The Statistical Content of Elementary School Mathematics Textbooks

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Abstract

This study reports on the statistical content of five U.S. textbook series written for elementary students in grades 1-5. The researchers examined 17,688 pages and coded 7445 statistical tasks to determine (1) the distribution of statistical topics within textbooks, and (2) the relative emphasis on the phases of the statistical problem solving process (Formulate Questions, Collect Data, Analyze Data, and Interpret Results). Different series contained markedly different

distributions of statistical content: two series located most statistical content near the end of the text, whereas two other series located statistical tasks more uniformly throughout the textbook. A large majority of statistical tasks required students to Analyze Data, with a heavy emphasis on the activities of reading displays and performing mathematical calculations.

1. Introduction

Over the past century, the need for statistical literacy has grown, as has the presence of statistics in K-12 classrooms (Schaeffer and Jacobbe 2014). Citizens and consumers are presented with data on a daily basis, which leads to a clear need for including statistics in the school curriculum. Initial efforts for a stand-alone course in statistics began in the late 1950s, with the creation of a course designed for mathematically-able high school students (College Entrance Examination Board 1959). Since that time, statistical topics have worked their way down through the grade levels and are now present in curriculum standards for elementary school (Jones and Tarr 2010). In particular, the *Common Core State Standards for Mathematics* (CCSSM, National Governors Association Center for Best Practices and Council of Chief State School Officers 2010) call for students to create and interpret data displays beginning in grade 1. The authors of the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K–12 Curriculum Framework* (Franklin et al. 2007) demonstrate how a teacher may guide students in elementary schools through the entire statistical problem-solving process.

The mathematics textbook is often the vehicle through which these standards and recommendations influence the daily activities of the elementary school classroom (Center for the Study of Mathematics Curriculum n.d., Fan 2013). For that reason, we sought to examine the statistical content of textbooks used in U.S. elementary schools. In this article, we describe our efforts to understand how statistics is treated within mathematics textbooks used in grades 1 through 5.

2. Review of Related Research

Our work is situated within the Center for the Study of Mathematics Curriculum (CSMC) framework (CSMC n.d.) as an analysis of the textbook curriculum. Several factors influence the textbook curriculum: market forces (Seeley 2003), recommendations of professional societies such as the American Statistical Association and the National Council of Teachers of Mathematics, and curriculum standards (Dossey, Halvorsen, and McCrone 2012). Forty-three U.S. states have adopted the CCSSM; the remaining seven states have their own sets of standards, such as the Texas Essential Knowledge and Skills (TEKS, Texas Education Agency 2013), to guide what is taught in classrooms at each grade level. Just as these forces affect the content of textbooks, textbooks have an impact on what is taught. Teachers use the textbook as resource for planning and teaching mathematics (Grouws and Smith 2000, Tyson-Bernstein and Woodward 1991), perhaps more in mathematical subjects than in others (Robitaille and Travers 1992).

According to the 2012 National Survey of Science and Mathematics Education [NSSME], “teachers in 81 percent of elementary mathematics classes using commercially published materials use the textbook/program to guide the overall structure and content emphasis in their most recent unit" (Malzahn 2013, p. 24). Interestingly, teachers in more than half of these classes in this same study incorporated activities from sources outside of the textbook. Furthermore, 30% of these classes were using materials that had been purchased within the past two years. About half of the elementary classes were using instructional materials that had been purchased in the past three to five years, and about one-fifth were using materials purchased more than five years prior to the study. This variability makes it difficult to determine how to measure which textbooks are most commonly used; adoption rates and purchase records do not tell the whole story. For this reason, it is important to examine both recently published textbooks and those that have been on the market for several years.

With respect to statistical content, the GAISE Report (Franklin et al. 2007) described the statistical problem-solving process as composed of four phases: Formulate Questions, Collect Data, Analyze Data, and Interpret Results. In an analysis of state curriculum frameworks, Newton, Horvath, and Dieteker (2011) found, "an overwhelming majority of these [grade-level expectations] (approximately 87%) were coded in either the Analyze Data or Interpret Results [phases] or both" (p. 156). Jones and Jacobbe (2014) reported a similar emphasis on the latter two phases within the CCSSM for students in grades K-8. Furthermore, in their investigation of the statistical content of six textbooks for prospective elementary teachers, "more than 70% of the tasks in [statistics chapters of] each textbook requested that students Analyze Data," (p. 10) while the other phases were addressed much less often.

In an analysis of the statistical content of U.S. textbooks for grades 6–8, Pickle (2012) examined two series that were commercially produced; one series developed with funding from the National Science Foundation, and the other series developed as a part of a university-based curriculum development project. The proportion of instructional pages containing statistical topics ranged from 8% to 13.6%, depending on the series. Additionally, statistics lessons tended to appear near the end of the textbook, and in 13 of the 14 textbooks in her sample, "the majority of the statistical topics were found in a single chapter" (p. 68).

As a part of the Third International Mathematics and Science Study, Valverde, Bianchi, Wolfe, Schmidt, and Houang (2002) conducted a study of 418 mathematics and science textbooks from 48 nations. They found that U.S. textbooks were among the largest in terms of number of topics and number of pages, and stated that the current structure of U.S. mathematics textbooks limits the coverage of the content.

> If all of the content were to be covered, then the amount of time available for doing this would be severely limited. On the other hand, if not all content is covered then there must be some basis on which to select the content that is to be covered. This is a good example of how physical characteristics can affect the learning opportunities developed in the textbook. (p. 37)

While the authors were not explicit in describing a possible basis for selecting content, one may speculate that topics near the end of the textbook (such as probability and statistics) are more frequently omitted than topics in the first part of the textbook.

To date, no studies have examined the statistical content of mathematics textbooks used in the elementary grades in the United States. Other analyses of mathematics textbooks have informed our work. Flanders (1987) examined three textbook series (grades K-8) and identified the percentage of pages that contained new content. Baker et al. (2010) also used pages as the unit of analysis when examining 141 elementary textbooks. By way of contrast, some studies used exercises as the unit of analysis without examining the entire book. Thompson, Senk, and Johnson (2012) identified and analyzed the content of purposefully-selected sections in 20 high school textbooks, while Bieda, Ji, Drwencke, and Picard (2013) examined every other lesson in seven elementary mathematics textbooks. In our study, we examined every page of each textbook in an attempt to answer the following research questions:

1. How is statistical content distributed within mathematics textbooks for grades 1-5?
2. What phases of the statistical problem solving process are addressed in these textbooks?

3. Methodology

3.1 Sample Selection

Our sample consisted of the student editions of textbooks from five different series used in the United States. We selected five textbooks from each series (those written for grade 1 through grade 5), for a total of 25 textbooks in our sample. The textbooks were chosen for various reasons. First of all, the curriculum standards of large textbook-adoption states, such as Texas, have a profound effect on the content of textbooks (Seeley 2003). For this reason, we included in our sample two series from the Texas adoption list. We selected the *Texas Go Math!* and *enVisionMATH Texas 2.0* because they were the only two series on the Texas textbook adoption list that addressed 100% of the Texas Essential Knowledge and Skills (Texas Education Agency 2014). Both of these series are commercially produced. For comparison, we also selected two other commercially produced series: *Math Connects* and *Saxon Math*. *Math Connects* is similar in structure to *Texas Go Math!* and *enVisionMATH Texas 2.0*; *Saxon Math* is designed using spiral review, where each lesson contains substantially more exercises relating to previous lessons than the current lesson. Finally, we selected *Math Trailblazers*, a series developed with funding from the National Science Foundation and based on *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics 2000). In Table 1, we provide the information on the five series in our sample, as well as the abbreviations that will be used to refer to these series throughout the analysis and discussion.

Table 1. Textbooks included in the sample, with abbreviations used in this article

Textbook Series	Publisher	Abbreviation
Texas Go Math! (Dixon et al. 2015)	Houghton Mifflin Harcourt	TGM
enVisionMATH Texas 2.0 (Charles et al. 2015)	Pearson	eVM
Math Connects (Altieri et al. 2009)	McGraw Hill	MC
Saxon Math (Hake and Larson 2008)	Houghton Mifflin Harcourt	SM
Math Trailblazers (Wagreich et al. 2008)	Kendall Hunt	MT

According to Dossey, Halvorsen, and McCrone (2008), the lack of a commonly accepted mathematics curriculum at the national level makes it difficult to obtain data on textbooks that are adopted or in use in classrooms across the country. Even in a textbook-adoption state like Texas, these records are not readily available. The 2012 NSSME reported that the top three publishers of textbooks used in elementary mathematics classrooms were Houghton Mifflin Harcourt with 35% of the market share, Pearson with 33%, and McGraw Hill with 29% (Banilower, Smith, Weiss, Malzahn, Campbell, and Weis 2013, p. 93). They also found that the first edition of *enVisionMATH* was one of the most commonly used textbooks for grades K-5.

3.2 Coding Scheme

3.2.1 Unit of analysis

To code the textbooks, we first defined our unit of analysis as a task. According to Doyle (1983, 1988), academic tasks are individual questions, exercises, or problems that students are asked to complete. For our study, we used the term *task* to refer to the smallest marked division in a set of problems or exercises that referred to statistical content, which agrees with the definition of a task used by Jones and Jacobbe (2014). A single task may be a numbered exercise, or a labeled portion of an exercise (i.e., part A and part B would be two separate tasks). We did not include worked examples given in the text that did not require the student to do anything further. In cases where there was no specified numbering system, we considered how the questions were partitioned on the page and broke them into tasks accordingly.

Next, we had to determine whether a task was statistical. Statistical tasks focused on a statistical topic and addressed variability in some sense. For example, items addressing survey questions and tables displaying the resulting data were considered statistical tasks. We did not, however, include all tables as statistical tasks; multiplication and division charts, place-value tables, or tables used to find a pattern served a different purpose. Probability and combinatorics were also not considered in our findings unless they were used in a statistical context, such as collecting data for an experiment. In Figures 1 and 2, we present examples of tasks to contrast statistical tasks against other tasks.

Figure 1. Example of a statistical task that uses probability (MT grade 4, p. 391)

Probability predicts that number greater than 4 (a 5 or a 6) will come up 1/3 (2/6) of the time when a number cube is rolled. Does your class's data agree with this?

Figure 2. Examples of two tasks that are not statistical tasks (MT grade 4, p. 391)

12 A.	What is the probability of rolling a 4? (Express your answer as a fraction.)
12 B.	Where would you place your answer to 12A on a probability line - nearer "1" or nearer "0"?

To answer our two research questions, we developed a coding scheme to consistently identify the location of statistical tasks within the textbook, and also classify these tasks according to the phases of the statistical problem solving process.

3.2.2 Location of statistical content

As an initial measure of the distribution of statistical content within a textbook, we compared the number of pages containing statistics to the total number of instructional pages in each textbook. Instructional pages only included those pages that contained lessons or tasks for the students. Therefore, the table of contents, index, and glossary were not included, but pages from other instructional sections (e.g., "Extra Practice" or "Step Up to Grade 6") were included in the total page count. A page was coded as a *statistics page* if it contained part of a statistics lesson or contained at least one statistical task.

To determine the distribution of statistical content within each textbook, we recorded the page number for each statistical task. We then divided the instructional pages into deciles–ten sections with equal numbers of pages. Next, we determined the number and percentage of statistics tasks in each decile for each textbook.

3.2.3 Phases of the statistical problem solving process

Each statistical task may address one or more of the following phases: Formulate Questions, Collect Data, Analyze Data, and Interpret Results. To classify statistical tasks by phases, we used the coding protocol in the appendix of Jones and Jacobbe (2014). Table 2 contains example tasks from each phase. Tasks coded as Formulate Questions required students to create one or more questions that could be answered with data, or identify types of variables, such as categorical or numerical. A Collect Data task involved designing or carrying out a plan to gather appropriate information. Tasks that addressed Analyze Data had students answer questions about data, which involved reading displays, performing mathematical calculations, constructing displays, and using other statistical reasoning. In tasks addressing the Interpret Results phase, students evaluated claims or made inferences or predictions based on given data. A task was also considered a part of Interpret Results if it asked students to make predictions without referring to previously collected data.

Table 2. Examples of tasks coded for each phase of the statistical problem solving process

Phase	Example
Formulate Questions	Write five number problems that use your data. Show how you solved each problem. (MT grade 1, p. 296)
Collect Data	Search for objects that have a cylinder shape, like the toilet paper core. Then, record the names of the cylinder shapes you see at your home. (MT grade 1, p. 315)
Analyze Data	How many sunny days did they record for December and November? Write a number sentence to show how you found your answer. (MT grade 1, p. 237)
Interpret Results	Look at the recommended amounts in the Servings Table. How does what you are compare to what you should be eating? Are there things you should change? Are there things you should keep the same? (MT grade 1, p. 341)

Because most curriculum standards address the Analyze Data phase, we created four codes for Analysis Activities that could be incorporated within these tasks. These are based on the categories for the nature of Analyze Data tasks used by Jones and Jacobbe (2014). (See examples in Table 3.) The first of these was Read a Display, where students were to read of tables and graphs. The second Analysis Activity was Perform a Mathematical Calculation, and included tasks that required simple arithmetic (e.g., addition or multiplication), as well as procedures to find the mean, median, or range of a set of numerical data. The third Analysis Activity was Construct a Display, where students were asked to make a graph or table, decide the type of display to use, or make and compare different displays of the same data. The final Analysis Activity was Use Other Statistical Reasoning, and was reserved for tasks that required students to do more than read a display or perform a mathematical calculation, such as making conclusions about data. While Jones and Jacobbe (2014) combined Read a Display and Construct a Display into a single category, we separated these two activities for the purposes of coding. Just as it was possible for a single statistical task to address more than one phase from the statistical problem solving process, a single Analyze Data task may be coded for more than one Analysis Activity.

Table 3. Examples of tasks coded for each Analysis Activity

Analysis Activity	Example
Read a Display	Which sport(s) team has the fewest teams? the greatest? (MC grade 4, p. 13)
Perform a Mathematical Calculation	A black bear weighs 25 pounds more than a gorilla. Use the information in the table to find how much a black bear weighs (MC grade 4, p. 41)
Construct a Display	Make a tally chart for each situation. Alexi took a survey to find out her friends' favorite colors. (MC grade 4, p. 91)
Use Other Statistical Reasoning	Give a possible explanation for an outlier in this situation. (MC grade 4, p. 99)

3.3 Interrater Reliability

A team of five researchers[1] planned to code the five textbook series in our sample, with each researcher coding a different series. Because we wished to ensure that each researcher was consistent in identifying statistics tasks and applying the coding scheme, we first coded lessons from three series that were structurally different: the traditional format of MC, the spiral review of SM, and the investigation format of MT. For a particular series, we randomly selected a lesson on a statistical topic, and each researcher independently identified the statistical tasks in the lesson and coded them by process phase and Analysis Activity. Next, we used Fleiss' Kappa (Fleiss 1971) to measure the degree to which the researchers' codes agreed. According to Landis

[1] This study was completed as a part of The Sam Houston State University Research Experiences for Undergraduates during the summer of 2014, and funded by the National Science Foundation. The first author directed the research project. At the time of the study, the other five authors were undergraduate students. They were selected for the team, in part, because of their excellent grades overall and within statistics courses. Three researchers were preparing to teach mathematics in high school, one was preparing to teach mathematics in the middle grades, and one was preparing to teach elementary school.

and Koch (1977), obtaining a Kappa of 0.80 or greater is evidence that the coding is almost perfectly identical. We calculated Fleiss' Kappa using software designed by Geertzen (2012). In those instances when Kappa was less than 0.80, we discussed the rationales for our codes, and then randomly selected another statistics lesson from that series and repeated the coding process until Kappa was at least 0.80 for each series. The column labeled Initial in Table 4 shows the Kappa values that were ultimately obtained at the beginning of the project.

Table 4. Interrater Reliability Measures

	Fleiss' Kappa	
Series	Initial	Intermediate
MC	0.818	0.887
MT	0.880	0.883
SM	0.921	0.757

After the Fleiss' Kappa scores were above 0.80 for each of the three textbook series, each series was assigned to a different researcher for coding. After each researcher had coded two textbooks from his or her series, we performed an intermediate reliability check. We randomly selected a statistics lesson from textbooks in the MC, MT, and SM series that had not yet been coded. Each researcher then independently identified the statistical tasks and coded them, and then Fleiss' Kappa was calculated for each lesson. As Table 4 shows, the intermediate Kappa for SM was slightly below 0.80. While a Fleiss' Kappa between 0.61 and 0.80 shows "substantial agreement" (Landis and Koch 1977, p. 165) among researchers, this did not meet our desired level of agreement. Upon discussion, we found that our codes were identical for all but one of the tasks in this lesson. One researcher had coded this task as Collect Data, one had coded it as Analyze Data, and the other three had coded it as both Collect Data and Analyze Data (which was treated as a third, different code by our software). After a thorough discussion of this task and its coding, the team reached a consensus and resumed coding the tasks in the remaining textbooks.

4. Analysis

4.1 Distribution of Statistical Content

We examined a total of 17,688 pages and coded 7445 statistical tasks across the 25 textbooks in our sample. We found that statistical content was present in every textbook in our sample, but to varying degrees depending on the series and grade level.

4.1.1 Statistics pages and statistical tasks by textbook series

Table 5 displays the results aggregated by series; each column describes the results from the five textbooks (for grades 1-5) within each series. Here, we report the total number of instructional pages, statistics pages, and statistical tasks for each series. We also provide some measures of the density of statistical content within each series, in terms of the proportion of pages that contain statistics, and the ratio of statistical tasks to statistics pages. Across the entire sample, 15% (n = 2646) of the instructional pages contained statistics, and the typical statistics page contained between 2 and 3 statistical tasks.

Table 5. Statistical pages and tasks by textbook series

	Textbook Series				
	TGM	eVM	MC	SM	MT
Instructional Pages	3522	4716	3445	3749	2256
Statistics Pages	480	767	587	404	408
Statistical Tasks	1524	1821	1648	907	1545
Proportion of Instructional Pages containing statistics content	13.6%	16.3%	17.0%	10.8%	18.1%
Statistical Tasks per Statistics Page	3.2	2.4	2.8	2.2	3.8

As Table 5 shows, eVM had the greatest amount of statistical tasks and statistics pages; at the same time, this series also had the greatest number of instructional pages. Therefore, eVM had the second lowest ratio of statistical tasks per statistics page. The least number of statistics pages, statistical tasks were found in SM; this series also had the lowest ratio of statistical tasks per statistics page (2.2). At the other extreme, the statistics pages in MT were the most dense with statistical tasks, and MT also had the greatest proportion of statistics pages (18.1%).

4.1.2 Statistics pages and statistical tasks by grade level

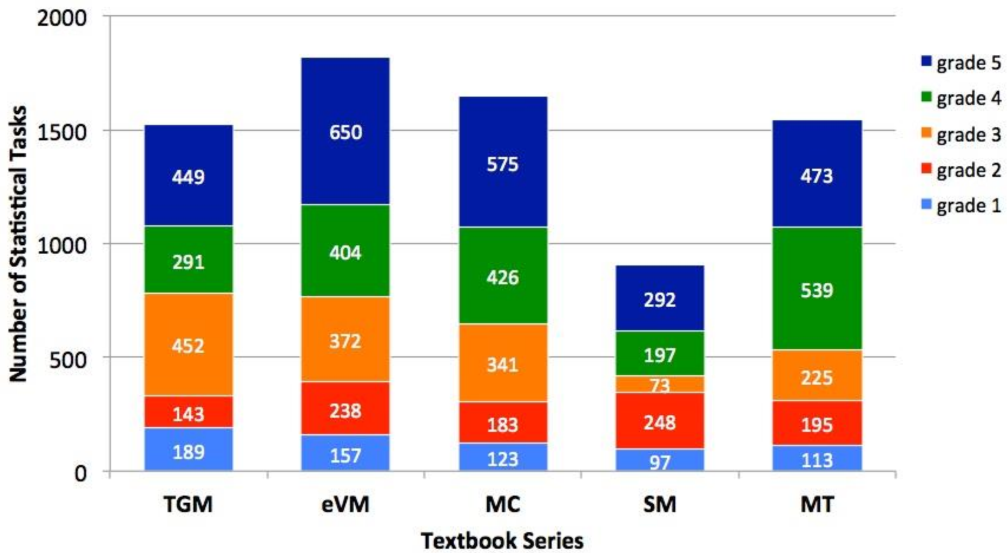
For a given series, the number of instructional pages remained relatively constant across the five grade levels. This was not the case with statistics pages. Table 6 shows the proportion of statistics pages within each textbook. Within four of the grade 1 textbooks, less than 10% of instructional pages contained statistical content. (The exception was MT with 14%.) This proportion was more than 20% for four of the grade 5 textbooks, with the exception of SM with 15%. Within a given series, the percentage (and number) of statistics pages tended to increase with the grade level, although a strict increase from grade 1 grade 5 was only present in MC. An examination of the proportion of statistics pages within each textbook revealed Simpson's Reversal does not occur here; the trends that appear in by aggregating data by series are similar to those that exist for each grade levels.

Table 6. Proportion of Statistics Pages in Each Textbook, by Textbook Series and Grade Level

	Textbook Series				
Grade Level	TGM	eVM	MC	SM	MT
grade 1	7%	6%	7%	8%	14%
grade 2	7%	9%	13%	11%	14%
grade 3	20%	19%	14%	6%	20%
grade 4	14%	19%	21%	13%	24%
grade 5	21%	27%	26%	15%	20%

While statistical tasks were found in each textbook, those for grade 1 tended to have fewer statistical tasks than those for the upper grades. Figure 3 displays the number of statistics tasks in each series, differentiated by grade level. Note that there tends to be an increasing number of statistical tasks as the grade level increases, but this is not strictly the case. For example, the grade 3 textbook in TGM has the greatest number of statistical tasks for that series, while the grade 3 textbook in SM has the least. Simpson's Reversal does not occur by aggregating these data by series.

Figure 3. Statistical tasks by series and grade level



4.1.3 Location in text

We also examined the location of statistical tasks within each textbook. Figures 4 through 8 below show the cumulative percentage of statistics tasks for a textbook plotted against the percentage of textbook pages. The five textbooks for each series have been shown in the same graph to help identify patterns within a series. For example, in Figure 4, there is a point on the grade 3 line located at 40% on the horizontal axis and almost 25% on the vertical axis. This means that in the grade 3 textbook for this series, almost 25% of the statistics tasks appear on the first 40% of pages. These figures also include a straight line to indicate a "uniform" distribution of statistics tasks throughout the pages of the textbook.

Two textbook series show a clear preference for including statistics content near the end of the textbook. In TGM (Figure 4), there is a trend of little to no statistics content in the first 80% of textbook pages, with almost all statistics content in the final two deciles of pages. This distribution is similar to eVM (Figure 5), which has less than half of the statistics tasks before the eighth decile of instructional pages for each grade level. For both TGM and eVM, the primary statistical content prior to the eighth decile relates to using tables to organize data; this topic is introduced in grade 3 and developed further in grades 4 and 5.

The distribution of statistical tasks across the pages of MC (Figure 6) is quite different, with large jumps occurring earlier. For example, at least half of the statistical tasks in the textbooks for grades 1, 2, and 4 appear in the first third of the book. In this series, a chapter devoted to statistics appeared in the first half of the books (with the exception of grade 3).

Figure 4. Location of statistics tasks in TGM by grade level

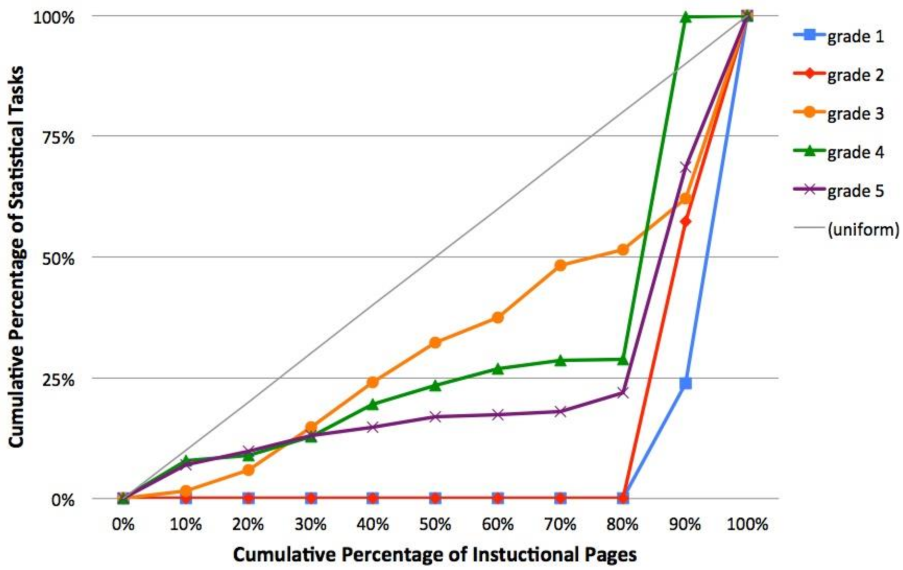


Figure 5. Location of statistics tasks in eVM by grade level

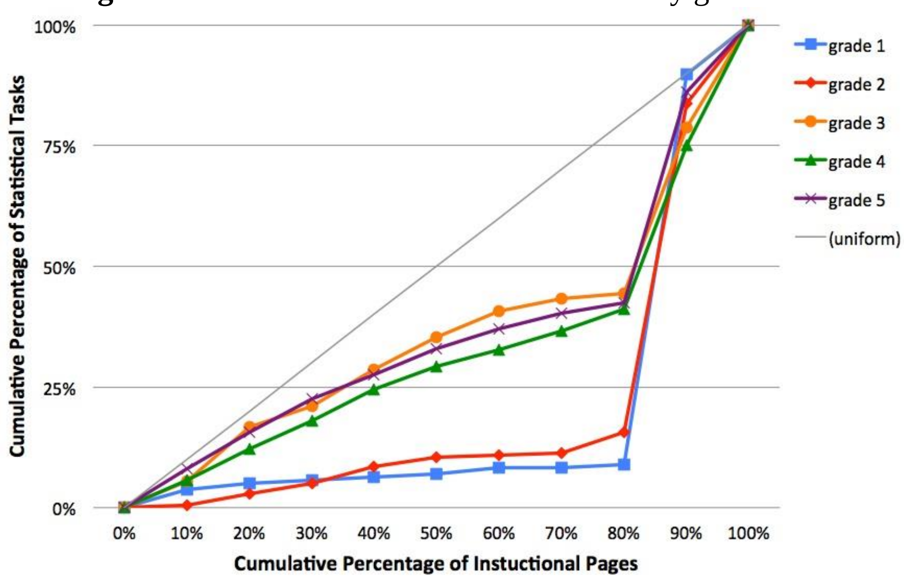


Figure 6. Location of statistics tasks in MC by grade level

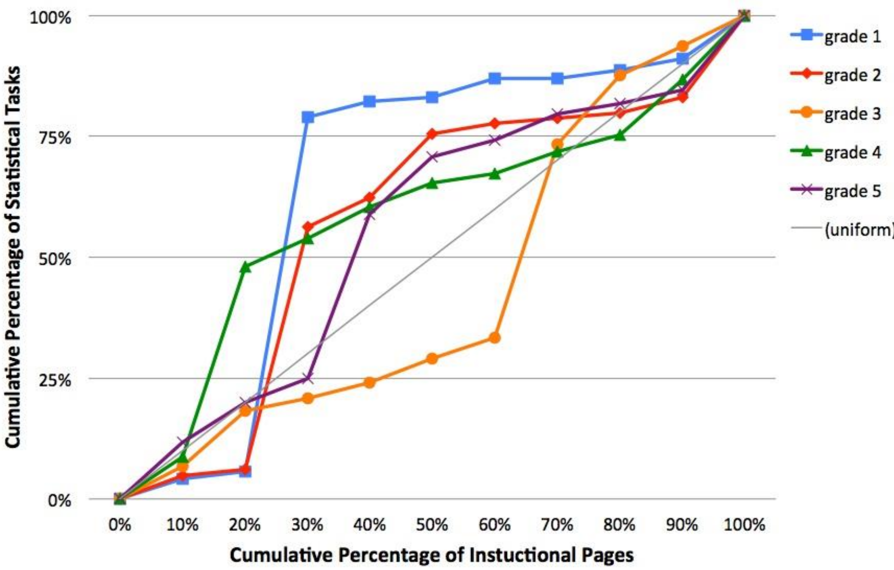


Figure 7. Location of statistics tasks in SM by grade level

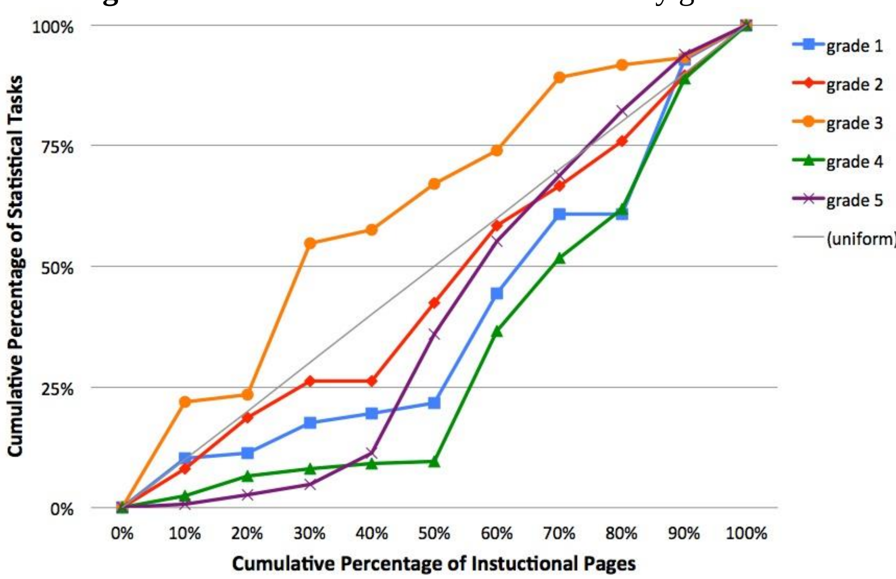
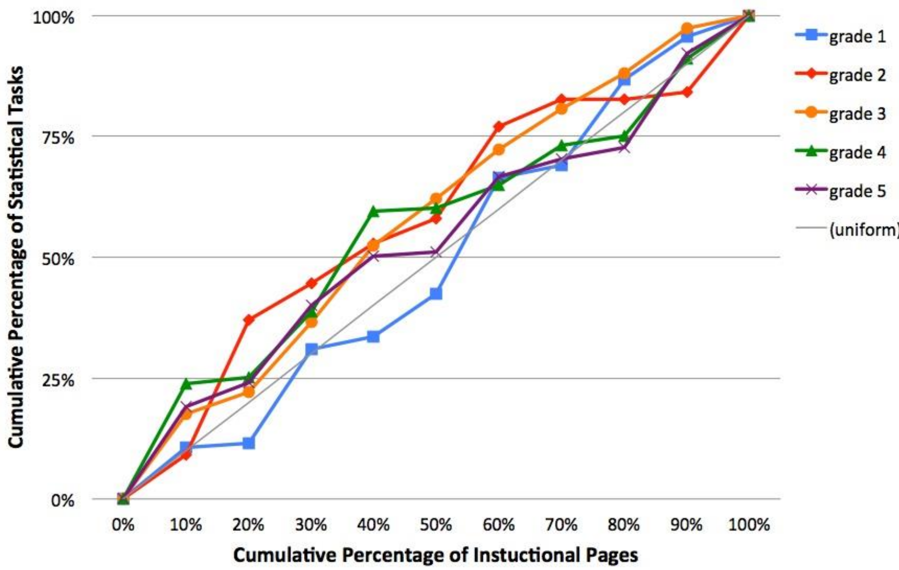


Figure 8. Location of statistics tasks in MT by grade level



The final two series display more uniform distributions of statistical tasks across the instructional pages. In SM (Figure 7), this may be explained by the spiral review feature of the textbooks. Lessons on statistical topics are placed throughout the textbook; the statistical tasks are likewise interspersed at regular intervals. At the same time, these statistics lessons and tasks are not integrated with the surrounding material in a salient manner. Instead, stand-alone statistics tasks appear within problem sets of otherwise unrelated mathematics lessons. We see MT (Figure 8) also has a roughly uniform distribution, but not because of spiral review. In stark contrast to SM, most lessons in MT integrate mathematical topics from different strands. For example, students may be asked to collect data and make a graph within a lesson that focuses measuring the perimeter of a rectangle.

4.2 Phases of the Statistical Problem Solving Process

Each textbook series, taken as a unit of books from grades 1 through 5, addressed the four phases of the statistical problem solving process (Formulate Questions, Collect Data, Analyze Data, and Interpret Results). Additionally, at least one series addressed each phase for every grade level. Having said this, not every textbook addressed all four of these phases. In the following sections, we will report our results by textbook series without disaggregating by grade level.

4.2.1 Phases addressed in each series

In Table 7, we report the proportion of the statistics tasks in a series that addressed each phase. All five series in our sample contained statistical tasks that addressed more than one phase. For that reason, the sum of percentages in each column is greater than 100. Furthermore, similar patterns were observed within the textbooks for each grade level, so results are reported as an aggregate for each series.

The major focus of statistical tasks within our sample was Analyze Data. As shown in Table 7, there was an overwhelming proportion (and number) of such tasks for every series. In four of the five series, relatively little attention is given to Formulate Questions, Collect Data, or Interpret Results. It appears that MT has a slightly different distribution; about three of every four tasks addressed Analyze Data, whereas more than 90% of the statistics tasks addressed this phase for the other four series. In the other phases, MT had the largest proportion (and number) of statistical tasks when compared to the other four series. In terms of relative emphasis, the number of statistical tasks addressing Analyze Data in MT is about 2.5 times the combined number of statistical tasks addressing the other phases. For the other series in our sample, this factor ranges from 8.7 in SM to 16.2 in TGM.

Table 7. Proportion (and number) of statistical tasks addressing each phase, by series

	Textbook Series				
	TGM	eVM	MC	SM	MT
Phase	*n* = 1524	*n* = 1821	*n* = 1648	*n* = 907	*n* = 1545
Formulate Questions	3% (52)	2% (41)	1% (19)	2% (19)	9% (142)
Collect Data	2% (31)	5% (90)	5% (78)	8% (73)	15% (237)
Analyze Data	98% (1491)	94% (1706)	97% (1600)	94% (849)	75% (1163)
Interpret Results	< 1% (9)	2% (40)	1% (20)	< 1% (6)	5% (82)

Note. Percentages for each column add to more than 100 because a task may be coded for multiple phases.

In four of the five series, Collect Data was the second most common phase to be addressed. Similarly, Interpret Results received the least attention in four of the five series. While every series addressed each level, some textbooks for particular grade levels did not address all four phases. Two textbooks (MC grade 1 and SM grade 1) did not contain any tasks coded as Formulate Questions. One textbook (TGM grade 4) did not contain any tasks coded as Collect Data. We found eight textbooks (TGM grades 1, 3, and 4; eVM grades 1 and 3; and SM grades 1, 2, and 3) that did not contain any tasks coded as Interpret Results.

4.2.2 Analysis Activities

We further classified the tasks coded as Analyze Data as including at least one of four Analysis Activities: Read a Display, Perform a Mathematical Calculation, Construct a Display, and Use Other Statistical Reasoning. For each series, Table 8 displays the proportion and number of Analyze Data tasks coded for each Analysis Activity. We report these results by textbook series, because the observed patterns still hold when data are disaggregated by grade level. The majority of Analyze Data tasks required students to Read a Display, such as a table or graph. In TGM and eVM, over 90% of these tasks involved this activity. The second most frequent Analysis Activity code assigned was Perform a Mathematical Calculation; the relative amount of Analyze Data tasks with this Analysis Activity ranged from about one third (in MT) to more than half (in TGM, eVM, and MC). Each series also contained tasks that required students to Construct a

Display; this was the third most common Analysis Activity in every series. A small proportion of tasks required students to Use Other Statistical Reasoning. By series, this ranged from 1% of the Analyze Data tasks in SM to 10% in MT.

Table 8. Proportion (and number) of Analyze Data tasks with each Analysis Activity, by series

	Textbook Series				
	TGM	eVM	MC	SM	MT
Analysis Activity	*n* = 1491	*n* = 1706	*n* = 1600	*n* = 849	*n* = 1163
Read a Display	93% (1383)	91% (1545)	82% (1311)	78% (662)	80% (925)
Perform a Mathematical Calculation	57% (851)	54% (922)	56% (892)	46% (392)	35% (406)
Construct a Display	15% (223)	14% (233)	21% (340)	20% (167)	14% (167)
Use Other Statistical Reasoning	5% (73)	3% (48)	4% (62)	1% (10)	10% (118)

Note. Percentages for each column add to more than 100 because a task may be coded for multiple Analysis Activities.

It was possible for these tasks to address more than one Analysis Activity, which is why the percentages for a given column in Table 8 have a sum of more than 100%. In fact, we found a majority of Analyze Data tasks addressed more than one Analysis Activity. The most common combination of Analysis Activity occurred in tasks that required students to Read a Display and Perform a Mathematical Calculation; this accounted for 47% of the Analyze Data tasks.

5. Discussion

5.1 Distribution of Statistical Content

Across the textbook series in our sample, between 10.8% and 18.1% of the instructional pages contained statistical content. These proportions are slightly greater than the proportions reported in other studies of the statistical content of textbooks. Pickle (2012) reported that statistics lessons comprised 8% to 13.6% of the instructional pages of series for grades 6–8, while Jones and Jacobbe (2014) noted that 6.3% to 14.2% of the instructional pages of textbooks for prospective elementary teachers comprised statistics chapters. This apparent difference may be due to the fact that our study examined all pages, while the earlier studies examined statistics lessons only.

We found it encouraging that statistical content was included within each textbook in our sample. For the commercially produced textbook series TGM, eVM, and MC, the statistical content was typically located within a single chapter. This is similar to what Pickle (2012) found with textbooks for grades 6–8. On the other hand, the textbooks series SM and MT contained a more uniform distribution of statistical content across instructional pages. In SM, a typical statistics page contained two statistical tasks surrounded by unrelated mathematical tasks. By way of contrast, a typical statistics page in MT contained three to four statistical tasks alongside tasks from other mathematical areas that related to a single context.

Three of the five MC textbooks placed the statistics chapter near the beginning of the textbook (chapter 4 in grades 1 and 2, chapter 3 in grade 4). The statistics chapters in each of the TGM and eVM textbooks was located in the final 20% of instructional pages. It is interesting to note that this may be by design, as these textbooks were written to address the TEKS and the order of the chapters follows the order of the topics listed in the TEKS (Texas Education Agency 2013). We do not note this to suggest that the authors of the TEKS intentionally placed statistics near the end to deemphasize its importance. In a list of content standards, something must come at the end. On the other hand, textbook authors are allowed to arrange material as they wish, and matching the chapter sequence to the sequence of standards may communicate the unintentional message that statistics chapters are not as important as the initial chapters of the book.

5.2 Phases of the Statistical Problem Solving Process

Our findings show that the overwhelming majority of statistical tasks address the Analyze Data phase of the statistical problem solving process. This agrees with the findings of Jones and Jacobbe (2014) with respect to tasks in textbooks for prospective elementary teachers. Furthermore, the emphasis on Analyze Data is not surprising, given the similar emphasis found in state curriculum frameworks (Newton, Horvath, and Dieteker 2011) and the CCSSM (Jones and Jacobbe 2014).

Across the series, the distribution of proportions of statistics tasks coded as Formulate Questions was similar to the corresponding distribution in textbooks for prospective elementary teachers (Jones and Jacobbe 2014); the same can be said for the Collect Data phase. On the other hand, textbooks for prospective teachers tended to include a greater proportion of statistical tasks addressing Interpret Results (ranging from 3% to 25.5%) when compared to the textbook series intended for elementary students (ranging from less than 1% to 5%). While it is encouraging that prospective elementary teachers have opportunities to engage in tasks from each phase, we would like to see increases in proportions for these phases in textbooks for prospective elementary teachers and elementary students.

The GAISE Report (Franklin et al. 2007) described three levels of increasing statistical sophistication, beginning with Level A. While we did not classify tasks according to these levels, it stands to reason that textbooks for grades 1–5 would include tasks at the initial level. Having said this, in the Formulate Question phase of Level A, teachers pose questions of interest to the student. Therefore, we may not expect to see evidence of teachers posing questions within the statistical tasks; instead, such information could appear elsewhere, such as the teacher’s edition of the textbook. That being said, we did code 273 tasks as Formulate Questions. The majority of these tasks (142) were located in MT, representing 9% of the statistical tasks in that series.

In terms of Analysis Activities, students were most often asked to Read a Display and Perform a Mathematical Calculation, and, to a lesser extent, Construct a Display. The relative proportions of these Analysis Activities was similar in all five series in our sample. Therefore, we believe this to be typical for most elementary mathematics textbooks. Viewed one way, most of the statistical content in mathematics textbooks for elementary students centers on data visualization in some way. On the other hand, the prevalence of such tasks may lead to the narrow view of statistics as looking at tables of data and "crunching numbers." Interestingly, there is not a similar consensus portrayed in textbooks for prospective elementary teachers. While Jones and Jacobbe (2014) found three textbooks that focused primarily on these Analysis Activities, two textbooks had more than 60% of Analyze Data tasks coded as Use Other Statistical Reasoning. It may be that prospective teachers who learn with these latter textbooks are better equipped to supplement their curriculum materials, if desired.

6. Conclusion

6.1 Implications and Recommendations

These results suggest that elementary mathematics textbooks do not place equal emphasis on the different phases of the statistical process. If textbooks predominantly focus on Analyze Data, they may inadvertently restrict opportunities for students to participate in other phases of the statistical problem solving process. Additionally, within Analyze Data, there is a large emphasis on procedural activities. As technology advances and the need for computation diminishes, the demand for statistical problem-solvers will overwhelmingly increase (Wild and Pfannkuch 1999). Therefore, students need more exposure to tasks involving statistical reasoning beyond reading and constructing displays and performing mathematical calculations.

We recommend that textbook authors incorporate more phases of the statistical problem-solving process throughout the entire textbook. In doing so, the textbook content would more closely adhere to the recommendations in the GAISE Report. The MT series demonstrates one way that this could be done: ask students to design and conduct surveys, and then analyze and interpret their results in context.

We also recommend that mathematics coordinators (at the state, district, and building level) become informed about the statistical content of their textbooks. The location and nature of this content may be quite different, depending on the series. A teacher using TGM or eVM will see very little statistics until the end of the text, while one using MC, SM, or MT will encounter statistical topics earlier. Teachers should also be prepared to supplement textbooks with statistical tasks using real data from the classroom or the larger world. Such content could be integrated with other mathematics topics or other subject areas.

6.2 Limitations

Our sample included five textbook series intended for students in grades 1–5. This is only a small portion of the total number of series available for these grade levels in the United States. At the same time, our sample was intentionally diverse, and included both commercially produced textbooks alongside those developed with funding from the National Science Foundation. While it is likely that other series may differ somewhat from those in our sample, we believe that the areas of agreement within the textbooks we analyzed paint a picture of the statistical content of U.S. elementary textbooks.

A second limitation arises in the fact that we are measuring the textbook curriculum – that which is intended (by textbook authors) to be implemented in classrooms. Ultimately, the classroom teacher may choose to include some portions of the textbook as written, include other portions after some modification, and omit the remaining portions (Malzahn 2013). Teachers may even include statistical content from resources outside of the textbook. With all of that said, our analysis provides one measure of students' opportunity to learn statistics, but it is by no means the only measure.

6.3 Future Directions

With this in mind, we call for future research to investigate the implementation of statistical content in the classroom, giving attention to the teachers' fidelity of implementation when using the text (Remillard 2005). A series of studies could also investigate the potential effect of the textbook on students' conceptual understanding of statistics, perhaps using the LOCUS Assessments (Jacobbe and Franklin 2013). Finally, it would be helpful to compare the statistical content of U.S. textbooks with those used in other countries.

As society advances, so does the need for statistical reasoning. The school classroom is a natural, and convenient, location to engage students in the statistical problem solving process. It is preferable, and perhaps necessary, to address statistical topics in a context, which leads to the application of statistical reasoning to science, social studies, and other areas of the school curriculum, beginning in the early grades. These efforts, if consistently applied within the elementary classroom, would serve as a bold first step in the development of a statistically literate society.

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