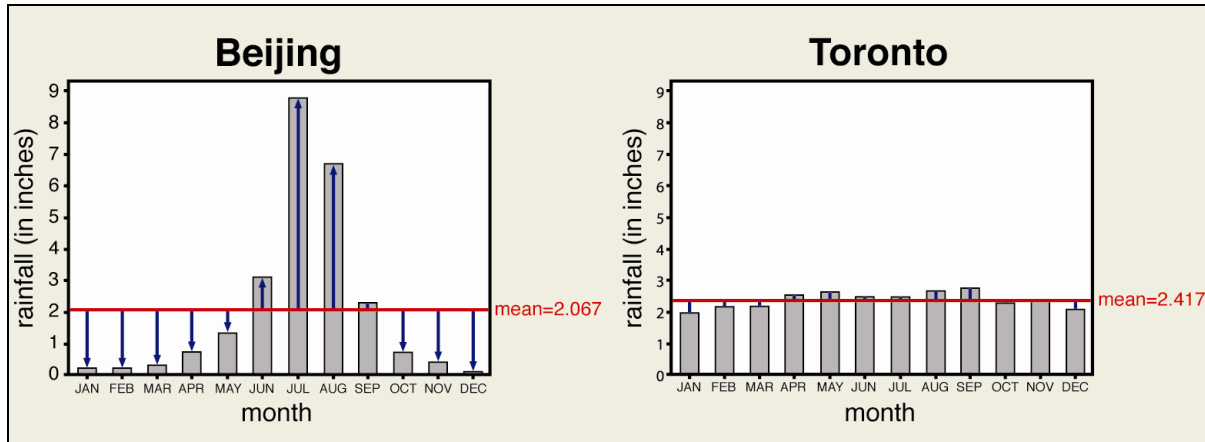
Figure 2. Time plots of monthly rainfall for Beijing and Toronto, arranged from more to less variable [data source: Buttle and Tuttle Ltd., 2007].

The important point for value bar charts and time plots is that variability in the data is perceived through variation in bar heights. In the next section, this correct perception will be juxtaposed with the misconception manifested when learners incorrectly interpret histograms.

2.2 Histograms

The two remaining types of graphs that we discuss, the histogram and the bar graph, both show the *distribution* of values of a single variable, the former quantitative and the latter qualitative. Their use of bars to show distribution necessitates a frequency axis. In order to interpret graphs that show a *distribution* of a variable, one's graph sense must be sophisticated enough to understand the interplay between the data values and their frequencies or relative frequencies. Difficulties have been noted in the research. Friel and Bright (1995) posited that interpreting graphs that utilize two axes (e.g., histograms, scatter plots) may present difficulties "if the functions of the x- and y- axes across these graphs are not explicitly recognized" (p. 3). This concern was heightened by results of their study that found that many middle school students confronted with data represented in a dot plot confused the frequencies (number of Xs) for the data values themselves, despite the fact that dot plots don't have an axis for frequencies. In their study on high school and college students, delMas, et al., (2005) also speculated that students don't actually understand what the axes represent. They reported that students had difficulty reading histograms of grouped data, in comparison to histograms in which bars represented frequencies of single variables. Furthermore, they noted that students confused histograms with value bar charts, incorrectly choosing a "bell-shaped" version of the latter, rather than a correctly corresponding histogram when asked to match a given set of statistics to a graph.

Histograms, along with dot plots and stem-and-leaf plots, share the same basic construction of showing the distribution of a quantitative variable through an explicit or implicit frequency axis. In the case of the histogram, frequency or relative frequency is readily accessible on the vertical axis; in contrast, unless the graphic was computer-generated, the reader must *count* to determine frequencies for dot plots and stem-and-leaf plots. Though variability is likewise expressed in these graph types, we chose to discuss histograms over the other distribution graphs for two reasons. First, its use of bars makes it visually similar to, and thus potentially confused with, the

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two other types of graphs discussed in this paper: value bar charts and distribution bar graphs. Secondly, for grouped histograms the raw data is no longer accessible, and therefore the reader is less likely to calculate a measure of variability; our interest is in perceiving variability from a graph and visually comparing magnitude of variability, rather than quantifying it.

Like other distribution graphs, histograms are classified by shape, center, and spread. Classic shapes that are identified are bell-shaped, uniform, positively skewed, and negatively skewed. If the shape is approximately symmetric, the mean is the preferable measure of center, which can be visualized as the balance point along the horizontal axis of the histogram, taking into account the "weight" and location of the bars above it. As in defining center, judging variability from a histogram depends to a degree on the shape. We focus our discussion of how variability is perceived within histograms through a comparison group of symmetric and approximately symmetric distributions, where variability in the data is commonly judged by how much the data is compressed about, or spread out from, the mean. The more compressed the data are to the mean, the less the variability; the more spread out, the greater the variability. This idea of magnitude of deviation of data values from the mean is analogous to the concept of variability in the case of the value bar chart, though the visual interpretation differs for the two.

Consider the two bell-shaped histograms of data sets (exam scores) in Figure 3. The notable difference between the graphs is that one histogram has a pronounced peak with narrow tails, while the other has bars of more similar height. In bell-shaped histograms, thicker tails indicate greater variability, while clustering toward the middle indicates little variability. Though one could construct exceptional counter-examples due to the grouped nature of the data, it is fair to say that in general, the raw exam scores of Class 2 are likely to be more spread out than those of Class 1, and thus are likely to be more variable.
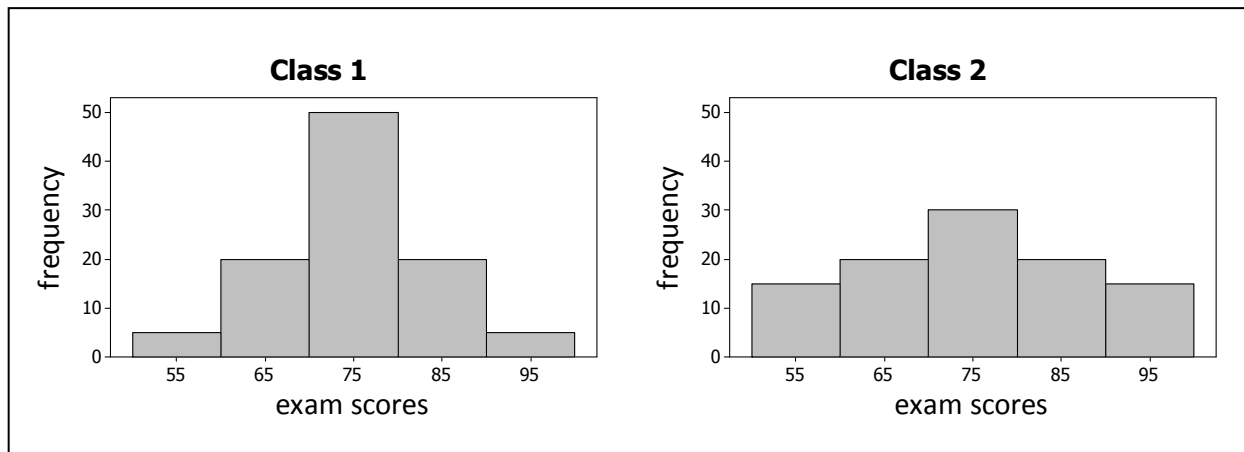

Figure 3. Histograms showing distribution of student exam scores for two classes, differing in their magnitude of variability.

When Cooper and Shore (2008) presented these histograms to 186 undergraduates, 50% indicated that the histogram with the higher peak and narrower tails was more variable.

Representative of subsequent interviews, one subject explained why she reasoned that Class 2 had less variability: "These [heights of bars in Class 2] were basically flat, while there was a peak here and small tails [in Class 1]" (p. 5). Thus, students mistook variability of bar heights to be an indication of variability in the data. This is consistent with the misconception noted earlier by Friel and Bright (1995) in that they failed to consider the interplay between the frequency axis and the data values on the horizontal axis. Simply put, the students were interpreting histograms as if they were value bar charts. This simple process of judging variability by "levelness of the bars" leads to a correct interpretation in value bar charts and an incorrect interpretation for histograms, pointing to the need to help students transition to more sophisticated graphs by juxtaposing the graph types and making key features explicit.

To encourage deeper understanding of variability in data represented by histograms, we again offer a visual aid (Figure 4) and compare the variability of four data sets that share a common mean and range and whose graphs are presented using a common set of bin intervals on a common scale. Supporting the idea of using a visual aid to enhance understanding of variability, delMas and Liu (2005) expressed interest in augmenting their computer-interactive histogram by illustrating deviations of data values from the mean; their goal was to help students understand the effect that positions of bars have on standard deviation. In our aid, variability of the scores of each class is visualized by a set of deviation lines. As the scores for each class are grouped in bins, it is impossible to recover the raw data values. In our discussion of variability, we choose to weight the data of each bin on its midpoint and thus the bin midpoints serve as proxies for the data in each interval. Beginning from the leftmost bin, lines extend from the balance point (mean) to the midpoint of the interval, illustrating deviation of data in that bin from the mean. The number of deviation lines drawn for each bin is determined by the frequency of the bin.
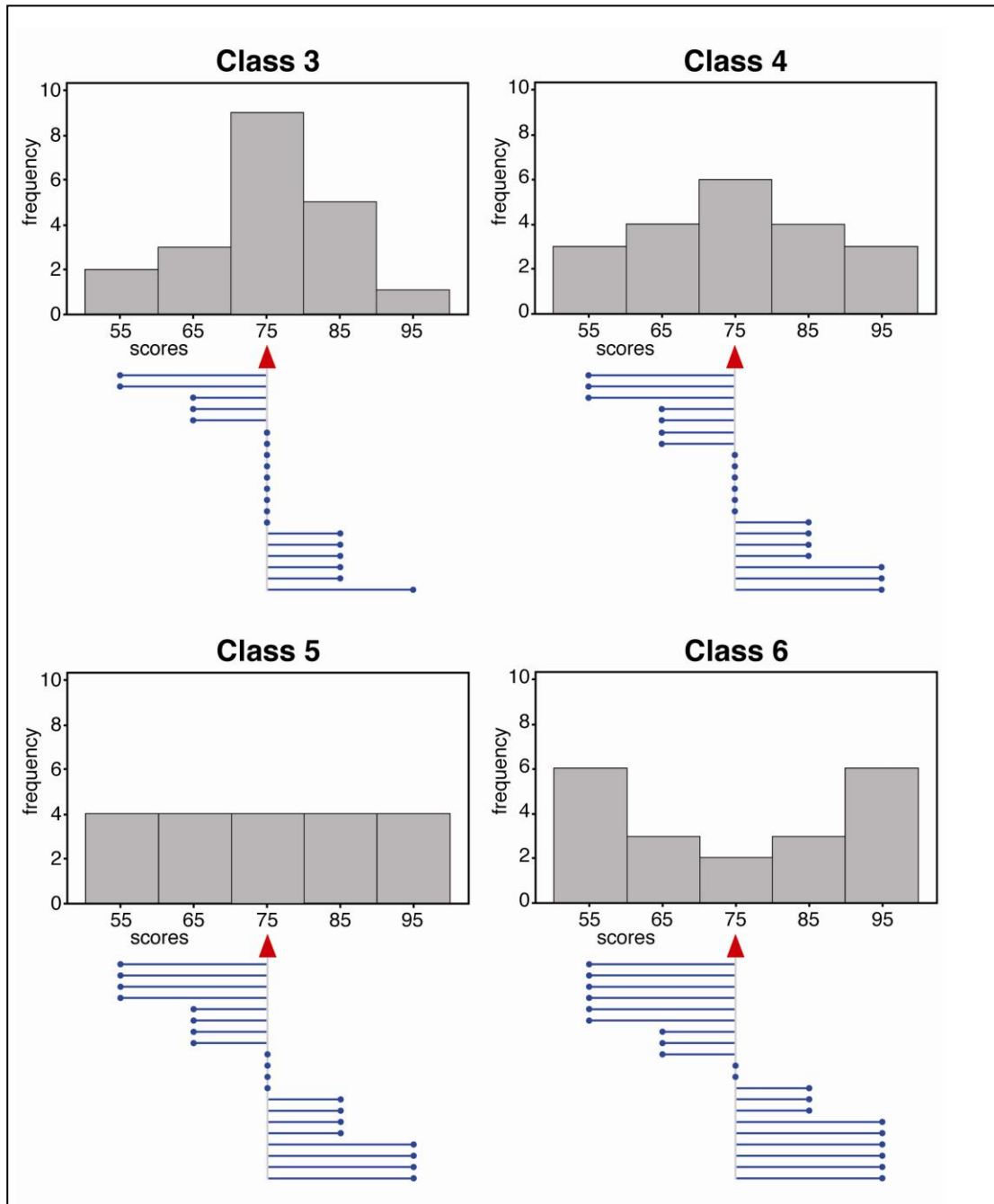
Figure 4. Histograms of distribution of student exam scores for four classes, arranged from least to most variable, along with accompanying graphics to aid comparisons of variability.

In Class 3, the bin centered over 85 has a frequency of five. Thus five deviation lines extend from the mean of 75 to the bin-midpoint 85. For data values within the bin centered over the balance point, a dot replaces an arrow indicating no deviation from the mean.

Longer deviation lines indicate greater spread from the mean and thus greater variability of the data. The classes in Figure 4 are numbered in order from least variable Class 3 to most variable Class 6. The variability ranking is perceivable simply by visually comparing the total lengths of

the deviation lines. But such a perception is valid only because the number of observations, n=20, is the same for each class. If the lengths of the deviations were quantified, students could find numerical estimates that measure the variability of each class, such as the mean absolute deviation (average length of deviation lines) and standard deviation, which do not require data sets of equal size for comparison. An alternative representation of this visual aid, perhaps more aesthetically pleasing due to its more balanced appearance, is to arrange the deviation lines simultaneously from the extreme left and extreme right bins, working toward the middle data values (see Figure 5). The result is a compact "variability graph" centered under the mean of the original graph.
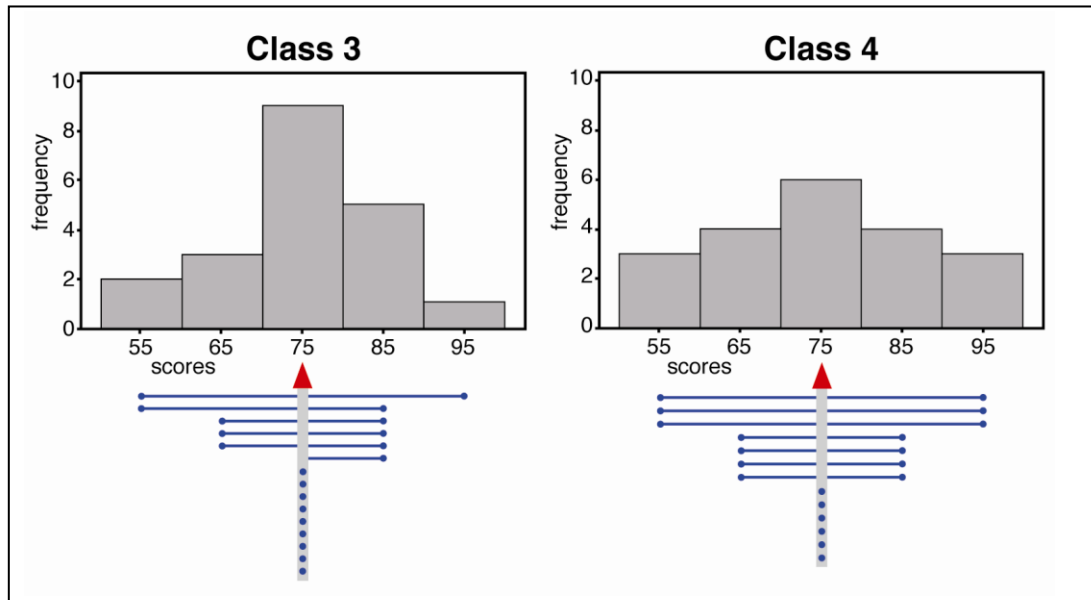

Figure 5. Histograms of distribution of student exam scores for two classes, arranged from less to more variable, along with alternative representation of graphic to aid comparisons of variability.

The benefit of the general model, and particularly as used with the four example histograms in Figure 4, is a) it simultaneously counters the misconception that one can compare variability by only looking at differences in bar height without regard for relative position to mean, while b) providing a concrete representation of variability relative to center.

2.3 Distribution Bar Graphs

Bar graphs are used to present the distribution of values of a qualitative variable and therefore may be aptly referred to as distribution bar graphs, further distinguishing them from value bar charts. Categories of the variable are typically marked on the horizontal axis. Unlike histograms, the order of the categories on the horizontal axis is not important, unless the data are ordinal. Bars of equal width are drawn for each category such that their height represents frequency or relative frequency for that category. Unlike the histogram, the bars do not touch.

The bar graphs of Figure 6 depict the distribution of blood types for modern Mayas, Scandinavian Lapps, and Japanese. There is almost no variability in blood type among the Mayas as roughly 98% have type O blood. There is more variability in blood type among the Lapps: Roughly 63% have type A blood while 29% have type O blood. Blood types among the Japanese are distributed more evenly and thus are the most variable. Furthermore, this is the case independent of the ordering of the bars.
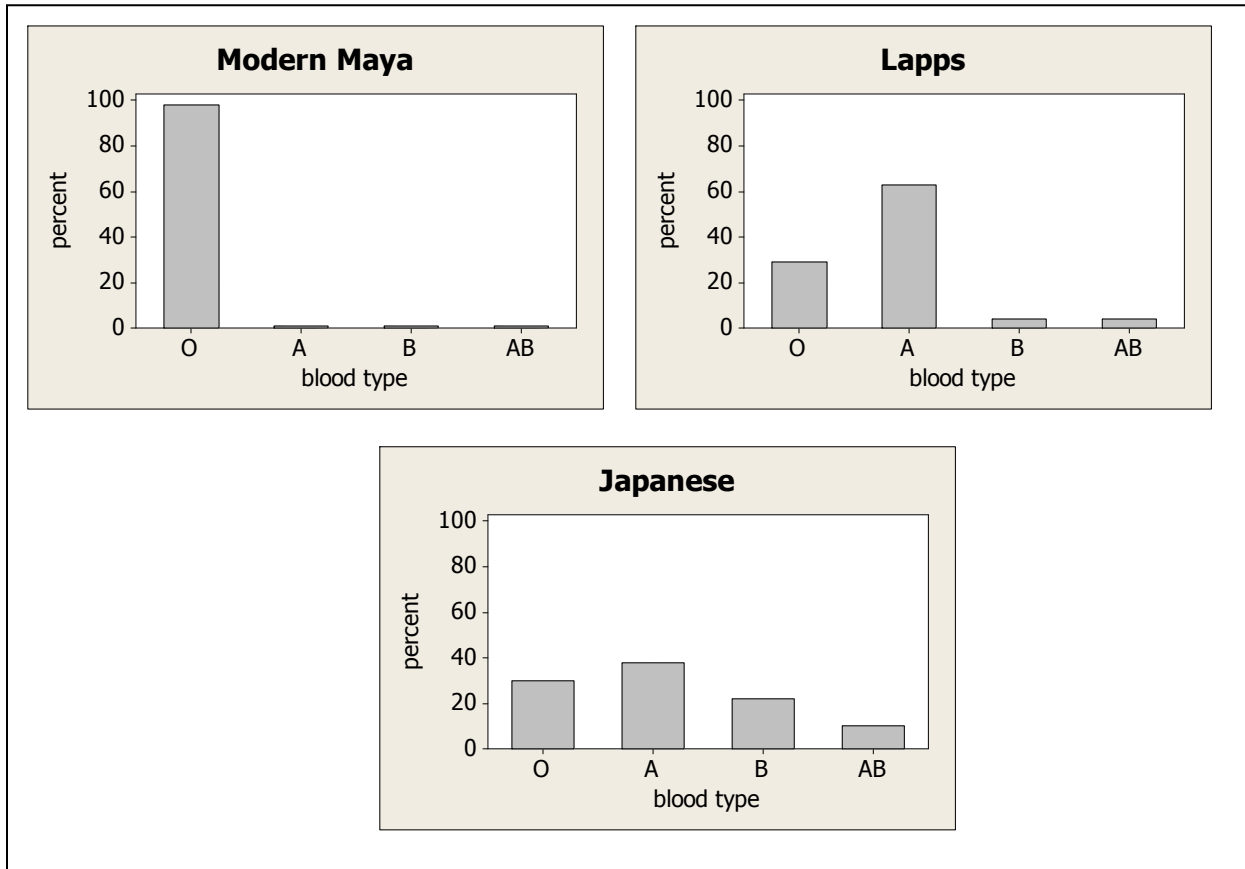

Figure 6. Distribution bar graphs of blood type for three ethnic groups, arranged from least to most variable. [data source: BloodBook.com 2006]

The example above demonstrates how the concept of variability differs for qualitative and quantitative variables. In contrast to variability judged as "variation about the mean" in the quantitative symmetric histograms of Figure 3 and value bar charts and time plots of Figures 1 and 2, variability in qualitative data is judged by unalikeability—"how often the observations differ from one another" (Kader and Perry, 2007)—the focus being on simply whether each pair of observations is different, rather than how much they differ as in quantitative data. Thus, in the Maya distribution of blood type, the pronounced mode of type O blood indicates virtually no unalikeability, i.e. variability, among blood types. To a lesser extent, the Lapps have a pronounced mode of type A. In contrast, the distribution of blood type for the Japanese has no obvious typical value. From a visual perspective, the more similar the heights of the bars, the more unlike the values and thus the more variable the data.

How does this compare to the quantitative setting in which we framed variability as deviations of data values from the center or mean (leveling line of value bar charts and balance point of histogram)? The only appropriate measure of "center" for qualitative data is the typical value. The mode of a data set is identified as the category whose bar has the greatest height. The extent to which the modal category's frequency stands apart from the frequencies of other categories determines the appropriateness to refer to the mode as a typical value. For data sets that have a typical value, the greater the frequency of that modal category compared to frequencies of other categories, the more alike the data is, and thus the less variable the data. The more the data differ from the modal category, to the extreme point that there is no longer a concentration of values, the more variable the data.

Students may erroneously perceive similarities in the methods to judge variability of distribution bar graphs and histograms. If exposure of histograms is limited to approximately bell-shaped (Figure 4. Class 3 and Class 4) and uniform distributions (Class 5), students may over-generalize that the more evenly distributed the data, the greater the variability. Indeed this is the case for the bar graph. However, a comparison of Class 5 and Class 6 offers a counterexample to confront the faulty thinking that the evenly distributed exam scores (level bars) of Class 5 indicate greatest possible variability. An examination of the overall length of deviation lines in the visual-aid below the graph makes it clear that exam scores of reverse bell-shaped Class 6 are more variable (deviate more from the mean) than those of Class 5. The important point to emphasize here is that there are differing concepts of variability—depending on data type—and subsequently, different methods to judge variability.

One final point to make is that, especially in K-12 education, value bar charts are often simply referred to as bar graphs, so that their distinction from distribution bar graphs is unclear. This is consistent with the data collected by the authors, and referred to in the introduction, where 64% of teachers identified a value bar chart as a "bar graph." Certainly these two types of graphs appear to be of the very same variety: Both have *qualitative values* (categories or case names) on the horizontal axis and a *numerical scale* on the vertical axis. However, the method to judge center is clearly very different, as "center" for a distribution bar graph implies a *typical categorical* value (if one exists at all), e.g., "type O blood," found on the horizontal axis, as opposed to a *central quantity*, e.g., "mean square footage," found on the vertical axis of a value bar chart. Furthermore, visually, the method to judge variability is exactly the opposite: For distribution bar graphs, bars of approximately equal height indicate great variability; whereas for value bar charts, the same visual display of approximately equal bar heights indicates little variability. In the interest of full disclosure, we acknowledge that the distinction between value bar charts and distribution bar graphs can in fact blur when the variable on the vertical axis is interpreted as frequency. For an example of such a case, see the Appendix. Suffice it to say here, that the context of a graph determines the questions that we ask. Those questions identify the data and implicate one of the two graph types, which in turn would *require* a particular approach to judging center and variability.

3. **Concluding Remarks**

The focus of this article is to bring teachers' attention to the differing methods by which to judge variability in value bar charts, histograms, and distribution bar graphs. While variability is manifested differently across graph types in general, our choice of graphs is particularly pertinent because their visual similarities often result in readers failing to distinguish among them, thus compounding difficulties perceiving variability. Students tend to encounter these graph types in somewhat different environments. Value bar charts and distribution bar graphs are seen in primary education classrooms when graphical displays are first introduced. They continue to make appearances throughout later elementary and secondary years even as more sophisticated graphical representations are introduced. Histograms and distribution bar graphs are among those graphs most prevalent in introductory statistics classes. Outside the school setting, time plots, other value bar charts, and distribution bar graphs are commonly found in the media. Students who haven't had adequate experience differentiating between graph types, due in part to the different environments in which they are generally found, may find it especially difficult to connect to the data and judge (or compare) variability appropriately.

For each graph type, we have framed our discussion of variability around the "center" of the data. Discussing variability in terms of center allowed us to develop visual aids—deviation lines from the mean—in graphs of quantitative data (value bar charts and histograms). In a value bar chart (see Figure 1 and Figure 2), the data of interest are represented along the vertical axis. Variability is judged in terms of deviations from the leveling line (mean) to each bar, so that greater variability is perceived by greater differences in bar heights. For histograms, (see Figures 3, 4, and 5), data values fall into bins along the horizontal axis; thus, variability is judged in terms of deviations (the number of which corresponds to the bar's frequency) from each bar to the mean or balancing point. Note that this approach is consistent with the use of both the mean absolute deviation and standard deviation. For qualitative data we look at the extent to which the data set has [does not have] a typical value. In a distribution bar graph (see Figure 6), greater variability is manifested by smaller differences in bar heights, or absence of pronounced typical value.

Although in terms of instruction we recommend that variability be framed in terms of "center," in reality the focus on center can be removed from discussions of variability for both quantitative and qualitative data. We can see this formulaically. Kader and Perry (2007) quantified variability for categorical data as the "proportion of possible data value comparisons (pairings) which are unlike" (p. 12)[1]. And Gordon (1986) offered an expression for standard deviation which is

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[1] Coefficient of unalikeability = $\dfrac{\sum_{i \neq j} c\left(x_i, x_j\right)}{n^2 - n}$, $c\left(x_i, x_j\right) = \begin{cases} 1, & x_i \neq x_j \\ 0, & x_i = x_j \end{cases}$

formulated on deviations between all possible data pairs, rather than deviations from the mean.[2]

What remains fundamental is examining "how often do observations differ" for qualitative data versus "how far apart are observations" in quantitative data.

Thus, for students to be able to perceive variability in graphically presented data they must be attentive to the underlying structure of the graph type, including being able to identify the type of data and on which axis it is plotted. In terms of instruction we recommend that teachers promote a conceptual grounding in how to visualize "how far apart" or "how different" the data values are from the "center" of the distribution. As variability is indisputably the most important concept in statistics, it is incumbent upon teachers to acknowledge the contrasting methods by which it is judged. When students confront questions about center and variability across different types of graphical displays, they deepen their understanding of these concepts, improve their graph sense, as well as gain insight that simply helps them make better sense of the data. If statistical literacy is the goal of the data analysis content strand in K-12 education as well as introductory undergraduate courses, it simply makes good sense to include rich discussions connecting an assortment of graphical displays to their corresponding data sets and methods to judge center and spread.

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[2] Standard deviation for a sample $= \sqrt{\dfrac{\sum_{i=1}^{n} \sum_{j=1}^{n} \left[\left(x_i - x_j\right)\right]^2}{2\left[(n)^2 - n\right)}}$

Appendix

The distinction between distribution bar graphs and value bar charts can blur if frequency is found on the vertical axis and the data itself is not well-defined. Are the values on the horizontal axis values of a variable such as grade level, or are they to be thought of as cases (each grade as a case)?

Consider the following two graphs:
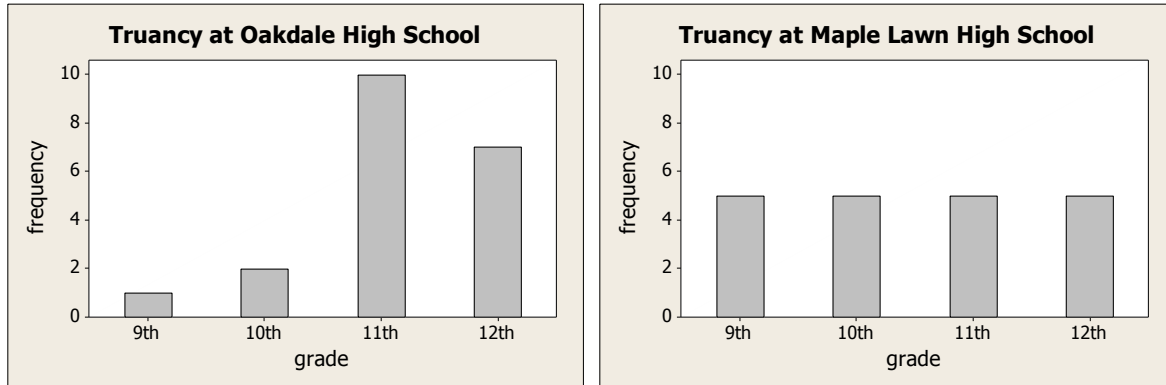
While the distinction between graph type may not be important for some questions [Which grade experienced the most number of truancy cases in Oakdale HS?], for interpretation of variability the question of what are the data of interest (and implicitly what is the graph type) is of critical importance.

Let's compare the two graphs in terms of variability. Which graph illustrates greater variability of data? Before we can answer that question, we need to know *what are the data of interest.*

If the data of interest are grades of truant student (raw data would look like 9th, 11th, 12th, 12th, 11th, …), then we have a distribution bar graph and we would say that there is little variability in Oakdale truant students' grade levels (most were in either 11th or 12th grade) while there is great variability in Maple Lawn's students' grade levels (equally distributed among 9th, 10th, 11th, and 12th grades).

However, if the data of interest are the number of truant students in a grade (the data set for Oakdale HS has four values: 9th grade–1; 10th grade–2; 11th grade–10; and 12th grade–7), then our graphs can simply be interpreted as value bar charts and we would say that there is great variability among the number of truancy cases by grade at Oakdale HS as compared to that of Maple Lawn HS (four data values: 9th grade–5; 10th grade–5; 11th grade–5; and 12th grade–5).

In conclusion, the two graphs can be viewed as either distribution bar graphs or value bar charts when no questions of interpretation are asked. However, it is important to note that some questions that we ask of the graphs force us to look at the data differently (grade of each student, or number of cases by grade). Before we declare which graph is more variable, we need to know *what are the data of interest.*

It is interesting to note that this dilemma exists only when frequency is found on the vertical axis. Had the graph been constructed with relative frequency instead, then the question of "What are the data of interest?" would be implied. That is, the "total" to which the relative frequencies are relative would reveal the data of interest. In a distribution bar graph, the relative frequencies would sum to 1 (100%) and the "total" would refer to the sum of the corresponding frequencies. In a value bar chart, not only would the relative frequencies most likely not sum to 1, the "total" number to which the frequencies are compared differs by category (in our case the total number of students in each grade). Lastly, in our example, one would be assuming relatively similar numbers of total students in each of the four grades. If this were not the case, another layer of complexity is added to the value bar chart and discussion of its variability.

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Linda L. Cooper
Towson University
8000 York Road
Towson, Maryland 21252
mailto:lcooper@towson.edu
Phone: 410-704-3817

Felice S. Shore
Towson University
8000 York Road
Towson, Maryland 21252
mailto:fshore@towson.edu
Phone: 410-704-4450

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