



Investigating student understanding of histograms

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Abstract

Histograms are adept at revealing the distribution of data values, especially the shape of the distribution and any outlier values. They are included in introductory statistics texts, research methods texts, and in the popular press, yet students often have difficulty interpreting the information conveyed by a histogram. This research identifies and discusses four misconceptions prevalent in student understanding of histograms. In addition, it presents pre- and post-test results on an instrument designed to measure the extent to which the misconceptions persist after instruction. The results presented indicate not only that the misconceptions are commonly held by students prior to instruction, but also that they persist after instruction. Future directions for teaching and research are considered.

1. Introduction

Histograms are commonly used graphical summaries of quantitative data. The use and study of histograms is included in introductory statistics texts, research methods texts, and in the popular press. For example, the popular introductory statistics text [The Basic Practice of Statistics \(Moore 2009, 5th edition\)](#) introduces histograms in the first chapter of the text. Other popular texts such as [Utts' and Heckard's Mind on Statistics \(2012, 4th edition\)](#) and [Agresti's and Franklin's Statistics: The Art and Science of Learning from Data \(2012, 3rd edition\)](#) also make prominent use of histograms. Histograms are adept at revealing the distribution of data values, especially the shape of the distribution and any outlier values.

[delMas, Garfield and Ooms \(2005\)](#) report that students “tend to confuse bar graphs and time plots with histograms. In addition, students have difficulty correctly reading information from histograms and identifying what the horizontal and vertical scales represent” (p.1). The authors hypothesize that this difficulty is because “students are most familiar with bar graphs or case value graphs where each case or data point is represented by a bar or a line, and the ordering of these is arbitrary” (p. 1). Similarly, [Meletiou-Mavrotheris and Lee \(2010\)](#) found that both U.S. and Cypriot college students had difficulty reading and reasoning about histograms. The difficulties that students have in reading and reasoning about histograms are especially problematic because histograms are important building blocks in student understanding of statistics. According to George Cobb and Robin Lock (cited in [delMas et al. 2005](#)), an understanding of histograms is an essential component necessary for students to develop understanding of density curves. Histograms, they claim, are the only graphical representations that use area to represent proportions and are, therefore, the cleanest way to make the transition to density as an idealized model of the histogram ([delMas et al. 2005](#)). The understanding of the connection between histograms and density functions or distributions is necessary to understand sampling distributions, which are, in turn, the foundation to an understanding of statistical inference ([Madden 2011](#); [Meletiou-Mavrotheris and Lee 2002](#)).

While there has been some research on student understanding of histograms, the literature contains specific calls for more work in this area. In particular, [delMas et al. \(2005\)](#) “suggest that future studies examine ways to improve student understanding and reasoning about graphical representation of distribution, and in particular, of histograms” (p. 6). Furthermore, [delMas et al. \(2005\)](#) encourage the use of items that appear in the literature for testing student knowledge so the results of new studies may be compared to the existing large set of baseline data.

The research presented in this paper seeks to answer the questions,

1. Is there evidence that students enter the first course in statistics with incorrect understandings of histograms related to the four problematic areas identified in the literature?
2. Is there evidence that these areas continue to be problematic for students after having taken a one semester course in statistics?

The research questions were addressed through a quantitative study with a pre- and post-test design. In Section 2, we present a literature review that illuminates the most common misconceptions students tend to develop about histograms. The instrument used in this study, which is provided in [Appendix C](#), is comprised of questions designed to test the misconceptions about histograms identified by the literature. The design and methodology of the study will be discussed in more detail in Section 3. The analysis of the data, given in Section 4, is descriptive rather than inferential due to the lack of random sampling: the students are a cluster sample of students from one institution. Nevertheless, we believe these students to be representative of typical introductory statistics students. The ultimate goal of the research project is to develop tools for teaching histograms that will improve student ability to reason about histograms. This is discussed in more detail in the final section of the paper.

2. Literature Review

The following misconceptions about histograms have been discussed in literature.

1. Students do not understand the distinction between a bar chart and a histogram, and why this distinction is important.
2. Students use the frequency (y-axis) instead of the data values (x-axis) when reporting on the center of the distribution and the modal group of values.
3. Students believe that a flatter histogram equates to less variability in the data.
4. For data that has an implied (though unobserved) time component, students read the histogram as a time plot believing (incorrectly) that values on the left side of the graph took place earlier in time.

2.1 Difference between bar chart and histogram

Many authors have noted the tendency for subjects to confuse bar charts and histograms. [delMas, et al. \(2005\)](#) reported that college students generally preferred “graphs where a bar represents a single value or case, rather than a frequency” (p. 5) and that certain college students concluded that any graph that used bars to represent data was a bar chart. [Meletiou-Mavrotheris and Lee \(2010\)](#) also reported the tendency of college students to “perceive bar graphs and histograms as case value graphs where each bar represents a single case or value” (p. 354). Note that these authors reference two types of bar charts: traditional bar charts of categorical data and case value graphs. In a traditional bar chart, each bar represents the number of elements in the class, for example, number of males who chose a particular answer on a survey or number of people who plan to vote for a certain candidate. These displays are part of the traditional undergraduate statistics curriculum (see for example, [Agresti and Franklin 2012](#) and [DeVeaux, Velleman and Bock 2009](#)). The second type, case value graphs, is typically not taught at the undergraduate level. In this type of graph, each bar represents a magnitude or value held by each of a set of similar cases. For example, the height of each bar might represent the number of goals scored by each team in a soccer league. The number of bars would represent the number of teams in the league. For the remainder of the paper, the first type will be called *bar charts* and the second *case value graphs*.

[Cooper and Shore \(2010\)](#) reported that among teachers “twenty-eight percent [of 75 in-service teachers, grades 4 – 12] failed to identify a histogram as a histogram, preferring to call it a bar graph, while fifty-one percent referred to all non-histogram graphs that use bars as simply bar graphs” (p. 2). They also noted results in the literature indicating a tendency of middle school students to confuse dot plots with case value graphs, thinking that the heights of the stacks of dots represented the value associated with one datum rather than an accounting of the number of items having that data value (ibid). In interviews with three elementary school teachers, [Jacobbe and Horton \(2010\)](#) found that when the teachers were given sketches of two graphs, one a bar chart and one a histogram, they tended to respond that there was no difference in the type of data that might be displayed by the two graphs. In other words, the teachers exhibited the tendency noted in other studies to see all graphs with bars as bar charts or case value graphs. In fact, the teachers noted only surface feature differences between the two graphs, for example whether or not the bars were touching and the fact that on one display the bars were all the same color and on the other they were all different colors. None of the teachers indicated that one display would be more appropriate for discrete and/or categorical data and the other for continuous and/or numeric data.

A consequence of thinking of histograms as bar charts is noted in a study of high school seniors ([Biehler 1997](#)). Biehler found that students had difficulty interpreting histograms because they were reading them with a "categorical frequency bar chart scheme in mind" (p. 176). The difficulties were manifested in two ways: students had difficulty reading histograms when the bins were labeled at the edges, rather than under the center and/or thought that a center label on a bin indicated that all observations in that bin took exactly the value labeling the bin.

2.2 Confusion between horizontal and vertical axes

Histograms represent a data reduction when compared to case value graphs ([Friel, Curcio and Bright 2001](#)). [Friel et al.](#) go on to explain that in a case value graph, the data are ungrouped and the horizontal axis contains an enumeration of the cases. The vertical axis, or height of the bars, contains the information on the magnitude of each case. In a histogram, data are grouped and the horizontal axis now contains the magnitudes, which had been displayed on the vertical axis in a case value graph. The vertical axis in a histogram tells us the number of cases that took the value (or set of values) represented on the horizontal axis and the information enumerating each case is no longer available to the reader. That readers find distinguishing the two axes problematic (ibid) should be no surprise, given the research cited above indicating people’s propensity to read all graphs with bars as bar charts or case value graphs and the manner in which the axes have changed their meaning. [Bright and Friel \(1998\)](#) observe that

It seems critical to observe that the roles of the y-axis for raw data and the x-axis for reduced data serve the same purpose; that is, providing information about the actual data values. If learners are to relate representations showing raw and reduced data, they need to be aware of the change of perspective required as these representations change (p. 65).

If students do not make this shift and consider a histogram to be a case value graph, they will read the heights of the bars as the values in the data set, finding summary statistics based on the heights of the bars and considering the size of the data set to be the number of bars. To our

knowledge, previous to this study, no one has examined this particular misconception directly. The misconception discussed in the next section is a logical extension of students' confusion between case value graphs and histograms and the issues that arise from a misreading of the axes of a histogram.

2.3 Flatter histograms show less variability

According to [Cooper and Shore \(2010\)](#), the misconception that flatter histograms indicate less variability than bumpy histograms is consistent with the misconception of the confusion between what is represented by the x- and y-axes of the histogram, discussed in Section 2.2. In particular, people make this error in judgments of variability because they fail “to consider the interplay between the frequency axis and the data values on the horizontal axis” (ibid, p. 7). [Meletiou-Mavrotheris and Lee \(2002\)](#) found that 25% of the college students they surveyed chose the bumpier histogram as the one with more variability, when in fact it had less variability than the other choice. Follow up interviews with a subset of the subjects confirmed the bumpiness as the main reason for this choice. These findings were corroborated by a larger scale study of U.S. students at the beginning of a statistics course in which nearly half of the students chose the bumpier histogram as that with more variability ([Meletiou-Mavrotheris and Lee 2010](#)).

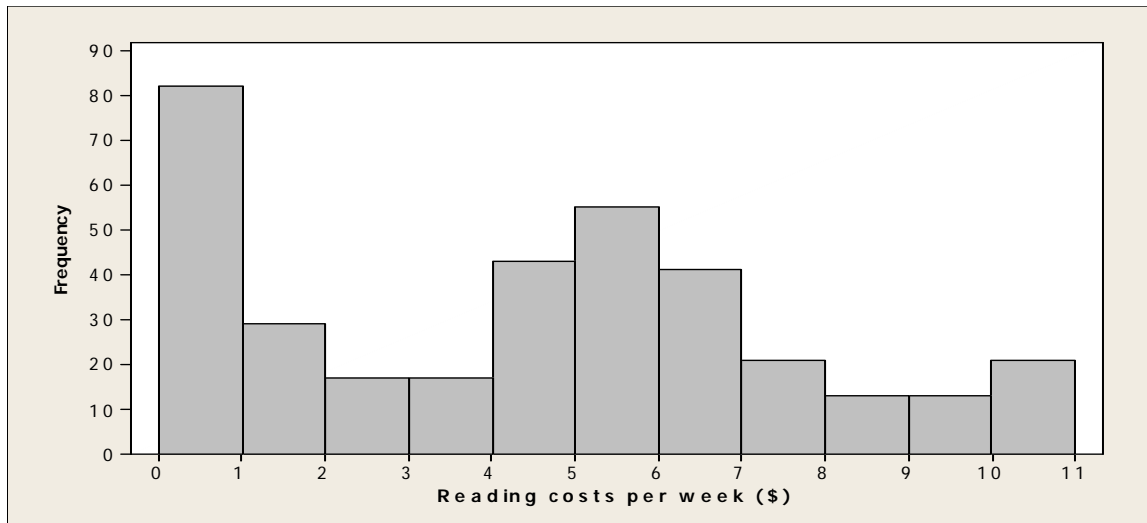
[Cooper and Shore \(2008\)](#) report similarly that nearly 50% of undergraduate students taking a statistics course who were surveyed after the descriptive statistics unit had been taught still chose the bumpier of two histograms as having more variability. These results are consistent with an earlier study completed by Shore (as reported in [Cooper and Shore 2008](#)) in which 56% of the pre-service high school mathematics teachers who were surveyed indicated that the bumpier data set had greater variability. A typical explanation for the choice was “These [heights of bars in Class 2] were basically flat, while there was a peak here and small tails [in Class 1]” (ibid, p. 5). Notice that this misconception is related to Misconception 1: confusing histograms and bar charts. The process of judging variability by the relative bumpiness of the distribution actually leads to a correct interpretation for case value graphs and is only incorrect once the data are grouped into histogram format or in a bar graph ([Cooper and Shore 2010](#)). “Simply put, the students [described in the studies above] were interpreting histograms as if they were value bar charts” ([Cooper and Shore 2010](#), p. 7).

2.4 Introduction of time component

There is relatively little research on this particular misconception, in which students exhibit a tendency to add a time component to the variable graphed on the horizontal axis. This is another misconception that stems from a lack of understanding of the meaning of the axes when reading histograms. [delMas et al. \(2005\)](#) state that students tend to confuse time plots with histograms, but do not substantiate the claim made in the abstract within the body of the paper. [Lee and Meletiou-Mavrotheris \(2003\)](#) asked students to identify the variable that would go on the horizontal axis of a histogram showing cholesterol level data of 100 individuals over the age of 40. Of the 162 undergraduate students enrolled in an introductory statistics class who responded to the question, 28% suggested that the variable age be represented on the x-axis. The authors take this as evidence of a tendency to add a time component to a histogram ([Lee and Meletiou-Mavrotheris 2003](#)). Finally, in unpublished research by the first author of this work, 517

undergraduate students who were either taking a statistics course or were undergraduate mathematics majors were asked, as part of a larger test of quantitative literacy to respond to the question shown in [Figure 1](#). A roughly equal number of respondents, 28%, gave the incorrect answer associated with reading the histogram as a time series plot (response a.) as gave the correct response (c.). These two answers were the most popular choices of the four listed, suggesting that the misconception with regard to time series interpretation of histograms may be more pervasive than the relative lack of literature on this subject would suggest.

Figure 1. This is a histogram of the average amount of money that was spent per week on reading material by each student in a random sample of 350 college students.



Select the statement that best describes the distribution.

- Students spend a lot of money on reading materials at the beginning of the semester. Then the amount they spend decreases. In the middle of the semester they spend some money, but then they don't spend very much toward the end of the semester.
- The distribution is normal, with a mean of about \$5.50 and a standard deviation of about \$2. The typical student spends between \$4 and \$7 and no student spends more than \$11 per week on reading material.
- About one-quarter of the students spend an average of \$1.00 or less on reading materials each week. For the students who spend an average of more than \$2.00 per week on reading materials, the majority spend between \$4.00 and \$ 7.00. No student in the sample spent more than \$11.**
- The distribution is left skewed with median roughly \$4.50 and range of \$0 to \$11.

3. Research Design and Methodology

3.1 The setting

The study was conducted during the winter and fall semesters of 2012 at a medium-sized (25,000 students) comprehensive university in the Midwestern United States. In 2012, 82% of applicants were admitted to the university. The interquartile range of SAT Math scores was from 510 to

620 ([Grove 2014](#)). The Department of Statistics offers roughly 55 sections of a three-credit-hour introductory statistics class each semester. Enrollments across sections are approximately 30 students and all sections are taught by faculty members. In addition to meeting in a traditional classroom, each section meets once per week in a computer lab. The course is a service course for students in a variety of majors including nursing and the social sciences. Topics covered include data collection, study design, descriptive statistics, confidence intervals and hypothesis testing for one-sample and two-sample proportions and means, correlation and simple linear regression, two-way tables and the chi square test of independence, and one-way analysis of variance.

3.2 The sample

During the winter 2012 semester, 278 students provided informed consent to use their data (see consent form in [Appendix A](#)). Students ranged in age from 18 years to 50 years with 96% of the students aged 23 or less. Students were roughly evenly split by gender (53% female). A plurality of the students (43%) were freshmen with sophomores comprising 34% of the sample, juniors 19%, seniors 3%, and other 1%. Students came from a wide variety of majors. There were 35 students claiming a STEM (Science, Technology, Engineering, Math) major, 104 Health Professions, 53 Business, 57 Social Sciences, Humanities or Arts, 35 Other, and 18 students were Undecided. (Total adds to more than 278 because of double majors.) The data were collected from students in 12 sections taught by a total of 4 different instructors. Instructors volunteered to participate in the study. Included were two tenured (PhD) faculty and two (MS) visiting faculty.

During the fall 2012 semester, 63 students provided informed consent to use their data. Students ranged in age from 18 years to 53 years with 94% of the students aged 23 or less. Females comprised 59% of the students. Unlike the winter 2012 semester, few students were freshmen (6%), a majority were sophomores (63%), with the remainder of the sample being juniors 19%, seniors 8%, and other 3%. There were only 7 students claiming a STEM (Science, Technology,

Engineering, Math) major, 21 Health Professions, 14 Business, 21 Social Sciences, Humanities or Arts, and 2 students were Undecided. Different from the data collected in winter 2012, the fall 2012 data were collected for students in 2 sections both of which had the same tenured PhD instructor. We expect that the students in the sample are representative of all students taking the course.

Students were asked whether they completed specific mathematics and statistics courses in high school and/or college. (See [Appendix B](#) Demographic Questionnaire for the exact wording of the questions.) [Table 1](#) shows the results. Differences between semesters are minor. More than one-fifth of the students had completed some calculus in high school. Many students had exposure to statistics in high school, but for most this exposure was limited to the attention given to statistics in a Functions, Statistics, and Trigonometry course. All of the students had taken the algebra course pre-requisite at either the high school and/or college level.

Table 1. Prior Mathematics and Statistics Courses Taken

High School Course	Percent Completing	College Course	Percent Completing
Pre-Algebra	79	Algebra	61
Algebra	97	Calculus I	7
Pre-Calculus	64	Calculus II	2
Calculus not AP	9	Introductory Statistics Algebra-Based	2
AP Calculus AB	17		
AP Calculus BC	1		
Functions, Statistics, and Trigonometry (FST)	35		
Introductory Statistics not AP	14		
AP Statistics	5		

N = 341

3.3 The instrument

The instrument used in this study was comprised of 10 questions. Questions 1 and 7 required an open-ended response by the students. All other questions were forced choice: either true-false or multiple choice. The results from the forced choice response questions are presented in this paper. [Table 2](#) shows the misconception addressed by each question, 2 through 10; Question 1 does not address a specific misconception. Questions 2 - 4 are modified versions of questions from the Assessment Resources Tools for Improving Statistical Thinking (ARTIST) database (see <https://apps3.cehd.umn.edu/artist/index.html>). Questions 1, 6, 9 and 10 are modifications of multiple choice items appearing on the Comprehensive Assessment of Outcomes in a First Statistics Course (CAOS, see <https://apps3.cehd.umn.edu/artist/caos.html> for more information). Questions 5 and 8 were created by the authors. [Appendix C](#) describes the complete instrument. The questions written by the authors or modified from questions in the ARTIST database are shown as they were given to the students. In order to preserve the security of the CAOS, those questions are described without the actual text or figures.

Table 2. Tabulation of Questions by Misconception

Misconception	Number of questions
1 – Students don't understand the distinction between a bar chart and a histogram, and why this distinction is important.	#5, #6, #7
2 – Students use the frequency (y axis) instead of the data values (x axis) when reporting on the center of the distribution and the modal group of values.	#3, #4
3 – Students believe that a flatter histogram equates to less variability in the data.	#8, #9, #10
4 – For data that has an implied (though not collected) time component, students read the histogram as a time plot believing (incorrectly) that values on the left side of the graph took place earlier in time.	#2

3.4 The timing of the administration

In the winter 2012 semester, the informed consent ([Appendix A](#)), demographic questionnaire ([Appendix B](#)), and instrument ([Appendix C](#)) were given to students by four different instructors in ten sections. Most of the instructors administered the forms on the first day of class prior to any instruction in statistics. One instructor (two classes) administered the forms on the second day of class having spent the first day discussing the syllabus and data collection issues. The questions were administered as a post-test to the same ten sections of students during the last week of the semester.

In the fall 2012 semester, the informed consent ([Appendix A](#)), demographic questionnaire ([Appendix B](#)), and instrument ([Appendix C](#)) were given to students by one instructor in two sections. The instructor administered the forms on the first day of class prior to any instruction in statistics. The questions were administered as a post-test to the same two sections of students on the last day of the semester.

4. Analysis/Results

The forced-choice response questions were analyzed separately to see if any of the demographic variables were associated with a difference in whether the student correctly answered the question on the pre-test. The results of the logistic regression analyses are reported in [Appendix D](#). While there were some differences noted, none were systematic. While there was some attrition of student from the pre- and post-test administrations of the instruments, no major differences were observed when the students for whom post-test data were not available were dropped from the data set. The analysis, therefore, includes the complete data set for all students who gave consent, whether or not the post-test data were available. Furthermore, the analysis of the data presented here is descriptive, rather than inferential, so the groups were combined across instructor, semester and demographics and the results are given for the entire group.

4.1 Misconception 1: Difference between bar chart and histogram

In item 5 students are given a bar chart showing the birthplaces of students in a large introductory statistics course and choose one of the following conclusions:

- A. The median is Michigan.
- B. The median cannot be told from the graph but could be if more information were given.
- C. The median cannot be found for this information even if we had the birthplace for each individual student.

The results for this item are shown in [Table 3](#). Prior to instruction, approximately 40% of the students indicate that the median could not be found for the data with 60% indicating that a median could be found, either from the data given or with more information. The proportion of students who answered correctly only rose to 50% after instruction. Most of the change appears to be from the groups of students who had previously thought that more information would allow them to calculate the median. Both pre- and post-instruction, about 25% of the students indicate that the median birthplace could be calculated from the bar chart given.

Table 3. Results Item 5: Median of Categorical Data

Answer Choice	Pre	Post	Change	Misunderstanding
A: The median is Michigan.	25.5%	25.9%	+0.4%	Median appropriate for categorical data
B: The median cannot be told from the graph but could be if more information were given.	35.2%	24.5%	-10.7%	Median appropriate for categorical data
C: The median cannot be found for this information even if we had the birthplace for each individual student.	39.3%	49.6%	+10.3%	Correct Answer

N = 341 Pre and 274 Post

In item 6, students are given a set of quantitative data in tabular form and choose from 4 graphs the one that best displays the distribution of the data. The choices given are:

- A. A case value graph with the bars in the order in which the data were presented in the table
- B. A case value graph with the bars ordered so the graph appeared unimodal and symmetric
- C. A histogram (correct answer)
- D. A case value graph with the bars in height order from smallest to largest (i.e. Pareto)

The results, shown in [Table 4](#), indicate that only 10% of students identify the histogram as correct prior to instruction and this number rises to 16% post-instruction. Prior to instruction, students choose the bell-shaped chart and the Pareto chart equally (roughly one-third of the group for each case) with 20% of the students choosing the chart with bars ordered based on the table. After instruction, more than 60% of the students choose the chart created to look bell-shaped. Consideration of the paired data indicates that most of the students who choose this option post-instruction had originally chosen one of the other case value charts. The small rise in the percent of students who correctly identify the histogram post-instruction seems to have drawn from the students who originally choose the height-ordered case value chart. It is discouraging to note the extremely high number of students who choose the bell-shaped case value chart after instruction, suggesting that students have learned to recognize the normal distribution and chose the “most normal” plot by shape rather than considering the information being presented on the graph.

Table 4. Results Item 6: Identifying a histogram

Answer Choice	Pre	Post	Change	Misunderstanding
A: Case value graph in order of data table	20.2%	7.3%	-12.9%	Bar graph appropriate for quantitative data
B: Case value graph ordered to look “bell-shaped”	35.5%	62.4%	+26.9%	Bar graph appropriate for quantitative data
C: Histogram	10.6%	16.1%	+5.5%	Correct Answer
D: Case value graph ordered to look “increasing”	33.7%	14.2%	-19.5%	Bar graph appropriate for quantitative data

N = 341 Pre and 274 Post

4.2 Misconception 2: Confusion between horizontal and vertical axes

In item 3 students are shown a unimodal and roughly symmetric histogram of Verbal SAT scores for 205 students entering a local college in the fall of 2002 and select the median of the data given the choices

- A. About 40 or 41 (the heights of the two modal, central bins)
- B. Between 19 and 26 (the median height of the bins)
- C. Between 525 and 625 (correct response)
- D. Between 425 and 525 (the values of the bins with median heights)

[Table 5](#) shows the results for item 3, which are relatively stable from pre- to post-test with over 70% of the students responding correctly. In hindsight, the easily understood context of item 3, i.e. SAT scores, may have allowed some students to determine the correct answer without consideration of the particular role of the x- and y-axis. On the other hand, our study suggests that some students do not understand the role of the x- and y-axis on a histogram. Consider as evidence the fact that even after instruction nearly 20% of the students report the frequency associated with the location of the center rather than the value of the variable at the center, indicating the persistence of this particular misconception for the students who do exhibit it.

Table 5. Results Item 3: Report a median from a histogram

Answer Choice	Pre	Post	Change	Misunderstanding
A: About 40 or 41	21.4%	19.3%	-2.1%	Confusing frequency and data value (height of middle bars)
B: Between 19 and 26	4.4%	2.6%	-1.8%	Confusing frequency and data value (median height of bars)
C: Between 525 and 625	72.1%	77.0%	+4.9%	Correct Answer
D: Between 425 and 525	2.1%	1.1%	-1.0%	Data value of bars in response B

N = 340 Pre and 274 Post

Item 4 requires students to extract the modal value from one histogram and compare it to the modal value from a second histogram. In both histograms, the modal category is the same height and is the fourth bin from the left, but the bin widths on one histogram are larger than on the other so the modal bar represents values of the variable that are larger than the values of the modal bar on the other histogram. The outcomes for item 4 are presented in [Table 6](#). Notice the decrease, from 54% to 43%, in the percent of students choosing the correct response from pre- to post-test. The reasons for this decrease are not well understood. One possible explanation is that on the pre-test students may simply be using their personal beliefs that girls tend to spend more than boys on jeans. In contrast, on the post-test students are attempting to use the evidence presented – albeit using the evidence presented incorrectly, as suggested by the large proportion of students who selected response C. This response suggests students are using the y-axis, i.e. frequencies, to determine the mode instead of the x-axis.

Table 6. Results Item 4: Comparing the modes when given two histograms

Answer Choice	Pre	Post	Change	Misunderstanding
A: Girls	54.0%	43.1%	-10.9%	Correct Answer
B: Boys	5.0%	3.6%	-1.4%	Unspecified Error
C: The modes for the two graphs are roughly the same.	41.0%	53.3%	+12.3%	Confusing frequency and data value

N = 339 Pre and 274 Post

4.3 Misconception 3: Flatter histograms show less variability

In item 8 students are asked to select the dot plot with the least amount of variability as measured by the standard deviation. Each plot had five data points. In graph A the points are grouped at the edges of the display, in graph B, they are grouped in the middle, and in graph C they are spaced uniformly along the horizontal axis. The results, in [Table 7](#), show that about one-quarter of the students correctly identify the dotplot with the least variation, with a small rise from pre- to post-instruction. Slightly more than one-half of the students incorrectly choose the most uniform (students often say “evenly spread”) dotplot as having least variability with little change from pre- to post-instruction. This outcome suggests that the grouping of data into classes when making a histogram is not necessarily the cause of the flatter histograms show less variability misconception. The misunderstanding regarding standard deviation as a measure of average distance of data points from mean even exists in dot plots which are one of the simplest visual displays of data.

Table 7. Results Item 8: Select the dotplot showing the least variability.

Answer Choice	Pre	Post	Change	Misunderstanding
A: Set A, because it has the most values away from the middle.	8.6%	4.7%	-3.9%	Unspecified Error
B: Set B, because it has the most values close to the middle.	22.3%	27.4%	+5.1%	Correct Answer
C: Set C, because it is the most evenly (i.e. uniformly) spread out.	55.8%	54.4%	-1.4%	Uniform equals low variability
D: All three datasets would have the same standard deviation.	13.4%	13.5%	+0.1%	Confusing center and variability

N = 337 Pre and 274 Post

Items 9 and 10 have students identify the lowest variability (item 9) and highest variability (item 10) from among five histograms, with results shown in [Tables 8](#) and [9](#). The percentage able to correctly identify the histogram with the large center peak and small tails as having the least variability went from just under 20% to just over 25% from pre- to post-test. More than half the students incorrectly choose the flattest histogram as having the lowest variability, though this figure dropped in the post-test. This misconception appears in item 8 as well. In fact, of the 123 students in the winter administration of the instrument who chose Class C as having lowest

variability of the dotplots in item 8 on the post-test, 78 of those students also choose the flattest histogram as having lowest variability of the histograms in item 9 on the post-test. This misconception exists for histograms and dot plots for many students even after instruction.

Items 9 and 10 were the two items that functioned differently in the fall administration when compared to the winter administration. In the fall the percentage of students choosing the correct response in item 9 increased from 18% to 40% pre-test to post-test. Most of the improvement came as students transitioned from choosing the flat histogram on the pre-test (62%, only 35% on the post-test). Again this appears to be a transfer of knowledge from item 8.

Table 8. Results Item 9: Identify the histogram with the lowest variability as measured by the standard deviation.

Answer Choice	Pre	Post	Change	Misunderstanding
A: The one with the large center peak because it has the most values close to the mean.	18.5%	26.2%	+7.7%	Correct Answer
B: The U-shaped one because it has the smallest number of distinct scores.	7.5%	3.3%	-4.2%	Equating variability with possible values
C: The uniform one because there is no change in scores.	58.8%	48.3%	-10.5%	Uniform equals low variability
D: Either the one with the center peak or the bumpy one, because they both have the smallest range.	9.2%	6.3%	-2.9%	Equating range and standard deviation
E: The bell shaped one because it looks the most normal.	6.0%	15.9%	+9.9%	Normal always equals lowest variability

N = 335 Pre and 271 Post

The results on item 10, identifying the histogram with the highest variability and shown in [Table 9](#), indicate little change from pre- to post-instruction. Disaggregating the results of the two semesters for this item also showed differences between the two administrations. In the winter administration the percentage of students able to correctly identify the U-shaped histogram as having the most variability increased from 40% pre-test to 50% post-test. The 50% of students choosing incorrectly on the post-test were not pre-dominantly choosing one particular incorrect histogram. In the fall administration the percentage of students correctly identifying the U-shaped histogram as having the most variability decreased from 46% in the pre-test to 9% in the post-test. Most of these students incorrectly chose the “flat” histogram on the post-test. Students choosing the flat histogram as having the highest variability increased from 10% on the pre-test to 63% on the post-test.

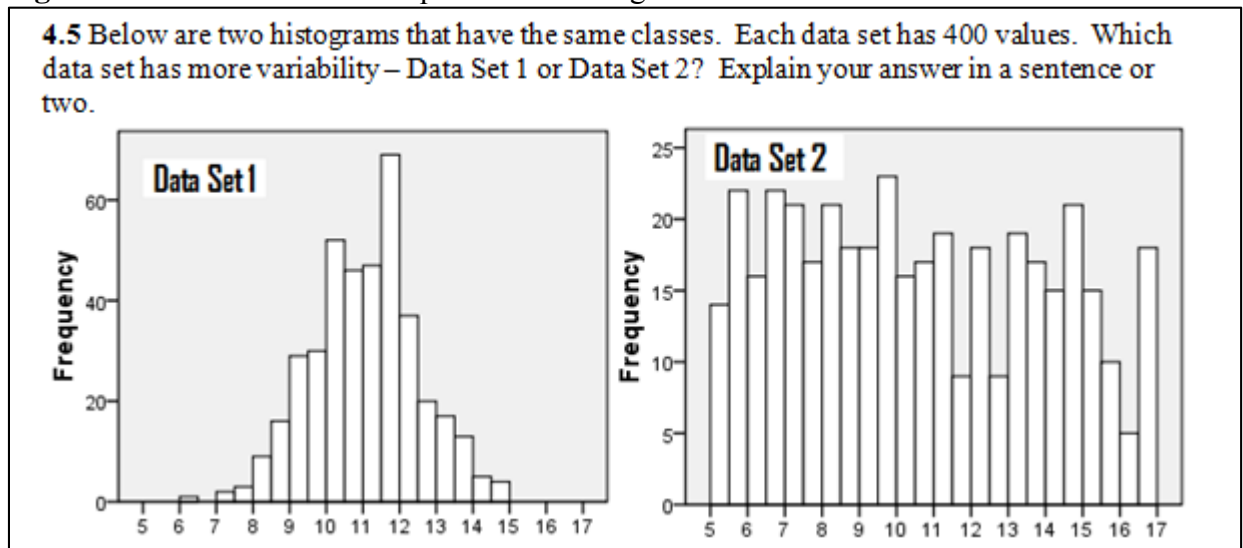
Table 9. Results Item 10: Identify the histogram with the highest variability as measured by the standard deviation.

Answer Choice	Pre	Post	Change	Misunderstanding
A: The one with the large center peak because it has the largest difference between the heights of the bars.	21.8%	16.3%	-5.5%	Equate variability to bar height differences
B: The U-shaped one because more of its scores are far from the mean.	41.5%	41.1%	-0.4%	Correct Answer
C: The roughly uniform one because it has the largest number of different scores.	14.6%	24.4%	+9.8%	Equating variability with possible values
D: The bumpy one because the distribution is very bumpy and irregular.	8.4%	5.6%	-2.8%	Equate variability to bar height differences
E: The bell shaped one because it has a large range and looks normal.	13.7%	12.6%	-1.1%	Normal always equals highest variability

N = 335 Pre and 270 Post

Recall that all of the students in the fall administration of the instrument had the same instructor. While the instructor did not spend time on dot plots, he did include homework problem 4.5 (Figure 2), which shows two histograms and asks students to identify the data set with more variability based on the histograms (Gabrosek and Stephenson 2014). In the discussion of this problem students were clued by the instructor to the fact that the “flatter histogram has more variability because it has more values further from the mean.” Clearly the fall students did not understand the concept of variability in either item 9 or 10 on the post-test. Instead they remembered the phrase from discussion of the homework problem that “the flatter histogram shows more variability.” They did not know how to apply the concept of “more values further from the mean.”

Figure 2. Fall 2012 homework question on histograms



4.4 Misconception 4: Time component introduced

Items 2a and 2c directly address the misconception that histograms contain a time component within the x-axis. Students are shown a histogram of printing costs per week in dollars for a sample of college students. Each of the items is a statement that students mark as true or false:

Statement 2A: There appears to be three times during the semester (beginning/middle/end) in which students spend a lot of money on printing at this college.

Statement 2C: This histogram suggests that students tend to spend the most on printing at the beginning of the semester.

The results for these items are shown in Tables [10](#) and [11](#) and the correct response for both parts is false. The results on question 2A indicate that over one-third of the students believe the histogram indicated a time component at the beginning of the semester in their response to item 2a. Moreover, this misconception persists after instruction as a similar percent of students respond true in the post-test. Item 2c, while done correctly by nearly three-quarters of the students in both the pre- and post-test conditions, exhibits the same persistence of the misconception. This item may have been answered correctly by a higher percent of students than item 2a not because students did not envision a time element, but because students did not agree that the mode at the left of the graph was significantly higher than the mode in the middle or on the right side of the graph.

Table 10. Results Item 2a: There appears to be three times during the semester (beginning/middle/end) in which students spend a lot of money on printing at this college.

Answer Choice	Pre	Post	Change	Misunderstanding
True	36.3%	33.8%	-2.5%	Introduce a time component
False	63.7%	66.2%	+2.5%	Correct Answer

N = 339 Pre and 272 Post

Table 11. Results Item 2c: This histogram suggests that students tend to spend the most on printing at the beginning of the semester.

Answer Choice	Pre	Post	Change	Misunderstanding
True	26.9%	24.8%	-2.1%	Introduce a time component
False	73.1%	75.2%	+2.1%	Correct Answer

N = 338 Pre and 270 Post

Item 2b asks students to consider how the graph would change if there were a rise in printing costs. Both sets of students answered this part correctly at a higher rate than either of the two other parts to the question at both the pre-test and post-test administration (see [Table 12](#)).

Table 12. Results Item 2b: If this college decided to increase drastically the amount students pay for printing, then the heights of the bars on this chart would get much taller.

Answer Choice	Pre	Post	Change	Misunderstanding
True	18.0%	16.8%	-1.2%	Confuse x and y axis
False	82.0%	83.2%	+1.2%	Correct Answer

N = 339 Pre and 273 Post

5. Discussion

5.1 Summary of Findings

The authors investigated four common misconceptions about histograms identified in the literature. The results presented in Section 4 substantiate the existence of three of the misconceptions: that students are confused about the difference between bar charts, case value graphs, and histograms, that students equate flatness of histograms with low variability, and that students have a tendency to introduce a time component to histograms even when one does not exist in the data. Students in the sample displayed these misconceptions prior to instruction. Even after the typical instruction in an applied statistics course, these misconceptions persisted.

With regard to the correct use of bar charts, case value graphs, and histograms, when given a choice between several case value graphs and a histogram and asked which best displayed the distribution of a quantitative variable, even in the post-test, fewer students chose the histogram than would be expected in random guessing of the answer. Even more alarming, 60% of students chose the case value graph in which the bars were arranged to most closely resemble a bell-shaped distribution as the best display of the data. In addition, more than half of the students, even on the post-test, responded that it was possible to compute a median when asked to consider a bar chart of categorical data.

Similarly, the data indicate that students equate flatter histograms with lower variability, with over 50% choosing the flattest histogram as having the lowest variability even after instruction. The students were somewhat better at identifying histograms that should have large variability, with nearly 50% choosing the correct option at the post-test. It should be noted, however, that nearly 25% of the students still chose a bumpier histogram as having high variability at the post-test. Furthermore, students appear to confuse the notion of equally spread out with variability when data are presented in a dot plot. The misconception of the introduction of the time component is less prevalent in the data, but persists from pre- to post-test at a rate of about two-thirds of the students. The persistence is disappointing, but perhaps not surprising.

The results on the questions designed to address misconceptions about the horizontal and vertical axes are less conclusive. The students tended to choose the correct answer in higher proportions than on the other items. Unfortunately, this may be due to the item construction, rather than actual student knowledge. The same issue arises for the last question designed to examine the misconception of the introduction of the time component. This limitation of the instrument will be discussed in more detail in the next section.

5.2 Limitations of the Study and Directions for Future Research

Some of the items used in this study may not have functioned optimally for investigating the misconceptions they were designed to target. In particular, the two items designed to assess whether students are confused by what is represented on the horizontal and vertical axes and one of the items designed to test whether students view histograms as having a time component had some construction issues. Using SAT scores as the context for item 3 may have led students to the correct response, not because they were inclined to read the axes correctly, but because the values on the horizontal axis were clearly in the range of SAT scores, which are between 300 and 800 points. In contrast, the values on the vertical axis of the graph, between 0 and 40 students, clearly were not in the appropriate range for SAT scores. Similarly, the results for item 2C may underestimate the prevalence of the time component misconception if students responded correctly not because they did not introduce time, but because they perceived the first and last bars to be of roughly equivalent heights. In future research on this topic, items 3 and 2C could be modified to address the issues raised: item 3 by changing the context to ACT scores, which would give similar scales to the two axes and item 2C by raising or lowering one of the bars on the display.

In item 4, in which students choose the graph corresponding to the data set with the larger mode, it is unclear why students chose option C, that the modes are roughly the same. It may have been because the height of the modal bars on the two histograms is the same, exhibiting the misconception being targeted, or because the modal bar is the fourth bar in both histograms. The second reason would indicate a lack of attention to the scaling of the horizontal axis, rather than confusion between the axes. The research team is currently collecting data from students to investigate the reason for the force-choice response. A second part has been added to the question in which students are asked to explain their choice. These data have not yet been analyzed and will be presented in future publications.

Even with the limitations of the research due to item construction, the fact remains that the data indicate not only that students entering a statistics course have certain misconceptions about histograms, but also that these misconceptions persist after instruction. This suggests as an avenue for future research the design of classroom interventions or curricular materials targeting student understanding of histograms. One specific recommendation in the literature to address the confusion between bar charts, case value charts and histograms is to have students translate from one type of graphical display to the other ([delMas et al. 2005](#)). For example, students might be given data in the form of a case value chart and asked to create the histogram that corresponds to the data set.

The authors also recognize that the results from this study are limited by the fact that students came from only one mid-sized, comprehensive, public university. Furthermore, instructors volunteered to participate in the study. While there is no reason to believe that students in the studied sections differed from students in other sections of the course, this cannot be stated with certainty. These two facts may limit the generalizability of the results to student populations at other universities or in Advanced Placement Statistics courses. Any future research on this topic should be expanded to include students from multiple institutions and settings and not rely on a voluntary sample of instructors.

5.3 Implications for Teaching

Mistakes in the interpretation of histograms are important not only because histograms are a commonly-used and very informative method to summarize the distribution of quantitative data, but also because histograms are a natural bridge to discussion of sampling distributions and statistical inference. How can we expect students to understand the idea of sampling variability in the sample mean when given a histogram of sample means from a simulation of repeated random samples drawn from a population, if they confuse the x-axis and the y-axis? If students always believe that the “center” of a histogram occurs at the middle of the x-axis, how can they understand the concept of skewness in the distribution of data?

As the instructor in fall 2012 learned, the ability of students to mimic what has been said in class does not imply understanding. The instructor cautioned students that a “flatter histogram means more variability” when comparing a roughly uniform distribution to a roughly normal distribution with the same binning of data. Students remembered this and incorrectly decided that a flat histogram had more variability than a U-shaped histogram. The students did not understand the concept of variability as distance from the mean as it is depicted in a histogram. The authors fear that faculty perceptions of student understanding of histograms are based more on their ability to recite a few key ideas rather than true understanding. When the authors presented results of this research to faculty at the participating university, they were met with surprised exclamations (and some dismay) at the results.

Faculty may have the view that students already have an understanding of histograms when they come into an applied statistics course at the collegiate level. After all, students are exposed to graphical summaries of data throughout their K-12 schooling. Faculty often reduce the time spent on histograms (and other data summaries) to allow more time to be spent on the perceived “hard part” of the course, namely statistical inference. This research suggests that such a decision would be foolhardy. The time spent on creating seemingly simple numerical summaries and graphical summaries of data is not wasted. Thorough discussions of graphical summaries (including histograms) and how they relate to numerical summaries may be appropriate in an introductory class especially in the age of big data where a reduction of information is necessary to gain insights into the data.

Appendix A – Informed Consent

Consent Form

INVESTIGATING STUDENT UNDERSTANDING OF HISTOGRAMS

Principal Investigator: John Gabrosek, Ph.D., Department of Statistics, Grand Valley State University
Co-Investigators: Phyllis Curtiss, Ph.D., Department of Statistics, Grand Valley State University
Jennifer J. Kaplan, Ph.D., Department of Statistics, The University of Georgia
Christopher Malone, Ph.D., Department of Mathematics and Statistics, Winona State University

Dear Introductory Statistics Student:

PURPOSE OF THE RESEARCH:

You are being asked to participate in a research project on student understanding of histograms. This study is being conducted collaboratively by Grand Valley State University (GVSU), The University of Georgia, and Winona State University. You have been selected as a possible participant in this study because you are a student in a class taught by one of the researchers or a colleague of one of the researchers. From this study, the researchers hope to learn more about student understanding of histograms. Your participation in this study will take about 15-20 minutes, both at the beginning of the term and toward the end of the term.

WHAT YOU WILL DO:

If you agree to participate, at the beginning of the term and toward the end of the term, you will be asked to answer a series of questions related to your understanding of histograms. Further, you will be asked to provide demographic information (age, gender, class standing, major, etc.). Because the study will not be finished at the end of the term, the de-identified summary findings will not be available to share with you while you are in the class. If you would like to learn of the results, you are encouraged to contact one of the researchers during the subsequent term.

POTENTIAL BENEFITS AND RISKS:

You will not directly benefit from your participation in this study, but it is hoped that this research will benefit those who take introductory statistics in the future. Your participation in this study may contribute to your understanding of histograms. There are no foreseeable risks associated with your participation in this study. Verbatim comments will be shared in publications and/or presentations given by the co-researchers. However, no actual names will be attached to any comment. Rather, comments will be credited anonymously such as, "A female at GVSU, who is a senior in Marketing, felt that..." Demographic information that could uniquely identify a student will not be used.

PRIVACY AND CONFIDENTIALITY:

All data entry will be done by the researchers at GVSU, Phyllis Curtiss and John Gabrosek. The data for this study will be kept confidential. You are being asked to provide your name on the Data Collection Form. This will allow the researcher to match responses at the beginning of the term with responses at the end. Each name will be assigned a unique identification number. Only the identification number – not the name – will be entered in the dataset. The file that links names with identification numbers will be stored on a secure, password-protected drive maintained by GVSU. Likewise, the dataset containing the responses and demographic information will be stored on a secure, password-protected drive maintained by GVSU. The researchers will need to share the dataset in order to analyze the data. The dataset will be shared electronically as email attachments, sent to secure, password-protected email accounts maintained by the three universities. The forms themselves will be stored in a locked file cabinet in John Gabrosek's faculty office. Upon receipt of the forms, the unique identification numbers will be assigned and the names will be removed. If your instructor is providing extra credit (see "Costs and Compensation for Being in the Study"), he or she will be provided with a list of participants at the end of the semester. Otherwise, your instructor will not know whether or not you have agreed to participate in this study.

YOUR RIGHTS TO PARTICIPATE, SAY NO, OR WITHDRAW:

Your participation in this research is encouraged and appreciated. However, your participation is completely voluntary. You are free to decide not to participate in this study or you may refuse to participate in certain procedures or answer certain questions or discontinue your participation at any time without consequence. Such a decision will not result in any loss of benefits to which you are otherwise entitled or affect your grade in any way.

COSTS AND COMPENSATION FOR BEING IN THE STUDY:

You will receive no compensation for participating in this study. The professor of the class may, at his or her discretion, award you extra credit points for participating. If points are awarded, participation in the research is not required to earn these points. Other activities will be available that can be used to earn the points if you prefer not to participate in the study. Whether or not points will be awarded (along with the number of points) and alternate activities will be explained in class at the time of the initial data collection.

CONTACT INFORMATION FOR QUESTIONS AND CONCERNS:

If at any time you wish to change the status of your consent prior to the end of the semester, you may contact John Gabrosek, gabrosej@gvsu.edu, (616) 331-3691. If you have any concerns or questions about this study or your participation in the study, please contact Dr. John Gabrosek, Statistics Department, MAK A-1-178, Grand Valley State University, Allendale, MI 49401, gabrosej@gvsu.edu, 616-331-3691. If you have any questions or concerns about your role and rights as a research participant that have not been answered by the investigator, you may contact the Grand Valley State University Human Review Committee Chair at (616) 331-3197.

DOCUMENTATION OF INFORMED CONSENT:

Note: If you are under 18 years old, stop now and return the consent form unsigned.

Choose one by placing an X in the blank:

_____ I voluntarily agree to participate in this study

_____ I do NOT agree to participate in this study

Printed Name: _____

Signature

Date

Appendix B – Demographic Questionnaire

First Name: _____ **Last Name:** _____

Institution: Grand Valley State Georgia Winona State

Age: _____ (years) **Sex:** Female Male

What is/are your academic major(s): _____

What is your class standing: Freshman Sophomore Junior Senior Other

Check EACH mathematics class that you completed in high school:

Pre-Algebra Algebra Pre-Calculus Calculus (Not AP)
 AP Calculus AB AP Calculus BC

Check EACH statistics class that you completed in high school:

Functions, Statistics, and Trigonometry (FST) Introductory Statistics (Not AP)
 AP Statistics Other: _____

Check EACH mathematics class that you completed in college:

Algebra Calculus I Calculus II

Check EACH statistics class that you completed in college (consider receiving an F as completing the class):

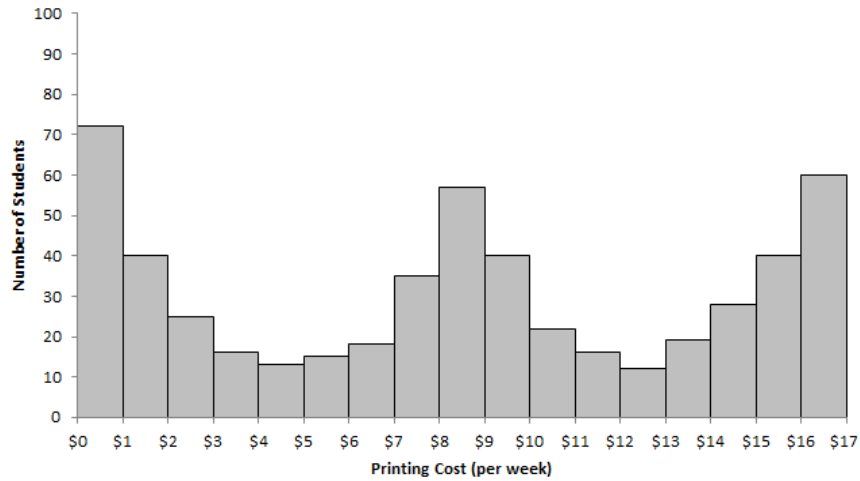
Introductory Statistics (Algebra-Based) Introductory Statistics (Calculus-Based)
 Other: _____

Appendix C – Testing Instrument

Directions: Please answer each question to the best of your ability.

Item 1 is a modified CAOS item that asks students to describe the distribution of a variable with the data given in the form of a histogram.

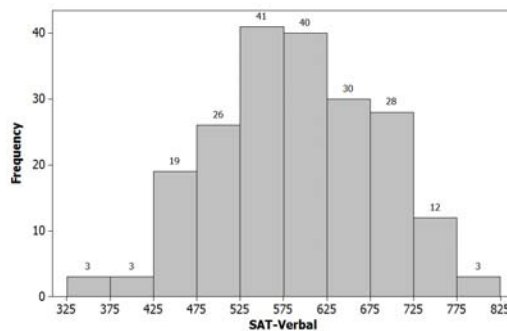
Item 2: The histogram below shows the Printing Cost (per week) for students at a nearby college.



Circle True or False for each statement about this histogram.

True	False	There appears to be three times during the semester (beginning/middle/end) in which students spend a lot of money on printing at this college.
True	False	If this college decided to increase drastically the amount students pay for printing, then the heights of the bars on this chart would get much taller.
True	False	This histogram suggests that students tend to spend the most on printing at the beginning of the semester.

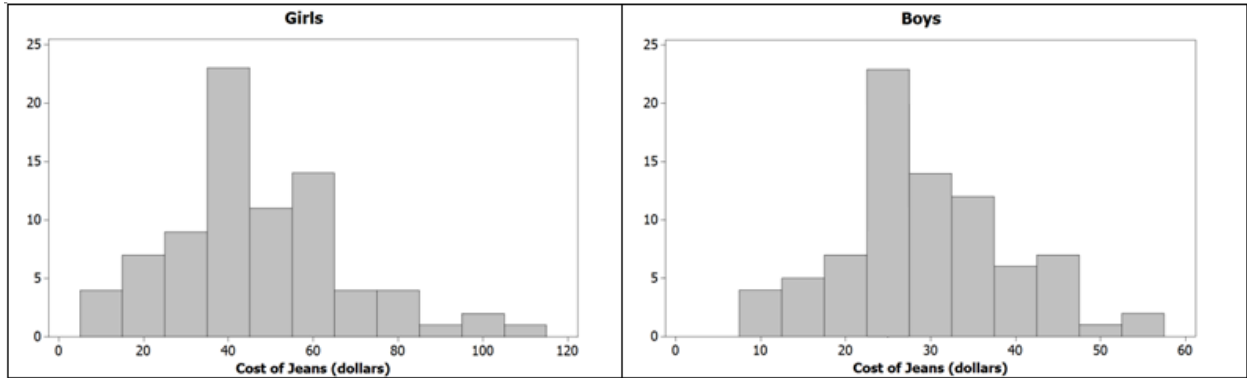
Item 3: The following histogram shows the Verbal SAT scores for 205 students entering a local college in the fall of 2002.



Circle the letter of your choice: The median score for these 205 students is:

- E. About 40 or 41
- F. Between 19 and 26
- G. Between 525 and 625
- H. Between 425 and 525

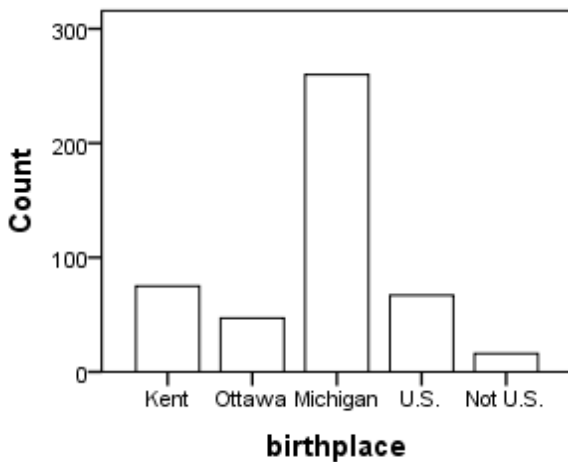
Item 4: The following two graphs represent the amount of money spent on a pair of jeans, one for a sample of high school girls, and one for a sample of high school boys.



Circle the letter of your choice: Which group has the larger mode?

- A. Girls
- B. Boys
- C. The modes for the two graphs are roughly the same.

Item 5: The following graph shows the birthplace of students in a large introductory statistics course.



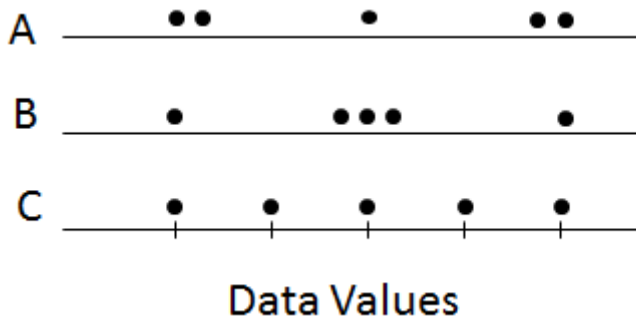
Circle the letter of your choice.

- A. The median is Michigan.
- B. The median cannot be told from the graph but could be if more information were given.
- C. The median cannot be found for this information even if we had the birthplace for each individual student.

Item 6: CAOS Item designed to test understanding that to properly describe the distribution (shape, center, and spread) of a quantitative variable, a graph like a histogram is needed. Students are given raw data and asked to identify the graph that best displays the distribution of the data. Three of the graphs are case value charts and one is a histogram.

Item 7: Explain why the graph you chose in the previous question is the best display of the distribution of the data given.

Item #8: Which of the datasets depicted in the graph below would you expect to have the least variability as measured by the standard deviation, and why?



Circle the letter of your choice.

- A. Set A, because it has the most values away from the middle.
- B. Set B, because it has the most values close to the middle.
- C. Set C, because it is the most evenly (i.e. uniformly) spread out.
- D. All three datasets would have the same standard deviation.

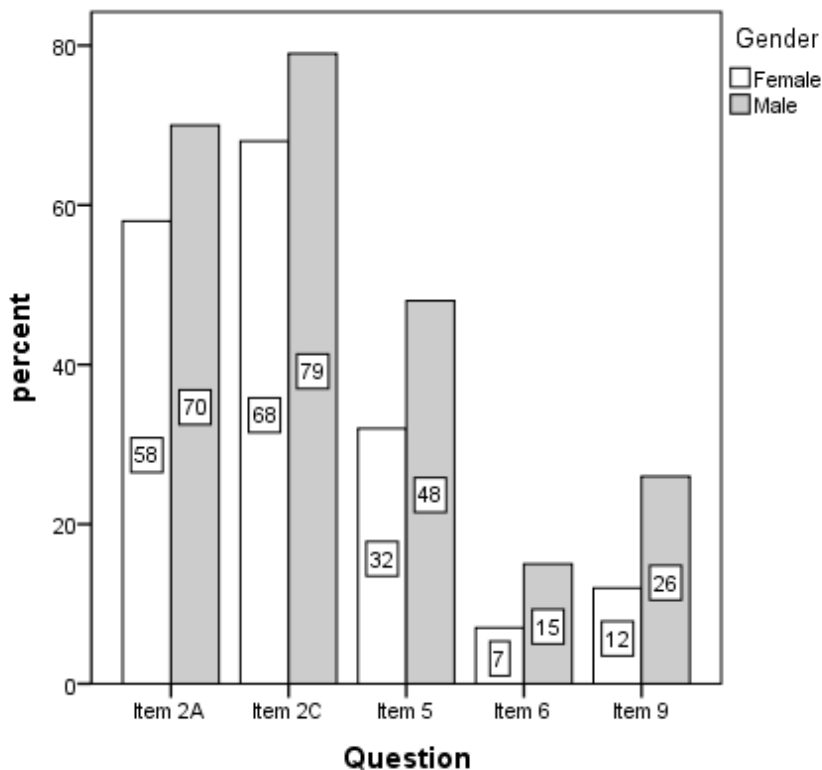
Items 9 and 10 are CAOS items designed to test the ability to correctly estimate and compare standard deviations for different histograms. In question 9, students were asked to select the histogram corresponding to the data that is expected to have the lowest variability and in question 10 the histogram corresponding to the data that is expected to have the highest variability, both as measured by the standard deviation.

Appendix D – Differences due to demographic factors

The questions 2A, 2B, 2C, 3 – 6 and 8 – 10 were analyzed separately to see if any of the demographic variables were associated with a difference in whether the person correctly answered the question on the pre-test. A logistic regression was used with the dependent variable whether the particular question was answered correctly or not. The independent variables in each logistic regression were the demographic variables: gender, age, class standing, whether the student previously had calculus or not (college or AP), whether the student previously had statistics or not (college, AP or high school but not AP), and the instructor who taught the class. The student's major was not tested due to the large number of categories generated by the variable along with the fact that many students had multiple majors creating overlapping categories.

Gender was significant on questions 2A, 2B, 5, 6, and 9. On all these questions, a higher percentage of males than females correctly answered the question on the pre-test. [Figure 3](#) shows bar charts indicating the percentages of males and females who correctly answered the question indicated. Note that significance was tested against $\alpha = 0.05$. While we realize that with multiple tests a Bonferroni-type adjustment is ideal, we use the significance testing more as a descriptive indicator of possible demographic differences rather than as evidence of differences in a population. After all, we do not actually have a random sample of students drawn from an identifiable population.

Figure 3. Percentage Correct on Pre-test Items for which Gender is Significant



After checking the questions for differences due to demographics on the pre-test, an analysis was done on the responses to the same questions on the post-test to see if any of the demographics were associated with whether the question was answered correctly or not. A logistic regression was used with the dependent variable being whether or not a particular post-test question was answered correctly and the independent variables being the same demographics included in the pre-test analysis along with the pre-test result on the particular question. The pre-test result entered into the model for all ten questions, being a significant predictor as to how the student did on the post-test.

There were only three questions out of the ten for which any of the demographics were significant. In particular, the gender difference only persisted from the pre-test for question 5 ($p = 0.025$). On this question, two other factors were significant: whether the student previously had taken calculus or not ($p = 0.040$) and an instructor ($p = 0.008$). Interactions were tested on these three terms but were not significant so they were dropped from the final model. It was found that male students have a significantly higher percentage correctly answering the question, students with calculus are more likely to get question 5 correct, and the instructor's students were less likely to get the question correct.

There were two additional questions for which the instructor variable was significant. On question 2C (p -value = 0.001), one of the instructors in the winter semester had significantly fewer students get the question correct than the other four instructors. On Question 10, the one instructor who collected data during fall semester had significantly fewer students get the question correct (p -value < 0.001). The three instructors whose students were identified as having significantly different results on each of three items were unique. That is, no one instructor was seen to have a significant effect on more than one item.

References

- Agresti, A., and Franklin, C. (2012), *Statistics: The Art and Science of Learning from Data* (3rd ed.). Upper Saddle River, NJ: Pearson Education, Inc.
- Biehler, R. (1997), "Students' Difficulties in Practicing Computer-Supported Data Analysis: Some Hypothetical Generalizations from Results of Two Exploratory Studies," in J. B. Garfield, and G. Burrill (eds.), *Research on the Role of Technology in Teaching and Learning Statistics: 1996 Proceedings of the 1996 IASE Round Table Conference* (pp. 169-190). Voorburg, The Netherlands: International Statistical Institute.
- Bright, G. W., and Friel, S. N. (1998), "Graphical Representations: Helping Students Interpret Data." In S. P. Lajoie (Ed.), *Reflections on statistics: Learning, Teaching and Assessment in Grades K-12*. Mahwah, NJ: Lawrence Erlbaum.
- Cooper, L. L., and Shore, F. S. (2008), "Students' Misconceptions in Interpreting Center and Variability of Data Represented via Histograms and Stem-and-Leaf Plots," *Journal of Statistics Education*, 16(2). <http://www.amstat.org/publications/jse/v16n2/cooper.html>
- Cooper, L. L., and Shore, F. S. (2010), "The Effects of Data and Graph Type on Concepts and Visualizations of Variability," *Journal of Statistics Education*, 18(2). <http://www.amstat.org/publications/jse/v18n2/cooper.pdf>
- delMas, R., Garfield, J., and Ooms, A. (2005), "Using Assessment Items to Study Students' Difficulty with Reading and Interpreting Graphical Representations of Distributions," in K. Makar (Ed.), *Proceedings of the Fourth International Research Forum on Statistical Reasoning, Thinking, and Literacy* (on CD). Auckland, New Zealand: University of Auckland.
- DeVeaux, R. D., Velleman, P. F., and Bock, D.E. (2009), *Intro Stats* (3rd ed.). Boston, MA: Pearson Education, Inc.
- Friel, S. N., Curcio, F. R., and Bright, G. W. (2001), "Making Sense of Graphs: Critical Factors Influencing Comprehension and Instructional Implications," *Journal for Research in Mathematics Education*, 32(2), 124 – 158.
- Gabrosek, J. and Stephenson, P. (2014), *Introductory Applied Statistics: A Variable Approach*, <http://redshelf.com/>.
- Grove, A. (accessed March 5, 2014), "Grand Valley State University – ACT Scores, Costs, and Admissions Data," <http://collegeapps.about.com/od/collegeprofiles/p/grand-valley-state-university.htm>.
- Jacobbe, T., and Horton, R. (2010), "Elementary School Teachers' Comprehension of Data Displays," *Statistics Education Research Journal*, 9(1), 27 – 45. http://iase-web.org/documents/SERJ/SERJ9%281%29_Jacobbe_Horton.pdf.

Lee, C., and Meletiou-Mavrotheris, M. (2003), "Some Difficulties in Learning Histograms in Introductory Statistics," *Joint Statistical Meetings*. San Francisco, CA, August 2003.

Madden, S. R. (2011), "Statistically, Technologically, and Contextually Provocative Tasks: Supporting Teachers' Informal Inferential Reasoning," *Mathematical Thinking and Learning*, 13, 109 – 131.

Meletiou-Mavrotheris, M., and Lee, C. (2002), "Student Understanding of Histograms: A Stumbling Stone to the Development of Intuitions About Variation," in B. Phillips (Ed.), *Proceedings of the Sixth International Conference on Teaching Statistics*, [CD-ROM] Cape Town, SA.

Meletiou-Mavrotheris, M., and Lee, C. (2010), "Investigating College-Level Introductory Statistics Students' Prior Knowledge of Graphing," *Canadian Journal of Science, Mathematics and Technology Education*, 10(4), 339 – 355.

Moore, D. S. (2009), *The Basic Practice of Statistics* (5th ed.). New York, NY: W.H. Freeman and Company.

Utts, J. M., and Heckard, R. F. (2012), *Mind on Statistics* (4th ed.). Boston, MA: Brooks/Cole: Cengage Learning.

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