An Analysis of the Statistical Content in Textbooks for Prospective Elementary Teachers

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Key Words: Statistics education; Mathematics textbooks; Elementary teacher preparation; Content analysis.

Abstract

We analyzed the statistical content within mathematics textbooks used in courses for preparing elementary teachers. Six textbooks commonly used in the United States comprised our sample. Each task in statistical sections was analyzed using both the levels of the GAISE framework (Franklin et al. 2007) and phases of the statistical problem solving process (Formulate Questions, Collect Data, Analyze Data, and Interpret Results). Tasks within the Analyze Data phase were also classified as creating or reading a display, performing a mathematical computation, or using statistical reasoning beyond mathematical computations. The majority of statistical tasks in each book required data analysis. Two textbooks primarily consisted of tasks addressing statistical concepts beyond computations, while three series focused on graphical displays and computational procedures.

1. Introduction

Each day, increasing amounts of data and information are made available through various sources. Individuals are challenged with the task of filtering, critiquing, analyzing, and interpreting this information in order to make decisions. Because of this, the ability to adeptly utilize and understand statistics is a critical skill for citizens and consumers, and will continue to be so in the future. It stands to reason that these skills should be taught in schools, beginning in

the early elementary grades. In fact, in the United States, students from kindergarten through grade 12 are required to study these topics in their mathematics classes. There are standards relating to statistics and data analysis in the state curriculum documents for each of the 50 states, as well as the *Common Core State Standards for Mathematics* (CCSS, National Governors Association Center for Best Practices and Council of Chief State School Officers 2010).

In order to teach statistics (and mathematics) well, teachers must understand the content – both at and beyond the level they are teaching. This means that teachers need the opportunity to learn statistics content at a conceptual, and not merely computational, level. According to *The Mathematical Education of Teachers*, "Statistics is the science of data, and the daily display of data by the media notwithstanding, most elementary teachers have little or no experience in this vitally important field" (Conference Board of the Mathematical Sciences [CBMS] 2001, p. 87). Furthermore, authors of *The Mathematical Education of Teachers II* stated, "Every elementary student deserves a teacher who knows, very well, the mathematics that the student is to learn" (CBMS 2012, p. 24). For these reasons, we focus on prospective teachers at the elementary level.

The CBMS recommends that preparation programs for elementary teachers contain at least twelve semester-hours of coursework in mathematics and statistics. The specific content organization of these courses is left to the institution, which means prospective elementary teachers' opportunities to learn statistics are likely to occur during a mathematics content course designed for elementary teachers. Typically, statistics is addressed within a 3-hour course that also focuses on probability and other mathematical content (e.g., rational numbers, measurement, or algebra). The textbook is a resource that is readily available, often utilized by the instructor of the course (Tyson-Bernstein and Woodward 1991; Grouws and Smith 2000; McCrory 2006), and may also serve as a source of activities, problems, and exercises that could be used in the classroom or as an assignment outside of class. Textbooks written for these courses contain content related to number and operation, geometry, measurement, algebra, probability, and statistics. Many institutions use a single book for multiple courses, and incorporate different chapters in each course. One possible reason for this practice is to keep costs low for students. With that said, instructors of such courses likely consider more than the statistical content when selecting a textbook.

At many institutions, there are some mathematics courses included in programs for teacher certification at the elementary and middle grades levels, covering topics such number and operation. In terms of statistical content, some programs may be completely identical for both levels. However, the CBMS (2012) recommends that programs for prospective middle school teachers include two stand-alone courses in statistics. This sequence could consist of an introductory statistics course (for a general audience) followed by a specialized course "designed for middle grades teachers to emphasize active learning with appropriate hands-on devices and technology while probing deeply into the topics taught in the middle grades" (p. 47). It is possible, therefore, that the textbooks examined in this study may be used by prospective middle school mathematics teachers, either in a stand-alone course or in a course that also focuses on other content.

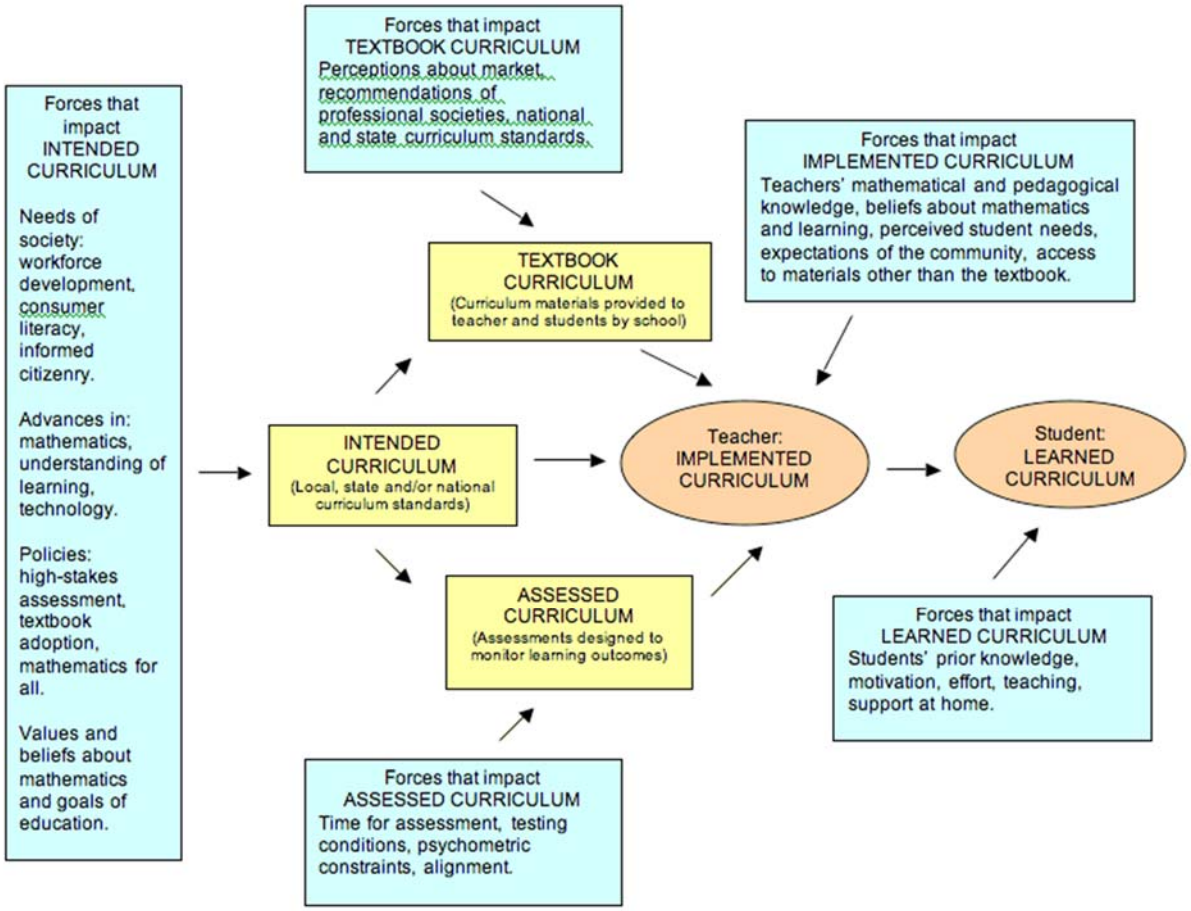
Having used a few different textbooks during our careers, we believed that not all textbooks were the same. We wished to examine current textbooks and describe the nature and extent of the

treatment of statistical topics. Below, we describe a rubric that we developed for this purpose, and provide details of analyzing six textbooks with this rubric. We begin with a discussion of the theoretical considerations that underpin our rubric.

2. Theoretical Considerations

We situate our work within the Center for the Study of Mathematics Curriculum (CSMC) framework (CSMC n.d.; see Figure 1). Our study focuses on the content of textbooks, which is one of several factors that ultimately impact student learning. While the CSMC framework was designed primarily for curricula at the school level (kindergarten-grade 12), portions of it also apply to textbooks used in college classrooms. In particular, the textbook curriculum, in the form of materials provided to the course instructor or department, influences the implemented curriculum (what is taught), which in turn influences the learned curriculum. Additionally, the textbook curriculum is influenced by factors such as perceptions of the market and recommendations from professional organizations and state and national standards documents.

Figure 1: Model depicting the forces and types of curriculum that influence teacher decision-making and student learning opportunities



As our study is related to statistics education in the United States, we utilized the American Statistical Association's *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K–12 Curriculum Framework* (Franklin et al. 2007). The GAISE framework identifies three levels of development students must progress through in order to become statistically literate (labeled A, B, and C). Within each level, the framework describes the statistical topics and reasoning needed according to the statistical problem-solving process. This process requires students to engage in four phases: Formulate Questions, Collect Data, Analyze Data, and Interpret Results. While this framework was developed for students in grades pre-K–12, it organizes the statistical content that must be contained in courses for prospective teachers.

We also considered the statistical content of state curriculum frameworks and the CCSS. In a recent study of state standards, Newton, Horvath, and Dietiker (2011) examined the K-12 mathematics curriculum frameworks from 41 states. A total of 1,711 grade level expectations (GLEs) were identified as addressing statistical topics, and these were classified according to the phases of the statistical problem-solving process as described in the GAISE framework. Across this set of GLEs, "an overwhelming majority of these (approximately 87%) were coded in either the Analyze Data or Interpret Results process components or both" (p. 156). This proportion is similar to that of the CCSS. For students in grades K-8, there are 20 distinct GLEs that reference statistical topics. When sorted into the four statistical processes, 85% (17 of the 20) relate to either Analyze Data or Interpreting Results.

Both the GAISE framework and the CCSS delineate content for students in school mathematics, but they may serve to inform standards for teacher preparation. For example, *The Mathematical Education of Teachers II*, "uses the CCSS as a framework for outlining the mathematical ideas that elementary teachers, both prospective and practicing, should study and know" (CBMS 2012, p. 25). For teachers of students in grades kindergarten through 5, the CCSS related to statistics (and data) are summarized as the essential ideas, "Using data displays to ask and answer questions about data. Understanding measures used to summarize data, including the mean, median, interquartile range, and mean absolute deviation, and using these measures to compare data sets" (p. 29). These topics, and many others, are contained within the GAISE framework. For this reason, we used the GAISE framework to serve as a more extensive list of statistical topics that may be included in textbooks for prospective elementary teachers. Furthermore, the GAISE also covers topics appropriate for middle school and high school students, particularly in Levels B and C.

Finally, we considered research literature on the use of textbooks in mathematics courses for prospective teachers. McCrory (2006) analyzed and characterized 20 mathematics textbooks for prospective elementary teachers. She found that "most of the books are encyclopedic, including every topic that might be covered in K-8 classrooms, and treating each topic as a separate entity" (p. 21). As a component of an effort to evaluate the mathematics preparation of elementary teachers in the U.S., the National Council on Teacher Quality (NCTQ 2008) examined the syllabi of 118 mathematics courses at 77 institutions and then reviewed the 19 mathematics textbooks listed on those syllabi. Twelve topics were considered in the reviews. One topic addressed statistical content, and included subtopics of drawing and interpreting graphs, tables, bar graphs, and pie charts; and understanding data characteristics such as the range, mean, and

median. The reviewers found that "all textbooks sections [*sic*] on data analysis and probability were determined to be 'adequate' in their initial screening," (p. 35) with 12 of the 19 textbooks earning perfect scores in their coverage of these areas. Because of the homogeneity of these findings, the NCTQ did not report any further results of the analysis regarding the statistical content of these textbooks.

We sought to explore the statistical sections of textbooks in more depth. For this study, we examine individual *tasks*, a term derived from Doyle's (1983, 1988) research on *academic tasks*, the individual questions, exercises, or problems set before students. Specifically, we focus on the tasks that prospective teachers may be asked to complete on their own as a part of an out-of-class assignment.

3. Methodology

3.1 Sample Selection

We selected six textbooks typically used in mathematics content courses for elementary teachers; these are listed in Table 1. Both McCrory (2006) and NCTQ (2008) analyzed earlier editions of five. These were the five most commonly-used textbooks in mathematics courses for elementary teachers (NCTQ 2008, p. 36). The sixth textbook (SSN) was also analyzed by NCTQ, and was not published at the time of McCrory's study. It was selected, in part, because of its use in courses at both of our institutions. Over 70% of the courses in the NCTQ study used one of the textbooks in our sample. We will refer to these textbooks by the authors' last initials, or a shortened form of the surname for single-authored textbooks.

Table 1. Textbooks included in sample

Bas	Bassarear, T. (2008), *Mathematics for Elementary School Teachers*, 4th Edition, Boston, MA: Houghton Mifflin.
Bec	Beckmann, S. (2011), *Mathematics for Elementary Teachers with Activity Manual*, 3rd Edition, Boston, MA: Addison Wesley.
BLL	Billstein, R., Libeskind, S., & Lott, J. W. (2010), *A Problem Solving Approach to Mathematics for Elementary School Teachers*, 10th Edition, Boston, MA: Addison Wesley.
LDM	Long, C. T., DeTemple, D. W., & Millman, R. S. (2009), *Mathematical Reasoning for Elementary Teachers*, 5th Edition, Boston, MA: Addison Wesley.
MBP	Musser, G. L., Burger, W. F., & Peterson, B. E. (2008), *Mathematics for Elementary School Teachers: A Contemporary Approach*, 8th Edition, New York, NY: John Wiley & Sons.
SSN	Sowder, J., Sowder, L., & Nickerson, S. (2010), *Reconceptualizing Mathematics for Elementary School Teachers*, 1st Edition, New York, NY: W. H. Freeman and Company.

Note. Each of the authors listed above earned a doctoral degree in either mathematics or mathematics education.

We did not analyze teacher's editions because they may not be readily accessible to prospective teachers. We also did not examine activity manuals or supplemental online resources, as these did not accompany all of the textbooks in the sample. Notably, Bec was designed to be accompanied by an activity manual; by not examining this manual, our methodology may undercount the statistical tasks in this textbook/activity manual package. (The fourth edition of Bec, released after data were collected for this study, includes the activity manual in the textbook.)

3.2 Unit of Analysis

Using the table of contents, we identified the sections of the textbook that related to statistics. Within these sections we examined the items located in the assignment portion under headings similar to *Assessment*, *Problems*, or *Learning Exercises*. Each item in the assignment portion was divided into one or more *tasks*. The size and content of a task was determined by the textbook's marked divisions of an exercise or problem. The smallest marked division was considered a task. To illustrate, Figures 2, 3, and 4 contain items with multiple tasks from three of the textbooks in the sample. In Figure 2, a total of three tasks are presented, labeled (a), (b), and (c). Similarly, Figure 3 contains two tasks, and Figure 4 contains four tasks.

Figure 2. Three tasks from SSN (p. 679)

2. Give examples of measurement data for the following contexts. If possible, also give examples of categorical data.
   (a) watching sports (b) swimming (c) shopping

Figure 3. Two tasks from BLL (p. 668)

10. (a) Use the following data to justify the amount of time that you expect to assign for weekly homework to classes in grades K-4 and grades 5-8.
    [A table summarizing the data is provided.]
    (b) How might the survey data be misused to justify assigning at least 2 hr of homework each week?

Figure 4. Four tasks from Bas (p. 453)

28. The following scenarios have been taken from the *Instructor's Course Planner* for the textbook *Children*. In each case, do you think the sample is a random sample or a biased sample? Justify your response.
    (a) The researchers asked teenage boys about their driving records and habits by going to a movie drive-in, a local bar, the beach, and a baseball park.
    (b) Parents at a PTA meeting were interviewed about the quality of the local public school system.
    (c) A telephone survey assessed a community's attitudes toward welfare recipients.
    (d) Children were asked their opinion of Santa Claus on December 28.

3.3 Rubric Development

An initial rubric was developed from the list of recommendations in the GAISE Report (Franklin et al. 2007, pp. 23-24, 37-38, 61-62), where statistical topics are arranged by GAISE Level and phase of the statistical problem solving process. For example, the recommendation "Students conduct a census of the classroom" (p. 23) is at GAISE Level A, Collect Data. We appended the rubric to indicate whether prospective teachers were to create or read data displays. The rubric was revised as necessary whenever the researchers identified statistical tasks that did not appear to fit into a given category on the rubric. In such cases, we examined the GAISE Report (and at times, consulted its authors) to determine the appropriate GAISE Level and phase of the statistical problem solving process. Our final rubric consisted of 61 statistical objectives, which is presented in the Appendix.

3.4 Coding Protocol

Each task was assigned one or more codes from the rubric according to the statistical topic (or topics) that were addressed. These codes identified the tasks by the phase of the statistical problem solving process, and by GAISE Level. For example, task 2a in Figure 2 was coded as Formulate Questions, Level B, as it pertains to the distinction between types of variables. Specifically, the code used was "B2: Students address questions involving a group larger than their group of interest and begin to recognize the distinction among types of variables, and a population, a census, and a sample." Both of the tasks in Figure 3 were coded under Interpret Results, Level A. Additionally, it was possible for a task to address multiple phases and levels. All four tasks in Figure 4 were coded as Collect Data at Level B and Level C with the following codes:

- B4: Students understand the role and nature of randomness.
- C1: Students understand what constitutes good practice in conducting an experiment, sample survey, and observational study.

In the case where a task addressed multiple GAISE Levels, the highest GAISE Level was recorded, as statistical understanding at a higher level assumes understanding at lower levels. Therefore, the tasks in Figure 4 were considered to be at Level C.

In order to account for the nature of the tasks within the Analyze Data phase (hereafter referred to as *analysis tasks*), we utilized an additional set of codes, listed in Table 2. Each task within this phase was assigned a single code, which helped to differentiate analysis tasks that were more procedural from those that were more conceptual and used statistical reasoning. Specifically, the codes "construct or read a display," and "perform a mathematical calculation" identified procedural tasks. Such tasks were algorithmic or computational in nature. The third category, "use statistical reasoning beyond mathematical calculations," was used to identify a task that addressed statistical concepts. Tasks in this category may also require calculations, but the calculation alone is not sufficient to answer the task.

Table 2. Codes used for the nature of analysis tasks

Code	Sample Task
Construct or read a display	Make a back-to-back stem and leaf plot for the following test scores. [Set of scores for Class 1 and Class 2 are provided.] (MBP, p. 460)
Perform a mathematical calculation	Determine the mean and standard deviation of the data obtained [from tossing seven pennies 30 times and recording the number of heads each time]. (LDM, p. 576)
Use statistical reasoning beyond mathematical calculations	Decide whether the mean or the median would likely give a better sense of what is typical or representative of the data for the purpose that is described. Discuss your reasoning in each case. (a) The starting salaries of a class of college graduates in which one of the graduates is drafted by the National Football League. Prospective students would like a sense of the starting salary they might expect when they get a job after they graduate. (Bec, p. 656)

3.5 Reliability Measures

Each researcher independently coded every task. After coding two or three sections, the assigned codes were compared. The researchers discussed each task that was assigned different codes in order to reach an agreement on the final code(s) that would be assigned to the task. At times, these discussions led to the revision of the rubric; the revised rubric was then used to re-evaluate tasks coded previously. This cycle of coding two or three sections and discussing the assigned codes continued until all sections had been coded.

During the coding process, we would occasionally come across statistical content that we had not anticipated. For example, standard deviation is not mentioned explicitly in the GAISE framework, so it was not included in our initial rubric. When we encountered this content, we noted that it did not appear anywhere in our rubric, but it should be incorporated somewhere. As a consequence, our assigned codes did not agree initially, but we revised the rubric to include standard deviation within Level C of the Analyze Data phase. Another more frequent discussion occurred about the nature of statistical tasks. Initially, we sought to sort all tasks into the distinct categories of “procedural” and “conceptual,” with procedural tasks focusing on computation and conceptual tasks highlighting the use of statistical reasoning. This proved to be difficult, as we found some tasks (particularly outside the Analyze Data phase) to be primarily “conceptual,” while there were many analysis tasks that were “procedural” but did not involve mathematical calculations (e.g., constructing a display). Ultimately, we restricted our rubric to examine the nature of analysis tasks, divided into the categories listed in Table 2.

Table 3 shows the percent of tasks with matching codes *before* discussion occurred. For the dimension of phases of the statistical problem solving process, we report two measures. For each task, we determined the proportion of four phases that were indicated (or not indicated) by both researchers. For example, if one researcher coded a task as Collect Data and Analyze Data, while the other coded it as Analyze Data only, the proportion of agreement for phases was 0.75. This is

because there was agreement that the task addressed Analyze Data and did not address Formulate Questions or Interpret Results, but there was disagreement on whether the task addressed Collect Data. Agreement on three of the four categories yielded the value of 0.75. These proportions were averaged to determine the *mean percentage agreement*. We also report the percent of tasks with complete agreement before discussion.

The measures for agreement in the dimensions of GAISE Level addressed and nature of the task are percentages of the total number of tasks in which there was complete agreement prior to discussion. These measures appear low due to the evolving nature of the rubric, as mentioned earlier in this section. In every case of disagreement, the tasks in question were discussed until the researchers agreed on how to code that particular item. The results presented in this article are based on the agreed-upon codes. At the end of the coding process, we agreed on the coding of 100% of the tasks.

Table 3. Interrater reliability measures

Coding dimension	Before discussion
Phases of the statistical problem solving process (n = 1846)	
Mean percentage agreement	94%
Percentage of tasks with complete agreement	85%
GAISE Level (n = 1846)	79%
Nature of analysis tasks (n = 1044)	73%

4. Results

4.1 Physical Features

Each of the textbooks in the sample contained a single chapter that addressed statistical topics, with the exception of SSN where statistical topics were divided into four chapters. The chapters of these textbooks were divided into sections, and at least two sections primarily addressed statistics. To gain a perspective of the relative amount of the textbook devoted to statistics, we measured the proportion of pages spanned by these sections when compared to the total number of pages within all chapters in the textbook (that is, only the pages included in chapters, which excluded the table of contents, preface, and appendices). These proportions ranged from 6.3% (Bec) to 14.2% (SSN) of the chapter pages. These data are contained in Table 4, along with the number of tasks in the assignment portions of these sections.

Table 4. Sections containing statistical content

Textbook	Bas	Bec	BLL	LDM	MBP	SSN
Number of sections	2	4	4	3	3	23
Percentage of total chapter pages	10.7	6.3	8.5	7.0	8.0	14.2
Number of tasks	267	92	369	275	333	510

Below, we report the results of our analysis along the dimensions of the GAISE Level, phases of the statistical problem solving process, and nature of the tasks within the Analyze Data phase.

4.2 GAISE Level

Although a statistical task may be assigned codes at different GAISE Levels, the highest GAISE Level of those codes was considered to be the GAISE Level of that task. As shown in Table 5, each textbook in our sample contained some tasks at each GAISE Level. Tasks within these sections that did not address statistical topics (e.g., those that were included to review a prior non-statistical topic) were coded as "not a statistical task." The distributions varied quite a bit from textbook to textbook, however. The largest proportion of tasks that did not extend beyond Level A was found in BLL; this was the most common GAISE Level for BLL. This is in contrast with the other textbooks in the sample, where the most common GAISE Level was Level B. In fact, more than half of the tasks were at Level B in both Bas (about one-half) and Bec (nearly two-thirds).

Table 5. Percentage of tasks in each textbook by GAISE Level

Textbook	Bas n = 267	Bec n = 92	BLL n = 369	LDM n = 275	MBP n = 333	SSN n = 510
Level A	15.0	14.1	43.9	29.5	32.1	22.4
Level B	51.3	64.1	33.3	37.5	38.4	43.3
Level C	32.6	18.5	14.4	18.5	23.4	33.5
Not a statistical task	1.1	3.3	8.4	14.5	6.0	0.8

Note: Percentages may not add to 100 due to rounding.

In two of the textbooks in our sample (Bas and SSN), about one out of every three tasks was at Level C. The statistical sections of these two textbooks also contained the smallest percentages of tasks that did not address statistics. In contrast, about one out of every seven tasks in the statistical sections of LDM did not address a topic in statistics.

4.3 Phases of the Statistical Problem Solving Process

4.3.1 Distribution of tasks across phases

All of the textbooks in our sample contained tasks that addressed each of the four phases of statistical problem solving, but these phases are not given equal attention. More than 70% of the tasks in each textbook requested that students Analyze Data, while fewer than 15% of the tasks dealt with any one of the other phases. There are two exceptions to this: about one-fourth of the tasks in Bas addressed Interpret Results; about one-sixth of those in SSN addressed Collect Data. Note that a single task could address multiple phases, while other tasks in these sections did not address statistics at all (e.g., a task designed to review a topic from an earlier section). For those reasons, the percentages for a particular textbook do not necessarily add to 100%. Table 6 displays the distributions of the different phases addressed in tasks contained in the analyzed sections of the six textbooks in our sample.

Table 6. Percentage of tasks addressing phases of the statistical problem solving process

Textbook	Bas n = 267	Bec n = 92	BLL n = 369	LDM n = 275	MBP n = 333	SSN n = 510
Formulate Questions	5.6	9.8	3.3	0.7	2.4	10.2
Collect Data	11.2	8.7	2.4	12.7	1.2	16.3
Analyze Data	75.7	89.1	76.7	72.0	90.7	73.3
Interpret Results	25.5	7.6	13.6	4.0	3.0	10.4

Note: Percentages may not add to 100 because a task may address multiple phases, or no phase.

The tasks in MBP focused almost exclusively on Analyze Data, with more than 90% addressing this phase and 3% or fewer addressing any other phase. Other textbooks contained only a handful of tasks in the phases of Formulate Questions (e.g., 0.7% of tasks in LDM) and Collect Data (e.g., 2.4% of tasks in BLL). On the other hand, SSN contained the largest percentage (and raw number) of tasks addressing the phases of Formulate Questions and Collect Data.

4.3.2 Statistical topics within phases

There were fifteen items on our final rubric that were addressed by each textbook in the sample. These statistical topics are displayed in boldface in the Appendix. Twelve of these topics were listed under Analyze Data, and none of them were listed under Formulate Questions. On the other hand, sixteen items on the rubric were addressed by only one textbook, and six items were not addressed by any textbook in the sample. These rubric items are also identified in the Appendix. Each phase contained at least one item that was addressed by only one textbook, and there were items under Collect Data, Analyze Data, and Interpret Results that were addressed by no textbook in the sample. It is noteworthy that among the 13 items associated with Interpret Results, two were not addressed by any textbook, and five were addressed only by SSN. In fact, SSN accounted for ten of the 16 items that were addressed by only one textbook. In other words, if SSN was not included in the sample, there would be 6 topics addressed by only one textbook and 16 topics addressed by none.

4.4 Nature of Analysis Tasks

We noted great variation among the six textbooks in our sample with respect to the nature of analysis tasks (see Table 7). For example, within Bas and SSN, about 64% of analysis tasks required students to use statistical reasoning beyond mathematical calculations; the remainder were similarly divided between the two more procedural categories of constructing or reading displays and performing mathematical calculations. By way of contrast, approximately 80% of the tasks in BLL, LDM, and MBP dealt with displays or mathematical calculations. There was also considerable variation within the textbooks that primarily contained procedural analysis tasks. Half of the analysis tasks in BLL and MBP addressed constructing and reading displays, while more than half of those in LDM were coded as requiring mathematical calculations. The Bec textbook had a different composition than the rest, with roughly 40% of analysis tasks coded in each of the two categories of “perform a mathematical calculation” and “use statistical reasoning beyond calculations.”

Table 7. Percentage of analysis tasks coded within each nature category

Textbook	Bas n = 202	Bec n = 82	BLL n = 282	LDM n = 198	MBP n = 302	SSN n = 374
Construct or read a display	16.8	18.3	50.0	22.2	50.3	15.8
Perform a mathematical calculation	19.3	37.8	36.2	58.6	29.1	20.1
Use statistical reasoning beyond calculations	63.9	43.9	13.8	19.2	20.5	64.2

Note: Percentages may not add to 100 due to rounding.

5. Discussion

The treatment of statistics varies greatly across the textbooks in our sample. All address statistics in some form, but some focus much more on the computational aspect of statistics. Below, we discuss the results of our analysis according to the GAISE Level, phases of the statistical process, and nature of the task.

5.1 GAISE Level

Each of the textbooks in our sample contained tasks at all three GAISE Levels. This is necessary, as some prospective teachers may need experiences at Levels A and B prior to attaining an understanding at Level C. Furthermore, tasks at the various levels may provide examples of tasks that could be posed to elementary students. While we do not prescribe an "appropriate" or "ideal" blend of GAISE Levels, we do recommend that textbooks include a range of levels for each phase of the statistical problem solving process. Furthermore, it may be appropriate to have fewer tasks at Level A, and more at Levels B and C, in order to prepare these prospective teachers for their desired careers. This would be particularly true for courses that include prospective middle school teachers.

5.2 Phases of the Statistical Problem Solving Process

The phase of Analyze Data gets the majority of attention in all books, with 70% to 90% of the tasks in the statistical sections addressing this phase. This appears to follow the trend in standards documents (National Governors Association Center for Best Practices and Council of Chief State School Officers 2010; Newton et al. 2011), where a similar proportion of standards related to Analyze Data. Perhaps this is necessary, due to the multiplicity of data displays (e.g., bar graphs, dot plots, and scatterplots) and statistical measures (e.g., median, mean, and interquartile range) that students must learn in the elementary and middle grades. It is also possible that the assignment portion of textbooks is a fitting place for tasks related to Analyze Data, where other process phases may be addressed during a class discussion, or with fewer tasks in the assignment portion.

Regardless of the reason, instructors may need to supplement or address other process phases during instruction, so that prospective teachers have sufficient experiences to effectively teach elementary students. For example, Formulating Questions was the lowest mean percentage of

statistical tasks for the textbooks in our sample, ranging from 0.7% (LDM) to 10.2% (SSN). This apparent lack of attention to Formulate Questions is similar to the state GLEs, where only 29 of 41 states explicitly addressed this phase. Unfortunately, a relative under-emphasis of a phase, such as Formulate Questions, may inadvertently give the message that this phase is not important. At the same time, the GAISE Report calls for teachers to pose questions at Level A. In order to meet this recommendation, prospective teachers must reach Level C and be able to pose their own questions, so that they can then utilize this skill in the elementary classroom. Instructors must provide these experiences, potentially by supplementing existing textbook tasks.

5.3 Nature of Analysis Tasks

Each of the textbooks in our sample contained tasks within the three categories (construct or read a display, perform a mathematical calculation, and use statistical reasoning beyond a mathematical calculation); however there was a clear difference in the nature of the tasks in some of the books. In some of the textbooks (MBP and SSN), there was significant emphasis on constructing displays, and in others, there was an emphasis on computational algorithms (BLL and Bec). As with the need to address all GAISE Levels in a textbook for prospective elementary teachers, it is also essential that textbooks sufficiently address the nature of statistics within their analysis tasks. For this reason, we recommend textbooks address all three types of tasks with heavier emphasis on the conceptual understanding in the analysis phase.

6. Conclusion

In order for prospective elementary teachers to teach statistical concepts, they must first learn about and use statistics. We have provided an analysis of the statistical tasks within commonly-used mathematics textbooks for prospective elementary teachers, and we assert that the following statements are true: (1) it is very unlikely that an instructor would assign every task within the textbook, and (2) it is very unlikely that an instructor would use only the tasks within the assignment portion of the textbook to teach statistical topics. We believe that in most courses, the instructor will use *some* of the statistical tasks, in addition to in-class activities from the textbook (which we did not analyze) or from sources outside of the textbook.

That being said, this article provides a tool for the analysis of the statistical content of a textbook, as well as the application of the tool to six textbooks. Instructors who use these, or other textbooks to help prospective teachers understand statistics, should be aware that all four phases of the statistical problem solving process are important, and should be addressed during the course. Additionally, students should be given tasks from a variety of GAISE Levels, and that address both the procedural (constructing graphs, performing calculations) and conceptual aspects of statistics.

Our previous experiences with different textbooks provided anecdotal evidence that not all textbooks were the same. This rubric provides a structure to describe the statistical content of textbooks, and consequently differentiates those textbooks along several dimensions. Our analysis does not present a “best” textbook. Rather, we provide a lens to help an instructor identify a textbook that best matches the goals and objectives for his or her course. On a personal note, we were delighted to see that the textbook used at our institutions, SSN, contained the

largest number of statistical tasks, covered a large number of statistical topics within nearly every level and phase, and encouraged statistical reasoning beyond (but also including) performing mathematical computations and constructing graphical displays.

The implications of this study extend beyond courses for elementary teachers. We believe all statistics educators should examine the materials used in classes for prospective middle school and high school teachers, as well as for other college-level courses, to ensure that all aspects of the statistical problem solving process are adequately addressed, and at various levels as appropriate. Additionally, we recommend that future studies focus on statistical content in textbooks for prospective middle grades or high school teachers, and compare the results to the statistical content of textbooks used in schools at the corresponding grade levels.

Using the lens of the content of textbooks, we see that prospective elementary teachers' experiences with statistics may not align with the recommendations of the statistics education community as set forth in the GAISE framework. This may have the undesirable result that these teachers will go on to teach students that statistics consists solely of mathematical computations and reading graphs. The worst possible scenario would be that these teachers ignore statistical content altogether. For this reason, we encourage our colleagues in statistics education to become involved in designing and teaching specialized courses for prospective teachers as described in CBMS (2012). Additionally, we may join in efforts working with practicing teachers through professional development courses related to statistics. Working with practicing and prospective teachers provides a unique opportunity to help shape the understanding and appreciation of statistical reasoning for these individuals and may have far-reaching effects for their current and potential students.

Appendix

Rubric for analyzing the statistical content by GAISE Level and Process Phase.

Formulate Questions

A1[a]: Teachers help pose questions (questions in contexts of interest to the student).
A2[b]: Students distinguish between statistical solution and fixed answer.
B1: Students begin to pose their own questions.
B2: Students address questions involving a group larger than their group of interest and begin to recognize the distinction among types of variables, and a population, a census, and a sample.
C1: Students formulate questions and determine how data can be collected and analyzed to provide an answer.

Collect Data

A1: Students conduct a census of the group of interest.
A2[b]: Students understand individual-to-individual natural variability.
A3[*]: Students conduct simple experiments with nonrandom assignment of treatments.
A4[*]: Students understand induced variability attributable to an experimental condition.
A5[b]: Students conduct an observational study.

A6: Students collect data by recording the results of a random process (e.g., rolling a die).
B1: Students conduct censuses of two or more groups of interest.
B2: Students design and conduct nonrandom sample surveys and begin to use random selection.
B3*: Students design and conduct comparative experiments and begin to use random assignment.
B4: Students understand the role and nature of randomness.
C1: Students understand what constitutes good practice in conducting an experiment, sample survey, and observational study.
C2: Students design and implement a data collection plan for statistical studies, including observational studies, sample surveys, and simple comparative experiments.

Analyze Data

A1: Students construct a display, such as a bar graph, dot plot [line plot], stem-and-leaf plot, scatterplot, or table.
A2: Students read the data in a display listed in A1.
A3: Students determine appropriateness of a data display listed in A1.
A4: Students observe advantages and/or make comparisons for different display types.
A5: Students determine measures of center (e.g., mean, median, mode/modal category); interpret the meaning of measure(s) of center (e.g., mean as "fair share"); or compare different measures of center for a group of interest.
A6: Students quantify variability within a group (using range).
A7[a]: Students compare individual to individual.
A8[a]: Students compare individual to a group.
A9[c]: Students become aware of group-to-group comparison.
A10: Students describe a distribution, using a center or "typical value," or describing atypical values.
A11: Students observe association between two variables.
B1: Students construct a display, such as a boxplot, circle graph (pie chart), grouped/relative frequency table, histogram, pictograph, or time series plot.
B2: Students read the data in a display listed in B1.
B3: Students determine appropriateness of a data display listed in B1.
B4: Students observe advantages and/or make comparisons for different display types.
B5: Students interpret the meaning and properties of measures of center (e.g., mean as "balance point," understand the effects on measures of center by transforming data), or determine a weighted mean.
B6: Students quantify variability within a group (using e.g., the interquartile range [IQR], mean absolute deviation, [MAD], a five number summary, or determining outliers within a group).
B7: Students compare two or more distributions using graphical displays and numerical summaries.
B8[a]: Students acknowledge sampling error.
B9: Students quantify the strength of association between two variables, develop simple models for association between two numerical variables, and use expanded tools for exploring association, including contingency tables for two categorical variables, the Quadrant Count Ratio (QCR) as a measure of strength of association, or simple lines for

modeling association between two numerical variables.

B10: Students generate a data set that fits certain conditions (e.g., mean = median < mode).

C1: Students identify appropriate ways to summarize numerical or categorical data using tables, graphical displays, and numerical summary statistics.

C2: Students understand the normal distribution and the empirical rule.

C3: Students quantify variability within a group (using standard deviation).

C4: Students understand the meaning and properties of standard deviation.

C5: Students calculate z-scores.

C6[b]: Students understand how sampling distributions (developed through simulation) are used to describe the sample-to-sample variability of sample statistics.

C7[*]: Students recognize association between two categorical variables.

C8[b]: Students recognize when the relationship between two numerical variables is reasonably linear, know that Pearson's correlation coefficient is a measure of the strength of the linear relationship between two numerical variables, and understand the least squares criterion in line fitting.

Interpret Results

A1: Students infer to the group of interest.

A2: Students acknowledge that results may be different in another group.

A3: Students recognize the limitation of scope of inference to the group of interest.

A4[d]: Students anticipate and/or explain inappropriate interpretations deriving from misuses of statistics.

B1: Students describe differences between two or more groups with respect to center, spread, and shape.

B2: Students acknowledge that a sample may not be representative of a larger population.

B3[b]: Students understand basic interpretations of measures of association.

B4[*]: Students begin to distinguish between an observational study and a designed experiment.

B5[b]: Students begin to distinguish between "association" and "cause and effect."

B6[b]: Students recognize sampling variability in summary statistics, such as the sample mean and the sample proportion.

C1[b]: Students understand the meaning of statistical significance and the difference between statistical significance and practical significance.

C2[*]: Students understand the role of p-values in determining statistical significance.

C3[b]: Students interpret the margin of error associated with an estimate of a population characteristic.

Note: Items in boldface were addressed by every textbook in the sample.
[*] Item not addressed by any textbook in the sample.
[a] Item addressed only by LDM
[b] Item addressed only by SSN
[c] Item addressed only by MBP
[d] Item addressed only by BLL

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