Conceptualizing a Framework for Advanced Placement Statistics Teaching Knowledge

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Key Words: Secondary statistics education; Statistics content knowledge; Statistics pedagogical content knowledge; Advanced placement statistics

Abstract

The purpose of this article is to sketch a conceptualization of a framework for Advanced Placement (AP) Statistics Teaching Knowledge. Recent research continues to problematize the lack of knowledge and preparation among secondary level statistics teachers. The College Board’s AP Statistics course continues to grow and gain popularity, but is a challenge for most secondary teachers to teach because of the emphasis on conceptual understanding and problem solving. Therefore, examining the components of teaching knowledge required for the course is of high importance. Using existing statistics teaching knowledge guidelines, previous research findings, national standards and College Board content requirements, a teaching knowledge framework for AP Statistics is proposed that will appropriately focus the teaching of AP Statistics courses to better prepare teachers and minimize current deficiencies.

1. Introduction

Recent research in statistics education continues to problematize the lack of knowledge and preparation among secondary level statistics teachers. This problem is compounded for The College Board’s Advanced Placement (AP) Statistics course. Although student enrollment has increased more than 20 times, as evidenced by national exam numbers of slightly less than 8,000 exams in 1997 to nearly 200,000 in 2015 (The College Board 2015), the number of capable teachers has not kept pace. In fact, recent feedback from The College Board calls for more training of AP Statistics teachers, especially in the areas of statistics content and active student learning (Franklin et al. 2011).
Furthermore, the newly released Statistical Education of Teachers (SET) report by the *American Statistical Association (ASA)* (2015) states that there are few research-based guidelines detailing what secondary-level teachers need to know to effectively teach statistics. Yet despite the need, no framework exists that describes or outlines the content or pedagogical content knowledge required to teach AP Statistics. The conceptual framework presented here seeks to fill in this gap by outlining specific components of AP Statistics teaching knowledge.

AP Statistics teaching knowledge is unique because AP teachers experience a heightened challenge beyond that of other secondary-level statistics teachers. The course requires them to teach not only basic, introductory statistical concepts, but also integrate statistical reasoning and literacy. Teachers must also incorporate ever-changing technological advances in data analysis software used by computers and calculators - tools with which many in-service and prospective statistics teachers feel inadequate (*Franklin 2010*). Research has also shown that teaching this course presents a unique challenge for most secondary teachers because of the emphasis on conceptual understanding and problem solving (*Gould and Peck 2004*). Therefore, AP Statistics teachers are faced with the task of acquiring and maintaining a high level of relevant content and pedagogical content knowledge.

## 2. Background of AP Statistics Course

First administered in 1997, the AP Statistics course has a strong emphasis on conceptual understanding and de-emphasizes probability (outside of the context of inference) and computation (*Gould and Peck 2004*). The course focuses on clear and accurate interpretation of statistical analyses, and heavily weighs a student’s ability to demonstrate clear, written communication skills in preparation for the AP Statistics exam. The AP Statistics exam consists of 40 multiple-choice questions and 6 free response questions. While 5 of the 6 free response questions pertain directly to course content, the sixth question is an investigative task that asks students to integrate statistical ideas and methods from the course to solve a problem of a type that they may not have previously encountered.

The major goal of the AP Statistics course is for students to obtain statistical literacy and a working knowledge of statistics that can be applied to a particular context. The AP Statistics course is not the traditional formula-oriented statistics often found in introductory statistics classes, and is intended to be taught at a conceptual level (*Roberts, Schaeffer, and Watkins 1999*). Thus, teachers cannot simply teach comfortable algorithmic formulas that are easy to memorize. Today's AP Statistics students are encouraged to discover key ideas and concepts with hands-on activities, or by using simulation guided by computers and calculators. So, these students need teachers who can support them in this endeavor.

The AP Statistics course scope and sequence was developed in conjunction with the *National Council of Teachers of Mathematics’ (NCTM) Principles and Standards for School Mathematics* (2000), and align with more recent recommendations of the *ASA* (2007) and the *Common Core Standards for Statistics and Probability* (2010) as shown below in Table 1. All of these recommendations require students to analyze and describe data, formulate questions, plan experiments and apply inference techniques to justify conclusions. Rules of probability and
simulation are included in all areas; however, the AP exam has a strong focus on probability as it pertains to inference (Gould and Peck 2004).

Specifically, the AP Statistics course is centered around four broad conceptual themes: exploring data, collection of data, anticipating patterns, and statistical inference (detailed below in Table 1). Teachers are free to teach course topics in any sequence, though most follow The College Board (2014) recommended timeline of first covering exploring data with graphs and numerical summaries, then association, sampling, probability distributions, inference, group comparisons, association between categorical and continuous variables, and finally an introduction to regression. These topics are most often taught over two semesters during the high school academic year.

Table 1 compares The College Board’s AP Statistics content categories with the NCTM (2000) and Common Core (2010) standards, as well as the ASA’s (2007) Guidelines for Assessment and Instruction in Statistics Education (GAISE). Compared to each of The College Board’s four main content categories, national guidelines and standards vary little in what students should be able to do. All suggest students follow a general process of formulating questions or predictions, selecting methods to analyze data, and evaluating and interpreting results.
<table>
<thead>
<tr>
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<tbody>
<tr>
<td><strong>Exploring Data:</strong></td>
<td>Select and use appropriate statistical methods to analyze data</td>
<td><strong>Analyse Data</strong></td>
<td><strong>Interpreting Categorical and Quantitative Data</strong></td>
</tr>
<tr>
<td>Describing patterns and departures from patterns</td>
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<tr>
<td><strong>Sampling and Experimentation:</strong></td>
<td><strong>Formulate the Question</strong></td>
<td><strong>Collect the Data</strong></td>
<td><strong>Making inferences and justifying conclusions</strong></td>
</tr>
<tr>
<td>Planning and Conducting a Study</td>
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<tr>
<td><strong>Anticipating Patterns:</strong></td>
<td><strong>Understand and apply basic concepts of probability</strong></td>
<td><strong>Formulate the Question</strong></td>
<td><strong>Conditional Probability and the Rules of Probability</strong></td>
</tr>
<tr>
<td>Exploring random phenomena using probability and simulation</td>
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<tr>
<td><strong>Statistical Inference:</strong></td>
<td><strong>Develop and evaluate inferences and predictions that are based on data</strong></td>
<td><strong>Interpret Results</strong></td>
<td><strong>Making inferences and justifying conclusions</strong></td>
</tr>
<tr>
<td>Estimating population parameters and testing hypotheses</td>
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</tbody>
</table>
3. Purpose of the Framework

Research on secondary-level, in-service statistics teaching knowledge is incomplete, and a framework of teaching knowledge for AP Statistics does not exist. The College Board (2014) outlines specific learning goals and content domains for student learning, but does not address or delineate categories of content or pedagogical content knowledge necessary to teach the subject. The conceptual framework outlined in this paper is an initial attempt at describing the components of content and pedagogical content knowledge for teaching AP Statistics.

The Advanced Placement Statistics Teaching Knowledge (or APSTK) conceptual framework will be used to develop items for an online assessment of AP Statistics teaching knowledge. A recent report from the ASA (2012) states that a top priority in the field of statistics education is the need for more assessment instruments to measure statistics teaching knowledge in order to improve teacher development and teaching practice. Furthermore, the ASA’s SET report (2015) calls for more efforts to assess teacher knowledge at all levels of the statistical problem-solving process. It is hoped that the APSTK assessment will measure subject-specific teaching knowledge of current AP Statistics teachers to gain a more comprehensive idea of what knowledge teachers possess or do not possess. Scores from this assessment will be used to establish a benchmark, or average level, of AP Statistics teaching knowledge. Research efforts to validate the constructs presented in this conceptual framework have taken place (Haines 2014) and a pilot study of the APSTK online assessment is already underway.

The APSTK conceptual framework and online teaching knowledge assessment can be used in a variety of ways to improve secondary-level statistics teaching and learning. For example, they can be used to focus college-level statistics instruction on specific content areas required of future AP Statistics teachers. Recently, Rossman, St. Laurent, and Tabor (2015) declared that teacher preparation is one of the main on-going challenges facing the AP Statistics program. Teacher preparation programs will find this framework useful in identifying and addressing knowledge deficiencies of pre-service statistics teachers. Addressing these areas at the college-level will better prepare teachers before they enter the secondary classroom.

Finally, those who provide professional development for in-service statistics teachers can use this framework, and subsequent assessment, as an online learning tool to focus and improve training in teacher knowledge areas that are lacking or deficient. The APSTK online assessment can also be implemented in a pre/post test format to analyze the impact of professional development on content and pedagogical content knowledge levels.

3.1 Overview

The conceptualization of a framework for AP Statistics teaching knowledge begins with a new, overarching construct entitled Advanced Placement Statistics Teaching Knowledge (APSTK). APSTK is grounded in research pertaining to mathematical and statistical content and pedagogical content knowledge, and is comprised of two smaller constructs. First, AP Statistics Content Knowledge (APSCK) that consists of The College Board’s (2014) content themes and specialized content knowledge unique to teaching the subject. Second, AP Statistics Pedagogical Content Knowledge (APSPCK) as it relates to subject-specific student learning (Shulman 1986;
Ball, Hoover, and Phelps 2008; Schoenfeld and Kilpatrick 2008), and the ability to adjust pedagogical strategies.

### 3.1.1 AP Statistics Content Knowledge

APSCK is defined as problem-solving content knowledge (Schoenfeld 1985) combined with a specialized content knowledge (Ball et al. 2008; Groth 2013) specific to teaching AP Statistics. This includes the ability to identify and recognize a variety of ways AP Statistics course content is organized, and the ability to present AP Statistics content as necessary to increase student understanding.

Shulman (1986) stated that “subject matter content knowledge” is a teacher’s ability to understand the variety of ways content of a discipline is organized as well as competing representations of the same concept. Schoenfeld (1985) recognized that mathematics teachers must use both “declarative knowledge” and “problem-solving knowledge” to not only memorize formulas, but also disentangle mathematical problems along the way. More recently, Ball et al. (2008) showed that “subject matter knowledge” for teaching mathematics is more than general mathematical knowledge or ability. Finally, Groth (2013) expanded on this idea in his Statistical Knowledge for Teaching framework by describing a “specialized content knowledge” unique to teaching statistics, and thereby distinctly different than teaching mathematics. This specialized content knowledge includes the ability to make statistics content understandable to students.

Therefore, APSCK is an intersection of Shulman’s (1986) concept of “subject matter content knowledge,” Schoenfeld’s (1985) “declarative” and “conceptual” levels of “content knowledge,” as well as Ball et al.’s (2008) and Groth’s (2013) components of content knowledge for teaching. Although APSCK is aligned with AP Statistics content it also includes additional components of content knowledge specific to teaching the course. Specifically, APSCK is meant to identify more than the required AP Statistics students’ content knowledge as outlined by The College Board (2014).

For example, an AP Statistics teacher who possesses APSCK not only knows the types and differences of hypothesis tests, but also how to organize instruction in a way that promotes student understanding of statistical inference at a conceptual level. This teacher provides students various opportunities to use inference to answer real-world research questions, draw conclusions, determine levels of uncertainty and evaluate the appropriateness of their conclusions.

Table 2 details the characteristics of APSCK by comparing AP Statistics content themes of Exploring Data, Sampling and Experimentation, Anticipating Patterns, and Statistical Inference with the three main components of APSCK: Problem Solving Knowledge, Organizing Content, and Student Learning.
Table 2. Characteristics of APSCK

<table>
<thead>
<tr>
<th>APSCK Definitional Components</th>
<th>Problem Solving Knowledge</th>
<th>Organizing Content</th>
<th>Student Learning</th>
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</thead>
<tbody>
<tr>
<td>AP Statistics Content Themes</td>
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</tr>
<tr>
<td><strong>Exploring Data</strong></td>
<td>Interpret information from graphical and numerical displays</td>
<td>Decide when and which graphical and numerical techniques should be taught</td>
<td>Explain how graphical and numerical techniques are used to visualize data</td>
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<tr>
<td></td>
<td>Analyze beyond the data to make generalizations beyond the given context</td>
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<td>Create explanations for why departures from patterns might exist</td>
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<td></td>
<td>Evaluate the benefits and shortfalls of choosing one technique over another</td>
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<td></td>
</tr>
<tr>
<td><strong>Sampling and Experimentation</strong></td>
<td>Understand sampling designs with random selection</td>
<td>Determine when and which sampling methods and experimental procedures should be taught</td>
<td>Model situations that use various types of sampling methods and experimental designs</td>
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<tr>
<td></td>
<td>Apply experimental designs using randomization</td>
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<td></td>
<td>Differentiate methods of data collection and analysis</td>
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<td></td>
<td>Develop appropriate conjectures based on a well-developed plan</td>
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<tr>
<td></td>
<td>Design experiments to test conjectured relationships among variables</td>
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</tr>
<tr>
<td><strong>Anticipating Patterns</strong></td>
<td>Measure uncertainty using probability models and theory</td>
<td>Decide when and which probability topics should be taught</td>
<td>Model the use of probability theory in real-world scenarios</td>
</tr>
</tbody>
</table>
Experiment with a variety of probability models

Compare and contrast continuous and discrete probability distributions

Use simulations to solve problems

<table>
<thead>
<tr>
<th>Statistical Inference</th>
<th>Recognize unique characteristics of each type of hypothesis test</th>
<th>Determine when and which statistical inference topics should be taught</th>
<th>Create an opportunity to use hypothesis testing in real-life situations that is meaningful to students</th>
<th>Model the use of probability rules in connection with inference procedures</th>
</tr>
</thead>
</table>

### 3.1.2 AP Statistics Pedagogical Content Knowledge

The second construct that constitutes APSTK is AP Statistics Pedagogical Content Knowledge or APSPCK. APSPCK is defined as a teacher’s knowledge of how his/her students learn or do not learn specific topics (Shulman 1986) within the AP Statistics curriculum. This includes being able to anticipate and interpret student thinking as it is happening. Furthermore, a teacher’s APSPCK allows for recognizing common errors or misconceptions students have (Shulman 1986; Ball et al. 2008; Schoenfeld and Kilpatrick 2008) and adjusting instruction, curriculum choices and assessment strategies accordingly (Ball 1988).

Research shows that components of pedagogical content knowledge have been refined over the past 25 years to emphasize a teacher’s knowledge of subject-specific student learning. The focus now is on how teachers combine content knowledge and knowledge of their students as learners in the context of a particular class. Today, the make-up of pedagogical content knowledge (including APSPCK) is complex, and encapsulates more than just recognizing student errors and misconceptions.

Furthermore, Ball et al. (2008), Groth (2008, 2013), Hill, Ball, and Schilling (2008), Schoenfeld and Kilpatrick (2008) and Shulman (1986) state that pedagogical content knowledge is inherently subject-specific. Specifically, pedagogical content knowledge describes the way a
teacher interprets a subject to make it more comprehensible to others. This includes identifying errors and misconceptions, changing explanations, and formulating the subject to accommodate shifting student interpretation, thus making the learning of specific topics easier or more difficult. A key point here is that pedagogical content knowledge is specific to one’s subject area.

Schoenfeld and Kilpatrick (2008) concur that pedagogical content knowledge includes knowledge of typical student understandings and misunderstandings and of ways to deal with them. They say that a teacher needs to recognize which errors are frequent occurrences among students and develop a set of responses to help the students see clearly what is wrong. Central to the idea of APSPCK is a teacher’s ability to not only recognize a wrong answer, but also know, in advance, which wrong answers students are most likely to make.

Finally, Ball et al. (2008) and Groth (2013) describe how pedagogical content knowledge is a teacher’s ability to arrange and deliver instruction in a way that facilitates student learning. This would include making appropriate curricular choices, sequencing topics in an order that makes sense, being familiar with how students think about a particular concept, and deepening student thinking to go beyond simple memorization. Therefore, pedagogical content knowledge requires the teacher to be able to anticipate and interpret students’ thinking about a concept as it is emerging or taking shape.

An AP Statistics teacher must not only recognize a student’s misconception that sampling error and bias are the same concept, for example, but also determine why the student has the misconception and decide how to adapt his/her instructional strategies to lessen the confusion. A teacher who possesses APSPCK will anticipate, recognize and accurately interpret the misconception, and adjust his/her instructional strategies to better facilitate student learning on a more conceptual level.

Conceptualization of APSPCK also includes Franklin et al.’s (2005) American Statistical Association Guidelines for Assessment and Instruction in Statistics Education or GAISE framework. This framework established recommendations for curriculum, instruction and student learning for K-12 statistics education based on the NCTM’s Principles and Standards for School Mathematics (2000). GAISE’s learning objectives were broken down into three levels, A, B, and C based on student development rather than age. Casey (2008) states that objectives for levels B and C are relevant to secondary students. These objectives are outlined in Appendix A.

Furthermore, the GAISE College Report (Garfield et al. 2005) provided six recommendations for statistics curricular choices: emphasize statistical literacy and develop statistical thinking, use real data, stress conceptual understanding rather than mere knowledge of procedures, foster active learning, use technology to develop conceptual understanding and to analyze data, and use assessments to improve and evaluate learning. Also, Groth (2008) outlined five GAISE pedagogical recommendations that build upon current discourse themes within the statistics education community. These themes include statistical literacy, variability and contextual problem solving. They are outlined in Appendix B.

Table 3 outlines the characteristics of APSPCK based on the ideas of Shulman (1986), Schoenfeld & Kilpatrick (2008), Ball et al. (2008), and Groth (2013) as well as GAISE
recommendations for learning objectives, curricular choices and pedagogical recommendations (Franklin et al. 2005; Garfield et al. 2005; Groth 2008).

### Table 3. Characteristics of APSPCK

<table>
<thead>
<tr>
<th>APSPCK Definitional Components</th>
<th>Adjust curriculum, instruction and assessment strategies</th>
<th>Recognize and interpret errors and misconceptions</th>
<th>Aid in student thinking and learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>APSPCK Characteristics</td>
<td>Change instructional strategies based on students learning or not learning</td>
<td>Identify errors and misconceptions on student work, quizzes, projects and tests</td>
<td>Analyze student learning in a way that encourages conceptual thinking</td>
</tr>
</tbody>
</table>

#### 3.2 Conceptual Framework

Figure 1, below, provides an illustration of the proposed framework for APSTK. The overarching APSTK construct is conceived by combining the ideas of APSCK and APSPCK. Components of APSCK are shown to be the four AP Statistics content themes and the components of APSPCK are shown to be the three definitional components of APSPCK previously described. In addition, there is assumed to be some overlap between APSCK and APSPCK based on the current definitions.

Figure 1. Conceptual Framework for APSTK
4. Conclusion

AP Statistics teachers must utilize a unique blend of content and pedagogical content knowledge to teach statistical content and reasoning as well as statistical problem solving and literacy skills. These teachers are required to teach beyond that of typical secondary statistics teachers, yet no specific teaching knowledge framework exists to describe the knowledge they need. Understanding the make-up of teaching knowledge for the subject is important in order to identify knowledge deficiencies and improve preparation and professional development of AP Statistics teachers. This paper proposes a framework to define and describe constructs that outline specific types of teaching knowledge unique to the subject of AP Statistics.
APPENDIX A

GAISE Learning Objectives

<table>
<thead>
<tr>
<th>Level B:</th>
<th>Level C:</th>
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<tbody>
<tr>
<td>Students become more aware of the statistical question distinction (a question with an answer based on data that vary versus a question with a deterministic answer).</td>
<td>Students should be able to formulate questions that can be answered with data.</td>
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<tr>
<td>Students make decisions about what variables to measure and how to measure them in order to address the question posed.</td>
<td>Students should be able to devise a reasonable plan for collecting appropriate data through observation, sampling or experimentation.</td>
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<tr>
<td>Students use and expand the graphical, tabular and numerical summaries introduced at Level A to investigate more sophisticated problems.</td>
<td>Students should be able to draw conclusions and use data to support these conclusions.</td>
</tr>
<tr>
<td>Students develop a basic understanding of the role that probability plays in random selection when selecting a sample and in random assignment when conducting an experiment.</td>
<td>Students should be able to understand the role that variability plays in the decision making process.</td>
</tr>
<tr>
<td>Students investigate problems with more emphasis placed on possible associations among two or more variables and understand how a more sophisticated collection of graphical, tabular and numerical summaries is used to address these questions.</td>
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<tr>
<td>Students recognize ways that statistics is used or misused in their world.</td>
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</table>
APPENDIX B

GAISE Pedagogical Recommendations

Statistical literacy should be a prominent curricular goal because of the central role it plays in democratic citizenship, personal choices, careers, and evaluating scientific findings.

Statistics and mathematics differ as disciplines. Statistics utilizes mathematics but should not be mistaken for a branch of mathematics.

The study of variability should have a central role in school statistics. Students should understand a variety of types of variability, including measurement, natural, induced, and sampling variability.

Statistical problem solving is heavily reliant upon context. It is not possible to give plausible interpretations of data without some knowledge of the context that generated them.

Pre-college experiences with statistics require an intuitive grasp of probability. Probability is an important tool in statistical analysis, but doing mathematical probability problems should not be mistaken for doing data analysis.

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