



Teachers and Textbooks: On Statistical Definitions in Senior Secondary Mathematics

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Abstract

The new *Australian Senior Secondary Curriculum: Mathematics* contains more statistics than the existing Australian Curricula. This case study examines how a group of Queensland mathematics teachers define the word “statistics” and five statistical terms from the new curricula. These definitions are compared to those used in some commonly-used Queensland mathematics textbooks and in the glossaries of new *Australian Senior Secondary Curriculum: Mathematics*. The findings of this study suggest that many teachers do not have a good understanding of statistical concepts and that they rely on procedural definitions (instrumental understanding). This is reflected in the presentation of statistics in Queensland senior secondary mathematics textbooks. Definitions in the glossaries of the new curricula are generally better but perhaps other simpler concepts could be introduced first to develop relational understanding.

1. Introduction

The new *Australian Senior Secondary Curriculum: Mathematics* was released in 2014 but to date is implemented in only one educational jurisdiction in Australia (the Australian Capital Territory). These new curricula consist of four subjects: *Essential Mathematics* ([ACARA 2014a](#)), *General Mathematics* ([ACARA 2014b](#)), *Mathematical Methods* ([ACARA 2014c](#)) and *Specialist Mathematics* ([ACARA 2014d](#)), each including some statistical content. For other

Australian jurisdictions (such as the state of Queensland), the statistical content in the new curricula is substantially greater than in the existing senior mathematics curricula and, in many cases, the new curricula include topics that do not appear in the current curricula. In Queensland, for example, topics such as sampling distributions and confidence intervals do not appear in the existing curricula (*Prevocational Mathematics* ([QSA 2004](#)), *Mathematics A* ([QSA 2014a](#)), *Mathematics B* ([QSA 2014b](#)) and *Mathematics C* ([QSA 2014c](#))), but do appear in the new *Specialist Mathematics Curriculum* ([ACARA 2014d](#)). Consequently, many teachers may be expected to teach content for which they are underprepared and even feel a measure of ambivalence ([Marshman, Dunn, McDougall, and Wiegand 2015](#)).

A part of teaching, learning and understanding any new topic is the need to master the language associated with that topic. The language of statistics presents many linguistic challenges ([Dunn, Carey, Richardson, and McDonald 2015](#)) that can make statistics difficult to teach, especially for teachers whose background is in teaching mathematics but not language. A case study of teachers in the Sunshine Coast region of Queensland, Australia ([Marshman et al. 2015](#)) showed a large proportion of teachers and pre-service teachers would like professional development to help them learn how to teach these new statistical topics, perhaps conceding that they realise their understanding is incomplete. As others have noted (for example, [Eichler 2011, p. 178](#)), what teachers are able to teach is not necessarily the same as what they are required to teach.

Because of this, teachers often rely heavily on textbooks, especially for topics that they are less familiar with. In the context of university lecturers, but easily extended to school teachers, [Utts \(2013\)](#) observes that:

Many instructors who teach introductory statistics were not trained in statistics, and may have little knowledge of the material or about what makes a good introductory course. For those instructors, the textbook is often their major source for learning the material they are teaching. (p. 4)

This means that the “textbook curriculum” is having a significant influence on the “teacher implemented curriculum” and teachers are deferring authority to the textbook rather than the discipline ([Jones and Jacobbe 2014](#)). In summary, the new *Australian Senior Secondary Curriculum: Mathematics* contains new topics in statistics that many mathematics teachers may feel unprepared to teach, and one reason for this unpreparedness may be that the teachers do not understand, and are unfamiliar with, the language of statistics. One consequence of this situation may be that teachers will defer to, and rely on, definitions provided by textbooks used in their classes.

In this paper, two research questions are addressed. The first aims to understand teachers’ present understanding of some statistical terms: “How well do teachers and pre-service teachers understand five statistical words related to the new curricula?” Since the authors expect teachers to have a poor understanding of these terms in general, the second research question aims to examine the definitions given in resources commonly, and primarily, used by Queensland senior secondary school teachers: “How well do some commonly-used textbooks, used as Queensland senior secondary school teaching resources in mathematics, define these same five statistical terms?”

2. Background

Numeracy is a focus of many of the mathematics curricular documents in OECD countries. The Australian Association of Mathematics Teachers (AAMT) defines numeracy as: “the disposition to use, in context, a combination of: underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic)...” ([AAMT 1997, p. 15](#)). This explicitly includes statistical literacy, and in Australian schools, statistics is taught within the mathematics curricula.

Since the publication of the American Statistical Association’s *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) for introductory statistics ([Aliaga et al. 2010](#)) and the corresponding pre-K–12 report ([Franklin et al. 2007](#)), the commentary and focus of statistics teaching has moved away from mathematical computations to statistical literacy (for example, [Bargagliotti 2012](#)). The Franklin report does not define “statistical literacy,” but begins by declaring “the ultimate goal: statistical literacy” ([Franklin et al. 2007, p. 1](#)). The GAISE College report defines “statistical literacy” as “understanding the basic language of statistics (for example, knowing what statistical terms and symbols mean and being able to read statistical graphs) and fundamental ideas of statistics” ([Aliaga et al. 2010, p. 14](#)). The former Australian Statistician, Dennis Trewin, defined statistical literacy as “...the ability to understand, interpret and evaluate statistical information” ([Trewin 2005](#)). These definitions are not sufficient for teachers because they do not make links to the context of a real-world application and the purpose of answering a question, which are explicit in the new curricula as the topics “statistical investigation” and “data investigation.” In addition, one aim of the *Australian Curriculum: Mathematics* is to ensure students are “able to investigate, represent and interpret situations in their personal and work lives and as active citizens” ([ACARA 2012](#)).

Statistical literacy is being recognised as an important skill for all members of the community:

There is no doubt that, in the age of information and computers, if we really want to render the citizens as independent as possible and free of influence and conditioning, the only real course of action is to have them attain a higher degree of statistical... literacy ([Biggeri and Zuliani 1999, sec. 2.2](#)).

Consequently, statistical literacy is recognised as a critical skill for students to master to engage with society ([Gal 2004](#)).

An important part of statistical literacy is the language of statistics. For teachers to be able to effectively teach their students to communicate with the language of statistics, they need to be able to use the language of statistics themselves. However, the language used in statistics presents many linguistic challenges ([Dunn et al. 2015](#)), and having misconceptions about the meaning of statistical language is not uncommon, even among teachers ([Batanero, Burrill, and Reading 2011](#)). The Australian Bureau of Statistics has observed that:

statistics is currently taught in a fragmented fashion in Australian schools and typically by teachers with little or no formal education in, or proper appreciation of statistics. Teachers also need to understand statistical subject matter at a depth greater than the content they need to teach if they are to feel confident in teaching statistics ([Ben-Zvi and Garfield 2004](#)). ([ABS 2013](#); reference appears in the original)

To master a new topic, teachers must be able to understand and appropriately *use* the language in all communication models: reading, writing, speaking and listening. Indeed, [Schield \(2004\)](#) notes that statistical literacy is “typically more about words than numbers, more about evidence than about formulas” (p. 9), and has a literacy demand that extends beyond just learning new words and their meaning ([Schmitt 2008](#)). However, statistics is traditionally taught with a focus on computations and formulae rather than the language and interpretation. Part of the reason, at least in Australia, may be that those who are given the task of teaching statistics are mathematics teachers who are more comfortable with numbers and formulae than with language.

Australian secondary mathematics teachers may have studied as few as one introductory statistics course at university. Studies have shown that students often emerge from these courses with a poor understanding of statistical terminology ([Kaplan, Fisher, and Rogness 2009, 2010](#)). [Richardson, Dunn, and Hutchins \(2013\)](#) came to a similar conclusion, but found that after taking an introductory statistics course many students were attempting to define words with statistical (rather than non-statistical) definitions after instruction, even if that definition was incorrect.

Teachers appear to have an instrumental understanding of statistical terms (“rules without reasons;” [Skemp 1978, p. 9](#)), because that is how they are represented in textbooks, rather than a relational understanding (“knowing both what to do and why;” [Skemp 1978; p. 9](#)). It is this relational understanding that will address the goal of statistical literacy.

As an indicator of teachers’ understanding the language of statistics, five terms were identified for the purpose of studying teachers’, pre-service teachers’ and textbooks’ understanding of statistical terms: “mean,” “standard deviation,” “sample,” “confidence interval” and “standard error.” Target definitions for these terms applied to the teachers and pre-service teachers’ definitions were focused on relational understanding rather than technical. It was expected that the ratings would be applied generously for four reasons: (1) teachers were not completing this survey under “examination” conditions; (2) they had not “studied” in preparation for this survey; (3) they were given no warning that such information would be sought; and (4) they were provided with limited (though sufficient) space and time to provide the definitions.

With this caveat in mind, the target definitions used were:

1. Mean: Responses such as “average” and providing a formula were both considered “correct” definitions. Note that no specific type of mean (such as arithmetic mean) is specified.
2. Standard deviation: A “correct definition” was allocated to definitions describing the “standard deviation” as a measure of the average or mean deviation of observations from the mean. Even though the precise definition is more technical, this definition was deemed to be accurate enough for the school context and the context in which the data were gathered. Providing a relational definition of the standard deviation can be difficult, as distinguishing the standard deviation from other measures of variation usually involves a description of the formula (usually a loose definition). For example, a definition such as “the amount of variation” is too general to differentiate between the standard deviation and interquartile range, so a useful definition of the standard deviation is akin to “the mean deviation of the observations from the mean.” However, even this definition is technically inaccurate (the definition actually describes the “mean absolute deviation,” or

MAD), though such definitions were coded as “Correct” for the purposes of this paper and is probably sufficient for use at school. Even more technical definitions, such as the “root-mean square error” are not completely accurate as the divisor when computing the sample standard deviation is $n - 1$ not n , but again coded as correct for our purposes. For this reason, [Franklin et al. \(2007, p. 44\)](#) explicitly discuss MAD through an activity and describe it as a “precursor to the standard deviation,” though they never provide a definition for “standard deviation” itself or an activity for developing the concept of “standard deviation.”

3. Sample: A correct definition indicated that a sample was a subset or part of the target population. Note that the term in question is a “sample,” but not necessarily a *random* sample.
4. Confidence interval: A correct definition described an interval in which a population parameter is likely to lie based on repeated sampling. Although this definition is not technically correct, this definition was based on relational understanding and was deemed sufficiently accurate for the given context in light of the four reasons identified above. Importantly, the raters were particularly looking for evidence in the definition that a confidence interval was formed for a population parameter, not for the data themselves.
5. Standard error: A correct definition described a measure of the amount of variation in a sample statistic when estimating a population parameter using sample information. The raters were particularly looking for evidence in the definition that a standard error measured variation in a sample statistic, not in the data themselves.

The term “standard error” does not appear explicitly in the curriculum, although the concept is implied. Some of these terms, or variations of these terms, have been identified as linguistically challenging ([Dunn et al. 2015](#)) or lexically ambiguous ([Kaplan et al. 2009](#); [Richardson et al. 2013](#)) and it is vital then to avoid issues of being statistically disingenuous in the promotion of understanding.

3. Methods

3.1 Participants

The Sunshine Coast region of Queensland hosts a support network for mathematics teachers (the MATHS network), which meets four times each year. At the meeting held in May 2014, speakers discussed the role of statistics in the new curricula, and attendees were asked to define the word “statistics.”

Following this meeting, the 2014 August MATHS meeting continued the focus on statistics. The meeting included a 30-minute plenary session to inform attendees of the current state of the new *Australian Senior Secondary Curriculum: Mathematics*, after which the attendees were asked to complete a short, anonymous survey. The survey consisted of three demographic questions (where the teacher was in terms of his or her career; his or her role in the school; his or her mathematics education), and attendees were also asked to define the five statistical words or terms previously identified.

The 25 teachers who responded to the August survey represented a cross-section of the mathematics-teaching community of the Sunshine Coast region ([Table 1](#)). Those attending varied

in their level of mathematics education, years of experience and current school role. The number of attendees exceeded the number who completed the survey, and more defined terms ($n = 25$) than provide demographic information ($n = 19$).

Table 1. The demographic characteristics of those responding to the survey ($n=19$; 25 responded to the remaining survey questions but not all complete the demographic questions). (HoDs: Heads of Departments; PSTs: pre-service teachers)

Characteristic	Role in school			
	Total	Teachers	HoDs	PSTs
<i>Levels of mathematics education</i>				
No or limited mathematics at university	3	2	1	0
With engineering degree	1	1	0	0
Minor in B.Ed	3	0	1	2
Major in B.Ed	6	4	1	1
Mathematics degree	6	3	3	0
<i>Where in career</i>				
Nearing retirement	3	2	1	0
Mid-career	10	5	5	0
Early career	3	3	0	0
Pre-service	3	0	0	3
Total for Role in school	19	10	6	3

3.2 Data analysis of attendees' definitions

After the data were collected from the teachers and the pre-service teachers, one of the authors and a research assistant (a former mathematics teacher who has been tutoring statistics at university, but who did not attend the MATHS meeting) studied each of the teachers' de-identified responses to the terms above. Before the data collection, the researchers agreed to code the teachers' response using the coding scheme used by [Richardson et al. \(2013\)](#) to evaluate university students' definition of statistical terms. The two raters together allocated each definition to one of the following codes:

1. Correct. In these cases, the supplied definitions were very close to the target definitions described earlier.

2. Incomplete, or some elements correct (“Somewhat correct”). In these cases, the definitions were close to the target definitions but were lacking important details or were incomplete.
3. Statistical but incorrect. In these cases, the definitions had statistical elements, but were incorrect.
4. Non-statistical. In these cases, the definitions were non-statistical, and possibly based on definitions used in general English ([Dunn et al. 2015](#)).
5. Without merit. In these cases, the answers were clearly wrong, or the respondents were clearly not making legitimate attempts at defining the term.
6. No response.

The two raters discussed the rating until both raters agreed with the allocation, remembering that the ratings would be applied generously.

While many allocations of codes to definitions were straightforward, a small number of allocations generated substantial discussion between the raters. Hence, while most definitions were relatively straightforward to allocate, the allocation of codes in a small number of cases is subjective. Part of the reason is that determining the intent and understanding behind a short, written definition is difficult.

For clarification, examples of how the ratings were applied are shown in [Table 2](#). For the reasons given above, we also decided not to classify the teachers’ definitions as relational or instrumental. A short definition does not demonstrate how teachers would develop the concept in the classroom.

Table 2. Examples of how the ratings were applied for three example terms, as given by teachers, Heads of Departments (HoD), pre-service teachers (PST) and other attendees.

Code	Mean	Standard deviation	Standard error
Correct (1)	Average of a given set of numbers (teacher)	The spread of the set of scores found from the difference from the mean and squared, averaged and square root (other)	
	Average (numerous)	The average distance between scores and the average (PST)	
Somewhat correct (2)	An exemplar value that generalises a set of data (PST)	The variance in a set of data (PST)	A measure of the precision of a sample in describing a population (PST)
	The value at which the centre of the data would lie if ordered canonically $\bar{x} = \sum x/n$ (PST)	How far from the mean a given set of data is (PST)	Average measure of error in a calculated value (HoD)
Statistical but incorrect (3)		Degree of deviation (teacher)	The error present in a set of data (PST) Residuals? (teacher)
Non-statistical (4)			Error within the measuring instrument (teacher)
Without merit (5)			An error that can never be gotten rid of (PST) Saying “specific” as “pacific” (teacher)

3.3 Textbook analysis

Definitions used for the terms above given in the curricula documents (in a Glossary) and the textbooks currently used in Queensland schools were also noted (but not classified as for teachers' definitions). The chosen textbooks were selected because they were commonly used in schools in the Sunshine Coast region for teaching senior mathematics (Table 3). These textbooks are the ones that contained the appropriate statistical content, and cover Year 11 Mathematics A and Mathematics B and Year 12 Mathematics A and the International Baccalaureate (IB). Queensland does not specify a particular order in which concepts need to be taught, hence schools and textbooks choose the order and year level, either Year 11 or Year 12, in which to teach the concepts. This means that statistical concepts are not in every textbook. For the terms “confidence interval” and “standard error,” which are not part of the current Queensland curricula, recently-used textbooks (Brodie and Swift 1994a, 1994b, 1996) were examined from when these topics were taught in Queensland schools.

Table 3. The textbooks whose definitions were examined, the mathematics curriculum for which they correspond, and which definitions were provided by each textbook. A “y” means a definition is provided, an empty cell means no definition is provided, and a ? means the definition is alluded to but not explicitly provided.

Curriculum	Textbook	Statistics	Mean	Standard deviation	Sample	Confidence interval	Standard error
11A	Brodie and Swift (2008a)		y	y			
	Elms and Simpson (2011)	?	y	y			
11B	Brodie and Swift (2008b)	?	y	y	y		
	Simpson and Rowland (2009)	y	y	y	y		
12A	Brodie and Swift (2009)		y	y	y		
	Elms, Simpson, and McPherson (2011)	?	?	?	?		
IB	Pimentel and Wall (2010)		y	y			
12A	Brodie and Swift (1994a)					y	y
12B	Brodie and Swift (1994b)					y	y
12C	Brodie and Swift (1996)					y	y

We now discuss the teachers’ definitions of these terms and how the textbooks help teachers to understand these terms.

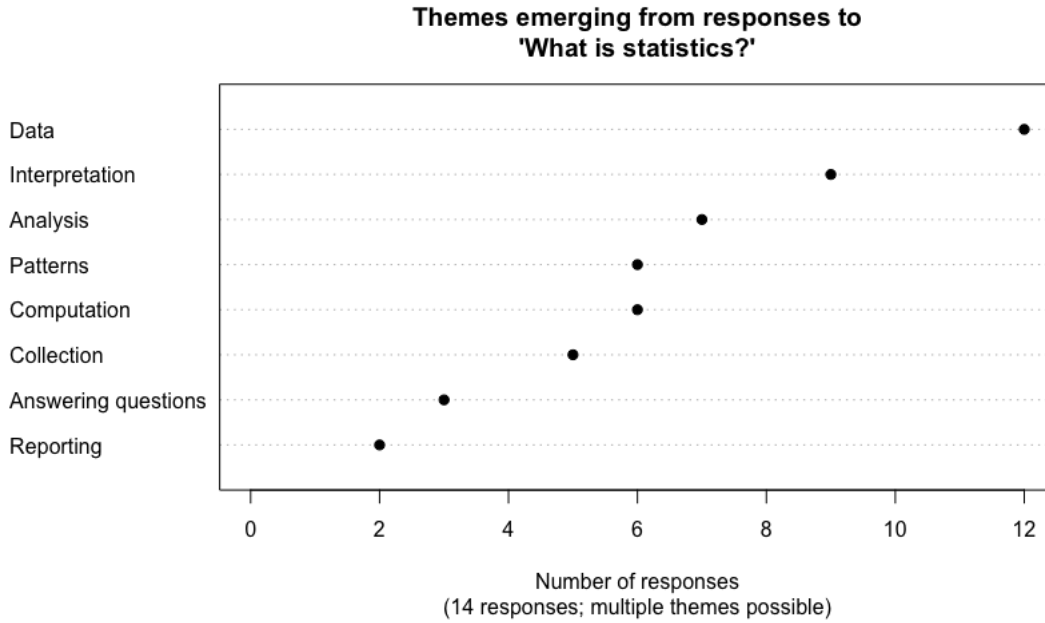
4. Results

In this section we detail the survey outcomes and compare and contrast text and reference document definitions in terms of their contribution to relational understanding.

4.1 Definition of “statistics”

Attendees to the May MATHS meeting were asked to define the term “statistics.” Fourteen attendees responded to this request. These definitions were read by two researchers to identify themes present in the definitions; definitions were often allocated to more than one theme. The most common themes were “data” and “interpretation” ([Figure 1](#)). This does not consider the evaluation of the statistical information, possibly the most crucial aspects for active citizens: statistical literacy.

Figure 1. Themes emerging from the definition of “statistics” from 14 respondents. Responses may have been allocated to more than one theme.



A theme of “data” was the most popular; examples responses include:

- “Making sense of a whole lot of data;”
- “Collecting and displaying, comparing and analysing data to identify trends (trends: mathematical representation of relationships).”
- “Using and analysing data in sophisticated ways to inform and identify trends.”

However, statistics was not often viewed as a means to answer a question, though teachers were trying to identify trends to understand the data (6 themed as “patterns;” 7 themed as “analysis;” 9 themed as “interpretation”) with having asked a question initially (3 themed as “answering questions”). Six responses had an explicit instrumental view that statistics was about “computation;” example responses include:

- “Number crunching.”
- “The mathematical analysis of discrete and continuous data.”

While mathematical computation is clearly involved in statistics (and statistics is taught within the mathematics curricula by mathematics teachers in Australia), statistics is more than computation:

The evidence that statistics is different from mathematics is not presented to argue that mathematics is not important to statistics education or that statistics education should not be a part of mathematics education. To the contrary, statistics education becomes increasingly mathematical as the level of understanding goes up. But data collection design, exploration of data, and the interpretation of results should be emphasized in statistics education for statistical literacy. These are heavily dependent on context, and, at the introductory level, involve limited formal mathematics. ([Franklin et al. 2007; p. 9](#))

An emphasis on computation in statistics on the part of teachers seems to be at odds with the *General Mathematics Curriculum* ([ACARA 2014b](#)) which explicitly includes topics such as the “data investigation process” (Unit 2, Topic 1) and the “statistical investigation process” (Unit 4, Topic 1). The new curricula are encouraging teachers to adopt and teach a relational

understanding, but teachers are viewing statistics more through a lens of instrumental understanding.

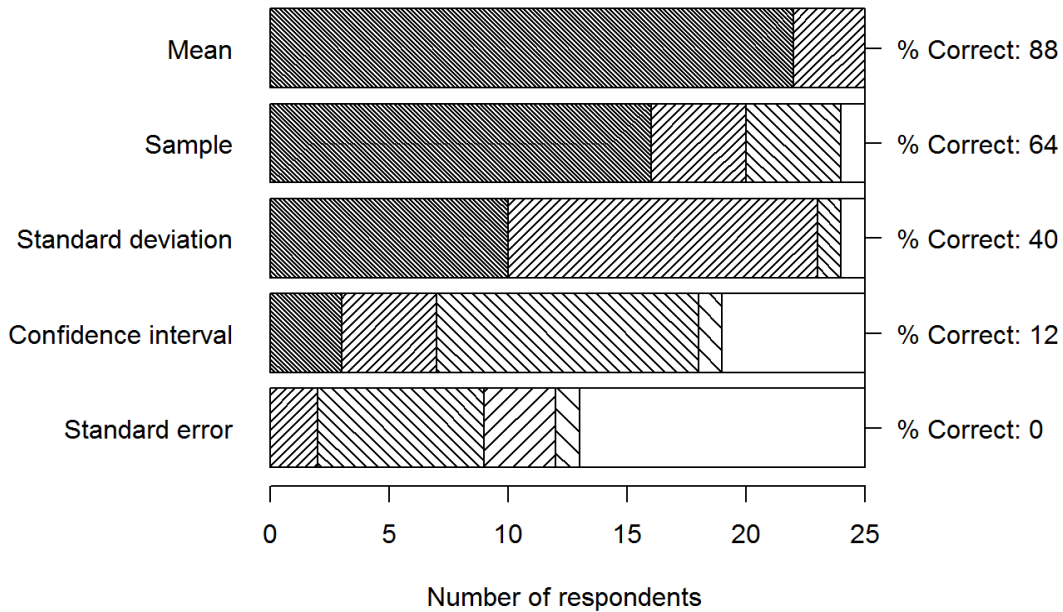
The current Queensland *Mathematics A* ([QSA 2014a](#)) and *Mathematics B* ([QSA 2014b](#)) curricula do not define “statistics,” but do define “summary statistics” in the Glossary as follows: “In descriptive statistics, summary statistics are used to summarise a set of observations, in order to communicate as much as possible as simply as possible,” then lists measures of central tendency, dispersion and the shape of a distribution. This definition implies that summary statistics are not for the purpose of answering a question or understanding a problem. The curricula itself mention statistics beyond summary statistics, including statistical modelling, but the only definition that appears is for “summary statistics.” “Statistics” is not defined in the new national curricula.

The textbooks studied do not all define “statistics” or related terms, and those that do offer a variety of definitions. [Brodie and Swift \(2008b, p. 38\)](#) only state that “the mathematical name for statistical information is **data**,” while [Brodie and Swift \(2008a, p. 320\)](#) make no attempt to define or introduce the term “statistics” but rather say that “statistical data is usually collected to help make decisions. In order to do this effectively, it is often useful to summarise the extensive information from a survey or statistical experiment. . . .” [Simpson and Rowland \(2009, p. 426\)](#) observe that “people have always been interested in the collection of information about themselves and their environment. The collection of such information in a systematic fashion is called *statistics*.” This definition explicitly states that statistics is about data collection in specific applications, but not summarising, analysis or interpretation. [Elms et al. \(2011, p. 177\)](#) use the example of the Australian Census to introduce statistics, and frequently use the term statistics, without formally defining what this means. [Elms and Simpson \(2011, p. 386\)](#) simply note that “When looking at a set of **statistics** we are often asked for the average,” which is confusing the terms “statistics” and “data”.

4.2 Definitions of statistical terms

The ratings given to each respondents’ definitions are summarised in [Figure 2](#). Unsurprisingly, “mean” is the term best understood ([Richardson et al. 2013](#)). Only two terms (“mean” and “sample”) were defined correctly by more than half of the respondents.

Figure 2. The distribution of ratings for each term. The terms are ordered by the percentage correct (Code 1). The most densely filled rectangles correspond to Code 1 (correct); rectangles with no shading correspond to No answers (Code 6).



Almost all respondents defined “mean” correctly. Interestingly, “standard deviation” was defined correctly by only 40% of respondents, but a further 52% somewhat correctly defined “standard deviation.” The most misunderstood terms were “confidence interval” and “standard error.” In addition, many teachers chose not to define “standard error,” suggesting that respondents were unsure of the meaning of the term (or perhaps that the term was unknown to them). The percentage of respondents leaving terms undefined tends to increase as the percentage of correct answer decreases.

Every respondent defined at least one term correctly (usually “mean”). None did better than correctly defining (Code 1) three of the five terms (one respondent, a teacher; [Table 4](#)). Extending this to include the “somewhat correct” answers (Code 2), five of the respondents (20%) could not define more than half of the five terms correctly ([Table 5](#)). The mean (median) number of terms defined correctly was 2.0 (2.0). One respondent obtained a Code 1 or 2 for every term (a pre-service teacher) and five respondents obtained a Code 1 or 2 for four terms (each misdefined “standard error”). The mean number of terms coded correctly or somewhat correctly was 3.0 (median: 3.0).

Table 4. The number of correct answers (Code 1) for the five terms being defined. No respondent defined more than three terms correctly.

Number defined correctly (Code 1):	0	1	2	3	4	5
Number of respondents:	0	6	12	7	0	0

Table 5. The number of correct answers (Code 1) and somewhat correct answers (Code 2) for the five terms being defined.

Number defined correctly (Code 1) or somewhat correctly (Code 2):	0	1	2	3	4	5
Number of respondents:	0	0	5	14	5	1

The terms are now discussed in the order presented in [Figure 1](#).

4.3 Mean

Only two respondents did not define the mean correctly; these definitions (both coded as “somewhat correct”) were:

- “An exemplar value that generalises a set of data” (pre-service teacher);
- “The value at which the centre of the data would lie if ordered canonically: $\bar{x} = \sum x/n$ ” (pre-service teacher).

In the second definition, the median is described yet the formula given is for the mean.

While providing a formula was coded as correct (demonstrating instrumental understanding), it may suggest that the respondents do not understand *what* the mean represents even though they can describe *how* the value of the mean is found. Twenty-two (88%) respondents use the word “average” in their definition of “mean.” Similarly, [Leavy and O’Loughlin \(2006\)](#) found that pre-service teachers often treated “mean” and “average” as synonyms. Defining the “mean” as the “average” has been coded as correct, even though the definition may be too vague (the median also fits this definition), but which may reflect the informal nature of the data collection process. Unsurprisingly, no respondents alluded to any other type of mean (such as geometric or harmonic mean).

This approach of defining the mean by the way in which it is computed is reflected in the textbooks. Every textbook examined, apart from [Elms et al. \(2011\)](#), includes a discussion of the mean. In every case, the mean is defined through a computation (usually explicitly as a formula); that is, the mean is defined by how it is *computed*, rather than what it *means*:

- “The formal definition of the mean is $\bar{x} = \frac{(\sum x)}{n}$ ” ([Simpson and Rowland 2009, p. 481](#)).
- “The correct term for the average is the **mean**” ([Elms and Simpson 2011, p. 386](#)) followed by a formula.
- “The **mean** is found by adding together all the values of the data and then dividing the total by the number of data values” ([Pimentel and Wall 2010, p. 257](#)). When discussing non-grouped data, the above definition is all that is provided with no formula; when discussing grouped data, a formula is given (p. 260, 263).
- “The **mean** (symbol \bar{x}) is the arithmetic average of the scores” ([Brodie and Swift 2008a, 2009, p. 320](#)). The formulae for computing the mean for ungrouped and grouped data follows.

- “The **mean** of the scores of the variable x is written as \bar{x} . It is the arithmetic average of the values of x ” (Brodie and Swift 2008b, p. 67). Then follows two formulae: one for ungrouped data, one for grouped data.

The “mean” is not defined in the current *Queensland Senior Mathematics Curricula* even though the term is used. The glossaries for the new *Essential Mathematics* (ACARA 2014a) and *General Mathematics* Curricula (ACARA 2014b) both define the mean (explicitly, the arithmetic mean) by explaining the computation of the mean, both as a formula and in words: “The arithmetic mean of a list of numbers is the sum of the data values divided by the number of values in the list.” This definition is followed by the statement “In everyday language, the arithmetic mean is commonly called the average,” consistent with the responses from the teacher survey. *Specialist Mathematics* (ACARA 2014d) does not provide a formula, but defines the “arithmetic mean” as “Sample mean the (sic) arithmetic average of the sample values.”

This formula-focus as the definition is used by teachers, textbooks and the curricula documents, consistent with other studies (Russell and Mokros 1991); none focus on the meaning as a “balance point” (O’Dell 2012) or as a “leveller of data” (Uccellini 1996). However, the informal way in which the data were collected may have contributed to the answers provided by teachers. These computational definitions exemplify Skemp’s (1978) instrumental understanding approach which is at odds with the aims of numeracy and statistical literacy. Teachers (and authors) who have a relational understanding of the mean should be expected to define the mean with some conceptual understanding. These results also confirm the observation of Jacobbe and Carvalho (2011) that a gap exists in teachers’ knowledge regarding the concept of the mean.

Definitions of the mean in common statistical dictionaries also eschew a relational definition; for example, Everitt (2006) defines the mean as “a measure of location or central value for a continuous variable,” and then provides a formula. In the context of school teaching, relational understanding is better developed using a definition such as the “balancing point of a set of data” (Uccellini 1996, p. 114). By itself, this definition seems terse, but it leads naturally to in-class exploration (Franklin et al. 2007, p. 41), and allows students to discover other characteristics of the mean (for example, that the mean does not need to be one of the values in the dataset) and even to discover the formula itself (O’Dell 2012).

In summary, teachers appear to understand that the mean is an average, but whether this understanding is relational or instrumental is unclear. However, the textbooks that teachers often defer to usually do not use a relational definition.

4.4 Sample

Most survey responses correctly identified a “sample” as part of a *population*. Many even alluded to a random sample: “Subset of a population, hopefully representative” (teacher).

A sample is defined in three textbooks, and defined sufficiently well. Simpson and Rowland (2009, p. 431) define a sample as “a selection of the target population... taken to be representative of the whole group.” Brodie and Swift (2008b, p. 40; 2009, p. 107) simply state that “a **sample** is part of the population;” the authors then define a random sample: “A random

sample has items selected so that every item has an equal chance of being selected” (p. 109). Their definition of a random sample excludes random sampling techniques such as multi-stage sampling, but is an acceptable definition at a senior school level and is within the scope of the curricula.

[Elms et al. \(2011\)](#) do not explicitly define a “sample,” but the authors discuss a feature of a sample: “A sample represents the population as closely as possible” ([Elms et al. 2011, p. 178](#)). The authors then proceed to define a specific type of sample: “A simple random sample is one for which each element of the population has an equal chance of being chosen” ([Elms et al. 2011, p. 179](#)). Note that the description of a simple random sample given by [Elms et al. \(2011\)](#) is incomplete (but sufficient for the context): Samples selected using, for example, a systematic sample also meet the definition given in the text. [Brodie and Swift \(2008a\)](#) do not define a sample, but different ways of collecting data from samples are discussed, such as surveys and questionnaires.

“Sample” is never defined in the current *Queensland Senior Mathematics Curricula* even though the term is used. The new *Australian Senior Secondary Curriculum: Mathematics* does not define “sample,” but the term is used nevertheless in other definitions. For example, the definition of “coefficient of determination” uses a “sample of mice;” the example for “scatterplot” uses data from a “sample of 24 countries” (both from *General Mathematics* ([ACARA 2014b](#))); “point and interval estimates” are defined as “the use of information derived from a sample” (*Mathematical Methods* ([ACARA 2014c](#))).

The *Specialist Mathematics Curriculum* ([ACARA 2014d](#)) defines a “random sample” as “a set of data in which the value of each observation is governed by some chance mechanism that depends on the situation.” This definition is incorrect: the randomness concerns the selection of the data, not the values which each observation takes on. The “values” of the observations are fixed (for example, an individual’s country of birth), but the observations that appear in the sample are chosen at random (any individual may or may not be selected in the sample).

In summary, teachers and textbooks seem to have sufficient understanding of the definition of a “sample,” though often the term is left undefined by the textbooks. The new curricula either do not define the term, or have an incorrect definition. In the context of school teaching, relational understanding can be developed using a definition such as “a selection of the target population” ([Simpson and Rowland 2009](#)). Defining a random sample requires more information (that the sample somehow is representative of the population), and specific types of random samples (such as the commonly-used simple random sample) need even more clarification.

4.5 Standard deviation

The “standard deviation” is acknowledged as being a difficult concept ([Sánchez, da Silva and Coutinho 2011, p. 214](#)), but nonetheless is one of the most popular measures of variation. Even experienced researchers can have difficulty with understanding standard deviation ([Vaux 2012](#)). Studies have shown that teachers often do not use the term “standard deviation” when describing the amount of variation in data ([Makar and Confrey 2005](#)), and the authors conclude that “the

notion of standard deviation as a measure of variation did not hold much meaning for the respondents” ([Makar and Confrey 2005, p. 38](#)).

The term exists in the current *Queensland Senior Mathematics Curricula* (but is not explicitly defined), so it is perhaps surprising that few survey respondents defined “standard deviation” correctly (Code 1). Many responses (52%) were coded as somewhat correct (Code 2); for example:

- “The distribution of the data s_x or σ ” (teacher);
- “Formula to compare scores with means” (teacher);
- “How far from the mean a given set of data is” (pre-service teacher).

These somewhat correct definitions (Code 2) show that the respondents have some understanding of the standard deviation and its connection with variation, but perhaps were not sure how to articulate this succinctly on the survey. In contrast to the definitions provided for the “mean,” no formula was presented by teachers (possibly because the formula is more complicated, rarely used in the classroom, and not committed to memory), but formulae were used in textbooks.

The textbook definitions were usually based on a formula with an attempt at explanation. [Simpson and Rowland \(2009, p. 502\)](#) state that “standard deviation σ can be calculated by using

the following formula $\sigma = \sqrt{\frac{\sum f(x_i - \bar{x})^2}{n}}$.” No further explanation of the meaning is given. The formula is confusing: the symbol σ is almost always used to represent a population standard deviation and the divisor of n suggests a population standard deviation, yet the formula itself uses the sample mean in the calculation.

Other textbooks also create confusion by mixing elements of sample and population standard deviations. For example, Brodie and Swift ([2008a](#); also see [Brodie and Swift 2009](#)) state:

The **standard deviation** measures how far every data item is from the mean. It is abbreviated as **SD**, has the symbol σ and is calculated using the formulas:

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \text{ for individual scores}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} \text{ for tables}$$

where Σ means the sum. ([Brodie and Swift 2008a, p. 328](#)).

In addition to using σ for the sample standard deviation, their attempt to provide a definition using relational understanding is incorrect: the definition implies the standard deviation is a distance-type measurement for *each* observation, rather than a single summary value for a dataset.

[Pimental and Wall \(2010, p. 264\)](#) use the text: “The **standard deviation** of data is a measure of dispersion that does take into account all of the data. It gives an average measure of difference (or deviation) from the mean of the data.” This definition, while not technically correct, is sufficient for school level and is using relational understanding. However, a formula follows on

p. 265: $s_n = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$. This formula uses a more standard symbol for the sample standard deviation, but the divisor of n is for computing a standard deviation from a population.

[Brodie and Swift \(2008b, p. 74\)](#) say that “the standard deviation (σ) is the square root of the mean squared deviation from the mean: $\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$ or $\sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$ for grouped data.” As other textbooks have done, using σ for a sample standard deviation is questionable (especially when the formula uses a sample mean), and using a divisor of n is incorrect for defining a sample standard deviation. [Brodie and Swift \(2008b\)](#) then follow with the computational formulae: $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$ or $\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$ which are also for computing population standard deviations, perpetuating the issues raised with their definitions. To further complicate matters, they also discuss the sample standard deviation explicitly ([Brodie and Swift 2008b, p. 75](#)), adopting the symbol σ_{n-1} without supplying a formula. This leaves the reader wondering if the formula for σ is indeed meant to be for computing a population standard deviation, but confused as to why (and how) this computation can be based on a sample mean.

[Elms and Simpson \(2011, p. 423\)](#) use a different approach, writing “the standard deviation is a measure of how much a typical score in a data set differs from the mean.” They provide no formula, but do provide instructions for finding the standard deviation on a calculator. In this definition, a “typical score in a data set” could easily and sensibly be interpreted as the mean, in which case their definition of the standard deviation becomes “how much the mean differs from the mean,” which would always be zero.

In summary, some definitions are purely based on instrumental understanding ([Brodie and Swift 2008b](#)), while some definitions are based on relational understanding ([Brodie and Swift 2008a, 2009](#); [Elms and Simpson 2011](#)) but are poor. Only [Pimentel and Wall \(2010\)](#) provide a useful definition based on relational understanding (but even then the formula provided is incorrect). The textbooks do a very poor job at explaining and defining the common concept of “standard deviation.” This makes it very difficult for teachers, and consequently for their students.

A definition for “standard deviation” is not given in the current *Queensland Senior Mathematics Curricula*, even though the term is used. The glossary in the new *General Mathematics Curriculum* ([ACARA 2014b](#)) defines the standard deviation as

...a measure of the variability or spread of a data set. It gives an indication of the degree to which the individual data values are spread around their **mean**.

This is followed by a correct formula: $s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$. The definition does not delve into discussion about how the “degree” of deviation is measured. The other curricula do not define “standard deviation.” This definition is a generic definition of variation for a specific way of measuring the amount of variation, but is a reasonable attempt at providing a definition based on relational understanding in the school context.

[Everitt \(2006\)](#) provides a definition that is technically correct, but based on instrumental understanding and unhelpful for teachers: “The most commonly used measure of the spread of a set of observations. Equal to the square root of the variance;” the variance is then defined as “in a population, the *second* moment about the mean,” and a formula then provided for computing an estimate of the variance in a sample.

In summary, the concept of “standard deviation” is elusive and generally poorly defined by teachers, and textbooks do not define or explain the standard deviation well either. However, students can be introduced to the general idea using the MAD, and hands-on activities could easily be used to develop relational understanding through an extension of [O’Dell’s \(2012\)](#) activities that develop the mean as a balance point, or using the ideas of [Kader \(1999\)](#).

4.6 Confidence intervals

Confidence intervals are known to be a difficult concept to understand and teach ([Chance, delMas, and Garfield 2004](#); [Fidler and Cumming 2005](#)), and are known to present language difficulties in statistics ([Richardson et al. 2013](#); [Dunn et al. 2015](#)). Of the 11 responses coded as 3 (statistical, but incorrect), seven (or 64% of Code 3s; 28% of all responses) were focused on the confidence interval being formed for the data rather than for a summary statistic:

- “Where the majority of data lies” (teacher);
- “The interval where there is a confidence that all of the data fits in between” (pre-service teacher);
- “Range of possible scores given restraints” (HoD).

No respondent mentioned the idea of repeated sampling when discussing confidence intervals.

Confidence intervals are not discussed in any current Queensland textbook as they are not in the current Queensland curricula. However, confidence intervals were part of the Queensland curricula until 2009, so many teachers have taught confidence intervals in the past and have used textbooks which teach confidence intervals. Since these textbooks may have influenced teachers, we also study three textbooks previously used in Queensland schools: [Brodie and Swift \(1996\)](#), and [Brodie and Swift \(1994a, 1994b\)](#). All three textbooks discuss confidence intervals, but never actually define confidence intervals; their discussion is based on one example. However, their example actually uses a prediction interval: “...with 95% confidence, next year’s sales will be between \$2.3 million and \$2.7 million.” The confidence interval is then described as the “upper and lower bounds of the estimate are stated, together with a stated degree of confidence for the interval or confidence interval.” No meaning is attached to the confidence interval apart from this, and there is no indication of the idea of repeated sampling in the explanation.

The new *Australian Senior Secondary Curriculum: Mathematics* only mentions “confidence intervals” in *Mathematical Methods* ([ACARA 2014c](#)). However, the term “confidence interval” is never defined, although terms associated with it (such as the “margin of error” and the “level of confidence”) are defined using the term “confidence interval” itself. Confidence intervals are alluded to in the Glossary, under the heading “Point and interval estimates”: “An interval estimate is an interval derived from the sample that, in some sense, is likely to contain the parameter” ([ACARA 2014c](#)). The concern here is that teachers have a poor understanding of the concept, most textbooks do not define the concept (and the one that does define the concept does so poorly), and the glossary in the new *Mathematical Methods* Curriculum ([ACARA 2014c](#)) does not supply a definition either.

[Everitt \(2006\)](#) defines a “confidence interval” as:

A range of values, calculated from the sample observations, that is believed, with a particular probability, to contain the true parameter value... Note that the stated probability level refers to properties of the interval and not to the parameter itself which is not considered a random variable.

In the context of school teaching, relational understanding is better developed using hands-on activities (such as those found in [Scheaffer, Watkins, Witmer, and Gnanadesikan \(2004\)](#)) and simulations ([Mills 2002](#); [Cumming 2014](#)).

4.7 Standard error

The concept of a standard error is difficult to grasp, even for those experienced with data ([Vaux 2012](#)), so it is not surprising that teachers in this survey had difficulty defining the term. Many respondents did not attempt to define “standard error” (12 out of 25, or 48%), far more than for any other term (“confidence interval” recorded six, or 24%), and no respondent defined the term correctly. The most common mistake made by the survey respondents in defining “standard error” was to assume that it referred to an actual mistake (see [Dunn et al. 2015](#)):

- “The errors we all make” (HoD);
- “Error within the measuring instrument” (teacher);
- “Uncontrollable error” (HoD);
- “The average of the typical errors made in data collection” (HoD).

No current textbooks defined standard errors, which may reflect the respondents’ lack of understanding. However, an older textbook ([Brodie and Swift 1996](#)), used when confidence intervals were part of the Queensland curriculum, states that “the standard error of a statistic is another name for the standard deviation of the sample mean” (p. 459). This definition states nothing explicit about repeated sampling, and is proposing a general definition of a standard error (“of *a* statistic”) but defines the term for one specific statistic (“*the* sample mean”). Defining generalities in term of particulars is bound to create gaps in understanding.

[Everitt \(2006\)](#) defines the “standard error” as “the standard deviation of the *sampling distribution* of a statistic.” Given the difficulties described above with explaining the term “standard deviation,” combined with the difficulty of understanding the concept of a “standard error,” it is not surprising that “standard error” is difficult to define. Nonetheless, the concepts can be developed through using relational understanding via hands-on activities ([Gnanadesikan, Scheaffer, Watkins, and Witmer 1997](#)) and simulation ([Mills 2002](#)).

4.8 Developing relational understanding of terms

Following the administration of the survey at the August MATHS meeting, the authors ran three sessions of hands-on activities for the attendees. In these sessions, teachers participated in activities that demonstrated the mean as a balance point (using ideas from [O’Dell \(2012\)](#)), regression as a means of minimising the sum of distances between points and the line (using least absolute deviations regression), and an in-class data collection exercise. While a need for professional development with terms such as “standard error” and “confidence intervals” is clear, insufficient time was available at this MATHS meeting, and these terms are also inherit in many other statistical concepts that the authors were not sure the teachers understood.

At the end of these activities, teachers were asked to self-rate the improvement in their confidence to teach these statistical terms. Thirteen of the 17 attendees (76%) who responded to this question reported that they were now more confident or a little more confident to teach these topics (the remaining 4 (24%) reported no change in their confidence levels). One teacher left this comment:

There is a lot more than mean, median and mode to stats. Great engaging ideas for the classroom.

That the teacher previously thought statistics was all about mean, median and mode is worrisome, but perhaps exemplifies the problems identified in this paper.

5. Conclusion

In this paper, the definitions of “statistics” and five statistical terms have been discussed. These terms appear, explicitly or by implication, in the new *Australian Senior Secondary Curriculum: Mathematics*, but some do not appear in current Queensland curriculum. This study shows that teachers’ and pre-service teachers’ understanding of these terms is often quite poor, and that the textbooks that they refer to generally provide poor definitions also. The definitions provided by teachers and in textbooks are often based around instrumental understanding rather than relational understanding. The concern is that these teachers and pre-service teachers will be teaching students about statistical terms and concepts when their own understanding is poor, and their primary resource (the textbooks) also provide poor definitions in many cases. As [Uccellini \(1996\)](#) states, “teaching for understanding is difficult” (p. 115).

The glossary in the new *Australian Senior Secondary Curriculum: Mathematics* generally provides definitions of the concepts in ways that could lead to a relational understanding. The obvious omission is the difficult concept of a “confidence interval” which is not defined at all; this could be rectified before wider adoption of the curricula.

A recurring theme is that the technically-correct definitions of some terms are difficult to access using relational understanding, yet similar concepts can be developed which do use relational understanding. For example, the MAD is easier to develop conceptually using relational understanding and hands-on activities than standard deviation. At a school level, the development of concepts using relational understanding and hands-on activities is considered more important by the authors and by [Franklin et al. \(2007\)](#) in the development of statistical literacy than knowing the correct technical definitions. However, the curricula are quite specific about the development of the technical terms, which may force teachers to miss opportunities to develop relational understanding in their students, and probably in themselves also.

We suggest that textbooks and curricula documents consider the steps or stages involved in getting students to develop an understanding of the concepts before embarking on the more technical definitions used in practice. At present, the curricula requires teachers to teach terms that are difficult to define with relational understanding and hands-on activities. In part, this may explain why teachers themselves have difficulty with the technical terms. Perhaps the textbooks and curricula could utilise terms which are easier to teach and understand relationally as stepping stones to the more technical definitions. This would enable teachers to develop relational

understanding in their students. The actual definitions of the terms in the curricula are too technical to be of much use at an introductory level, and may actually hinder relational understanding.

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References

Aliaga, M., Cobb, G., Cuff, C., Garfield, J., Gould, R., Lock, R., Moore, T., Rossman, A., Stephenson, B., Utts, J., Velleman, P., and Witmer, J. (2010), "Guidelines for assessment and instruction in statistics education: College report," Technical report, American Statistical Association.

Australian Association of Mathematics Teachers (AAMT) (1997), "Numeracy equals everyone's business," Adelaide.

Australian Bureau of Statistics (ABS) (2013), Statistical Literacy Background Paper [online]. Available at <http://www.abs.gov.au/websitedbs/CaSHome.nsf/Home/Statistical+Literacy+Competencies.es>.

Australian Curriculum, Assessment and Reporting Authority (ACARA) (2012), <http://www.australiancurriculum.edu.au/mathematics/aims>

Australian Curriculum, Assessment and Reporting Authority (ACARA) (2014a), *Essential Mathematics* [online]. Available at <http://www.australiancurriculum.edu.au/seniorsecondary/mathematics/essential-mathematics/curriculum/seniorsecondary#page=1>

Australian Curriculum, Assessment and Reporting Authority (ACARA) (2014b), *General Mathematics* [online]. Available at <http://www.australiancurriculum.edu.au/seniorsecondary/mathematics/general-mathematics/curriculum/seniorsecondary#page=1>

Australian Curriculum, Assessment and Reporting Authority (ACARA) (2014c), *Mathematical Methods* [online]. Available at <http://www.australiancurriculum.edu.au/seniorsecondary/mathematics/mathematical-methods/curriculum/seniorsecondary#page=1>

Australian Curriculum, Assessment and Reporting Authority (ACARA) (2014d), *Specialist Mathematics* [online]. Available at <http://www.australiancurriculum.edu.au/seniorsecondary/mathematics/specialist-mathematics/curriculum/seniorsecondary#page=1>

Bargagliotti, A. E. (2012), “How well do the NSF Funded Elementary Mathematics Curricula align with the GAISE report recommendations?” *Journal of Statistics Education* [online], 20, 3.

Batanero, C., Burrill, G., and C. Reading (eds.), *Teaching Statistics in School Mathematics—Challenges for Teaching and Teacher Education: A Joint ICMI/IASE Study*, Springer, 2011.

Ben-Zvi, D. & Garfield, J. (2004), 'Goals, Definitions, And Challenges', *The Challenge of Developing Statistical Literacy, Reasoning and Thinking*, edited by Ben-Zvi, D. and Garfield, J., Kluwer Academic Publishers, pp.3-15

Biggeri, L., and Zuliani, A. (1999), “The dissemination of statistical literacy among citizens and public administration directors,” [online], paper presented at the ISI 52nd Session, Helsinki, Finland. Available at <http://www.stat.auckland.ac.nz/~iase/publications.php?show=5>.

Brodie, R. and Swift, S. (1994a), *QMaths 12A*, South Melbourne: Moreton Bay Publishing.

Brodie, R. and Swift, S. (1994b), *QMaths 12B*, South Melbourne: Moreton Bay Publishing.

Brodie, R. and Swift, S. (1996), *QMaths 12C*, South Melbourne: Moreton Bay Publishing.

Brodie, R. and Swift, S. (2008a), *New QMaths 11A*, third Edition, South Melbourne: Nelson Cengage Publishing.

Brodie, R. and Swift, S. (2008b), *New QMaths 11B*, third Edition, South Melbourne: Nelson Cengage Learning, South Melbourne.

Brodie, R. and Swift, S. (2009), *New QMaths 12A*, third Edition, South Melbourne: Nelson Cengage Publishing.

Chance, B., delMas, R., and Garfield, J. (2004), “Reasoning about sampling distributions,” in *The Challenge of Developing Statistical Literacy, Reasoning and Thinking*, eds. D. Ben-Zvi and J. Garfield, Dordrecht, The Netherlands: Kluwer Academic Publishers, pp. 295–323.

Cumming, G. (2014), “The new statistics: Why and how,” *Psychological Science*, 25(1), 7–29.

Dunn, P.K., Carey, M., Richardson, A. M., and McDonald, C. (2015), “Learning the language of statistics: Challenges and teaching approaches,” in *Statistical Education Research Journal*. In press.

Eichler, A. (2011), “Statistics teachers and classroom practices,” in *Teaching Statistics in School Mathematics—Challenges for Teaching and Teacher Education: A Joint ICMI/IASE Study*, eds. C. Batanero, G. Burrill, and C. Reading, Springer, pp. 247–258.

Elms, L., and Simpson, N. (2011), *Maths Quest Year 11 Maths A for Queensland*, second edition, Milton: Jacaranda.

Elms, L., Simpson, N., and McPherson (2011), *Maths Quest Year 12 Maths A for Queensland*. Milton: Jacaranda.

Everitt, B. S. (2006), *The Cambridge Dictionary of Statistics*, Third edition, Cambridge: Cambridge University Press.

Fidler, F., and Cumming, G. (2005), “Teaching confidence intervals: Problems and potential solutions,” in *Proceedings of the International Statistical Institute 55th Session*, Sydney, Australia. International Statistical Institute.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., and Scheaffer, R. (2007), “Guidelines for assessment and instruction in statistics education (GAISE) report: a pre-K–12 curriculum framework,” Technical report, American Statistical Association.

Gal, I. (2004), “Statistical literacy: meanings, components, responsibilities,” in *The Challenge of Developing Statistical Literacy, Reasoning and Thinking*, eds. D. Ben-Zvi and J. Garfield, Dordrecht, The Netherlands: Kluwer Academic Publishers, pp. 47–78.

Gnanadesikan, M., Scheaffer, R. L., Watkins, A. E., and Witmer, J. A. (1997), “An activity-based statistics course,” *Journal of Statistics Education* [online], 5, 2.

Jones, D.L., and Jacobbe, T. (2014), “An analysis of the statistical content in textbooks for prospective elementary teachers,” *Journal of Statistics Education* [online] 22(3). Available from: <http://www.amstat.org/publications/jse/>

Jacobbe, T., and Carvalho, C. (2011), “Teachers’ understanding of averages,” in *Teaching Statistics in School Mathematics—Challenges for Teaching and Teacher Education: A Joint ICMI/IASE Study*, eds. C. Batanero, G. Burrill, and C. Reading, Springer, pp. 199–209.

Kader, G. D. (1999), “Means and MADs,” *Mathematics Teaching in the Middle School*, 4(6), 398–403.

Kaplan, J. J., Fisher, D. G., and Rogness, N. T. (2009), “Lexical ambiguity in statistics: what do students know about the words association, average, confidence, random and spread?” *Journal of Statistics Education* [online], 17(3). Available from: <http://www.amstat.org/publications/jse/>

Kaplan, J. J., Fisher, D. G., and Rogness, N. T. (2010), “Lexical ambiguity in statistics: how students use and define the words: association, average, confidence, random and spread,”

Journal of Statistics Education [online], 18(2). Available from:

<http://www.amstat.org/publications/jse/>

Leavy, A., and O'Loughlin, N. (2006), "Preservice teachers understanding of the mean: Moving beyond arithmetic average," *Journal of Mathematics Teacher Education*, 9, 53–90.

Makar, K., and Confrey, J. (2005), "'Variation-talk': Articulating meaning in statistics," *Statistical Education Research Journal*, 4(1), 27–54.

Marshman, M., Dunn, P. K., McDougall, R., and Wiegand, A. (2015), "A case study of the attitudes and preparedness of secondary mathematics teachers towards statistics," *Australian Senior Mathematics Journal*. 29(1), 51–64.

Mills, J. D. (2002), "Using computer simulation methods to teach statistics: A review of the literature," *Journal of Statistics Education* [online], 10(1). Available from:

<http://www.amstat.org/publications/jse/v10n1/mills.html>

O'Dell, R. S. (2012), "The mean as a balance point," *Mathematics Teaching in the Middle School*, 18(3), 148–155.

Pimentel, R., and Wall, T. (2010), *Mathematical Studies for the IB Diploma*, Hodder Education.

Queensland Studies Authority (QSA) (2004), *PreVocational Mathematics*, [online], Spring Hill: State of Queensland. Available from

http://www.qcaa.qld.edu.au/downloads/senior/snr_prevoc_maths_04_sas.pdf

Queensland Studies Authority (QSA) (2014a), *Mathematics A*, [online], Spring Hill: State of Queensland. Available from

http://www.qcaa.qld.edu.au/downloads/senior/snr_maths_a_08_syll.pdf

Queensland Studies Authority (QSA) (2014b), *Mathematics B*, [online], Spring Hill: State of Queensland. Available from

http://www.qcaa.qld.edu.au/downloads/senior/snr_maths_b_08_syll.pdf

Queensland Studies Authority (QSA) (2014c), *Mathematics C*, [online], Spring Hill: State of Queensland. Available from

http://www.qcaa.qld.edu.au/downloads/senior/snr_maths_c_08_syll.pdf

Richardson, A. M., Dunn, P. K., and Hutchins, R. (2013), "Identification and definition of lexically ambiguous words in statistics by tutors and students," *International Journal of Mathematical Education in Science and Technology*, 44(7), 1007–1019.

Russell, S. J., and Mokros, J. R. (1991), "What's typical? Children's ideas about average," in *Proceedings of the Third International Conference on Teaching Statistics*, ed. D. Vere-Jones, Voorburg, The Netherlands: International Statistical Institute, pp. 307–313.

Sánchez, E., da Silva, C. B., and Coutinho, C. (2011), “Teachers’ understanding of variation,” in *Teaching Statistics in School Mathematics—Challenges for Teaching and Teacher Education: A Joint ICMI/IASE Study*, eds. C. Batanero, G. Burrill, and C. Reading, Springer, pp. 211–221.

Scheaffer, R. L., Watkins, A., Witmer, J., and Gnanadesikan, M. (2004), *Activity-based Statistics: Instructor Resources*, Second edition, CA: Key College Publishing.

Schmitt, N. (2008), “Instructed second language vocabulary learning,” *Language Teaching Research*, 12(3), 329–363.

Schild, M. (2004), “Information literacy, statistical literacy and data literacy,” *IASSIST Quarterly*, 2/3, 6–11.

Simpson, N., and Rowland, R. (2009), *Maths Quest Year 11 Maths B for Queensland*, Second edition, Milton: Jacaranda.

Skemp, R. R. (1978), “Relational understanding and instrumental understanding,” *The Arithmetic Teacher*, 26(3), 9–15.

Trewin, D. (2005), “Improving statistical literacy: The respective roles of schools and the National Statistical Offices,” in M. Coupland, J. Anderson & T. Spencer (Eds.), *Proceedings of the Twentieth Biennial Conference of The Australian Association of Mathematics Teachers* (Keynote paper), 11–19, The Australian Association of Mathematics Teachers Inc..

Uccellini, J. C. (1996), “Teaching the mean meaningfully,” *Mathematics Teaching in the Middle School*, 2(2), 112–115.

Utts, J. (2013), “Comment: The future of the textbook,” *Technology Innovations in Statistics Education*, 7(3), 2013.

Vaux, D. L. (2012), “Know when your numbers are significant,” *Nature*, 492, 180–181.

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