If you're doubting yourself then, what's the fun in that? An exploration of why prospective secondary mathematics teachers perceive statistics as difficult

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Journal of Statistics Education Volume 21, Number 3 (2013),
www.amstat.org/publications/jse/v21n3/leavy.pdf

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Key Words: Statistics; Teacher education; Perceptions of difficulty; Attitudes, Beliefs

Abstract

Most research into prospective secondary mathematics teachers' attitudes towards statistics indicates generally positive attitudes but a perception that statistics is difficult to learn. These perceptions of statistics as a difficult subject to learn may impact the approaches of prospective teachers to teaching statistics and in turn their students' perceptions of statistics. This study is the qualitative component of a larger quantitative study. The quantitative study (Hannigan, Gill and Leavy 2013) investigated the conceptual knowledge of and attitudes towards statistics of a larger group of prospective secondary mathematics teachers (n=134). For the purposes of the present study, nine prospective secondary teachers, eight of whom were part of the larger study, were interviewed regarding their perceptions of learning and teaching statistics. This study extends our understandings garnered from the quantitative study by exploring the factors that contribute to the perception of statistics as being difficult to learn. The analysis makes explicit the tensions in learning statistics by highlighting the nature of thinking and reasoning unique to statistics and the somewhat ambiguous influence of language and context on perceptions of difficulty. It also provides insights into prospective teachers' experiences and perceptions of *teaching* statistics and reveals that prospective teachers who perceive statistics as difficult to learn *avoided teaching* statistics as part of their teaching practice field placement.

1. Background

Attitude toward statistics is the extent to which students hold positive or negative feelings towards statistics and, as a result, their perception of its relevance, value and difficulty (Evans 2007). Attitudes are rooted in experience, and as a result of their 'apprenticeship of observation' in schools (Lortie 1975), prospective teachers enter teacher education programmes with pre-existing attitudes towards the content they encounter.

Attitudes are not passive, they influence behaviour. This relationship between attitudes and behaviour has motivated research examining the role played by teacher attitudes in teaching behaviours. Research has shown that there is a well-established relationship between the attitudes of a teacher and the effectiveness of his or her teaching of mathematics (Gal and Ginsburg 1994; Ma 1999; Pajares 1992). Teacher attitudes influence pedagogical practices (Ball 1988; Cooney 1988; Fennema, Peterson, Carpenter and Lubinski 1990; O'Shea and Leavy 2013; Thompson 1984), the organization of content (Nespor 1987; Pajares 1992) and classroom ethos (Ernest 1989; Goulding, Rowland and Barber 2002). The established relationship between teacher attitude and self-efficacy and their influence on teaching style has led to a focus on developing positive teacher attitudes. Teachers with strong mathematics self-efficacy and positive attitudes towards mathematics are more likely to utilise experiential and less controlled teaching methods (Wilkins 2008), approaches which have been shown to positively impact student achievement (Jarvis, Holford and Griffin 2003).

In contrast, negative attitudes impact teachers and their teaching in a multitude of ways. Those with less self-efficacy in mathematics avoid presenting their students with higher level thinking tasks (Ross 1998; Ross and Bruce 2007) and allocate less instructional time to mathematics (Bromme and Brophy 1986). Teachers with negative attitudes have been shown to use more rule-based, teacher-directed strategies than teachers with positive attitudes whose focus is on understanding, exploration and the discovery of relationships (Wilkins 2008). Several studies have attributed instrumental teaching methods to the development of students' negative attitudes towards mathematics (Haylock 1995). Teachers who hold beliefs that contrast with reform or curricular innovation tend to resist curricular and methodological innovations, dilute curricular reform efforts (Burkhardt, Fraser and Ridgway 1990; O'Shea and Leavy 2013) and become obstacles to curricular change (Prawat 1990).

Attitude towards learning mathematics, then, has a critical role in determining the nature and quality of educational provision. For this reason, it is important to have insight into prospective teacher's attitudes to mathematics and mathematics teaching as they progress through teacher education programs. However, the construct of 'attitude' is multi-dimensional and definitions of attitude encompass affective, cognitive and behavioral dimensions. A number of instruments have been developed to measure attitudes towards statistics (e.g. Wise 1985; Roberts and Saxe 1982; Schau, Stevens, Dauphine and del Vecchio 1995) and these instruments distinguish a number of different components of attitude to statistics (for example, valuing, liking and enjoyment, difficulty, self-efficacy, anxiety). Recent research on the area of attitude towards statistics reveals the interesting finding that prospective teachers may hold positive attitudes on some of these components and less than positive attitudes on other components. This is particularly the case for the construct of 'difficulty.' There is research to indicate (for example, Estrada, Batanero, Fortuny and Diaz 2005) that prospective mathematics teachers value statistics, enjoy statistics and find it interesting while concomitantly *perceiving statistics as difficult* to learn. These perceptions of statistics as being difficult are a source of considerable concern for teacher educators as teacher attitudes play a large part in the formation of students' attitudes (Estrada and Batanero 2008; Lester, McCormick and Kapusuz 2004).

In any discussion of teacher attitudes towards statistics we need to remain cognisant that most teachers of statistics at the secondary level have more experience teaching and learning mathematics than teaching statistics. The potential impact that holding a mathematical perspective may have on attitudes towards the teaching and learning of statistics cannot be overlooked. In the following section we explore the differences between mathematical and statistical thinking and reasoning.

2. Theoretical perspective

As mentioned in the previous section, when undertaking the study of statistics at the college undergraduate level, prospective mathematics teachers often perceive statistics as difficult to learn. In an effort to gain insights into these difficulties, in the following section, we explore what is entailed in thinking and reasoning statistically. We also examine some of the factors that have been posited as contributing to these perceptions of difficulty.

2.1 Statistical thinking and reasoning

Statistics is a discipline distinct from mathematics with its own core concepts and ideas; consequently, statistical thinking is fundamentally different from mathematical thinking (Ben-Zvi and Garfield 2004; Cobb and Moore 1997; Moore 2004; Rossman, Chance and Medina 2006). Developing competency in statistical reasoning and thinking requires the acquisition of a range of skills and dispositions unique to statistics. Great strides have been made in establishing our definitions and understandings of what it means to engage in statistical activity. Distinctions have been made, importantly, between statistical *thinking* and *reasoning*. While statistical thinking and reasoning are often used synonymously to refer to the same type of cognitive activity, delMas (2004) provides a helpful distinction between the two by referring to the nature of the task. Statistical thinking, he argues, is demonstrated when a person knows when and how to apply statistical knowledge and procedures. Statistical reasoning, however, is indicated when a person can justify conclusions and make inferences.

Developments have been made in describing what is entailed in *thinking* statistically. Moore (1997, p. 178) outlined four elements of statistical thinking, which were subsequently approved by the board of the American Statistical Association, as consisting of (a) the need for data, (b) the importance of data production, (c) the omnipresence of variability, and (d) the measuring and modeling of variability. Elements of this framework are visible in Wild and Pfannkuch's (1999) four dimensional framework for statistical thinking in empirical enquiry. Arising from interviews with statisticians and those involved in statistical investigations, the Wild and Pfannkuch framework organizes some of the elements of statistical thinking during data-based inquiry. The framework encompasses an Investigative cycle (dimension 1) which describes how one thinks and acts during a statistical investigation; Types of Thinking (dimension 2) which outlines the ways of thinking which lay the foundations for statistical thinking; an Interrogative cycle (dimension 3) which delineates the thinking processes in use during statistical thinking, and Dispositions (dimension 4) which are personal qualities or characteristics that are engaged during the context of statistical problem solving. Dimension 2 identifies five types of thinking (recognition of the need for data, transnumeration, consideration of variation, reasoning with statistical models, and integrating the statistical and contextual) which are fundamentally statistical.

Statistical *reasoning*, argues delMas (2004), is akin to Galotti's (1999) definition of reasoning. This definition refers, in part, to reasoning as mental activity that transforms given information and makes inferences and draws conclusions. Thus, delMas (2004) argues, a person demonstrates statistical reasoning when they can, for example, justify particular methodologies, defend the selection of a particular representation or model, explain a result and test models for fit. These explanations are predicated on an understanding of processes that produce data leading to, for example, understandings of samples and statistics arising from those samples. Difficulties in statistical reasoning are abundant and stimulate research into developing pedagogical practices which support the development of statistical reasoning. Shortfalls in prospective and practicing teachers' statistical reasoning have been reported specifically in relation to reasoning about data displays (Jacobbe and Horton 2010; Leavy and Sloane 2008), samples and sampling (Estrada and Batenero 2008; Noll 2011), distribution (Leavy 2006; Makar and Confrey 2005), probability (Stohl 2005), and inference (Arnold 2008; Leavy 2010). Recommendations arising from these studies identify the need to support the development of statistical reasoning skills. These recommendations range from engaging prospective teachers in the design and implementation of statistical investigations, incorporating active strategies such as cooperative group work, using innovative pedagogies and authentic assessments and utilizing technology to enhance statistical reasoning.

In summary, statistical thinking and reasoning require skills and dispositions unique to statistics. There is evidence to suggest that some learners, even those who are confident in mathematics, find statistics challenging. Some of this challenge is rooted in the unique nature of statistical thinking and reasoning; however, there is evidence that other factors also contribute to the perceived difficulty of statistics.

2.2 Challenges posed by the role of context in statistics

As the teaching of statistics at the secondary level has been traditionally incorporated into the teaching of mathematics, the possibility exists that students may equate statistics with mathematics and approach statistics with the mind-set, skills and tools that they use for mathematics. Approaching the study of statistics from a mathematical perspective poses considerable obstacles for beginning learners of statistics. One major challenge posed by statistics relates to the role played by context – a role that differentiates statistics from mathematics. Dealing with context requires a different kind of thinking to mathematical thinking because the numbers which form the basis of all statistical activity are not just isolated numbers; they are numbers that are derived from and intimately connected to a context. Cobb and Moore (1997, p. 803) identify the distinctly different roles played by context in the disciplines of mathematics and statistics:

> "Although mathematicians often rely on applied context both for motivation and as a source of problems for research, the ultimate focus in mathematical thinking is on abstract patterns: the context is part of the irrelevant detail that must be boiled off over the flame of abstraction in order to reveal the previously hidden crystal of pure structure. *In mathematics, context obscures structure.* Like mathematicians, data analysts also look for patterns, but ultimately, in data analysis, whether the patterns have meaning, and whether they have any value, depends on how the threads of those patterns interweave with the complementary threads of the story line. *In data analysis, context provides meaning.*"

Students of mathematics, it can be argued, place little emphasis on context as, in their experiences, context is often irrelevant to the mathematics. It may not be surprising then to find that the context in statistics problems often misleads students (Ben-Zvi and Garfield 2004) and may result in reliance on naïve intuitions rather than statistical processes to address statistical problems. Context also poses additional challenges and implications for the teaching of statistics. In addition to possessing an understanding of mathematical theory and statistical ideas, Cobb and Moore (1997) argue that teachers of statistics also require a repertoire of authentic real world examples which produce data of the kind that lends itself to the development of students' statistical interpretation and critical judgment.

Context in statistics problems also gives rise to challenges relating to language and literacy. There are particular challenges in diverse classrooms, especially for classrooms with language learners. Lesser and Winsor (2009), in their study of English language learners in introductory statistics courses in the United States, identified the use of context as problematic for these students due to the language requirements necessary to make sense of the statistical contexts presented. As context is a core component of statistics, these same students experienced more problems with context in statistics than mathematics. The same challenges, we presume, present themselves for students with reading difficulties and/or dyslexia.

2.3 Challenges posed by the specificity of statistical language

Another language-related challenge is the preponderance of words in statistics which are also used in everyday English. Words such as random, spread, average, association and confidence are used differently in statistics giving rise to lexical ambiguity (Kaplan, Fisher and Rogness 2009). Lexical ambiguity may result in students making incorrect associations between words that have specific meanings in statistics that are different from the common usage definitions. Moore (1990, p. 98) defined the term 'random' in the context of statistics as

> "Phenomena having uncertain individual outcomes but a regular pattern of outcomes in many repetitions … 'Random' is not a synonym for 'haphazard,' but a description of a kind of order different from the deterministic one that is popularly associated with science and mathematics."

However, Kaplan et al. (2009) found that the most common use of the word 'random' by students is exactly the opposite of Moore's definition, i.e., students view an occurrence that is unplanned, unexpected or haphazard as random. Another example of lexical ambiguity relates to use of the word 'spread.' In statistics, 'spread' is used to describe the range of values in a set of data to give an indication of variability. However, spread in its everyday usage can also mean 'an even coating'. Kaplan et al. (2009) found that while the majority of students in their study interpreted spread as to disperse or scatter, a significant minority of students interpreted spread as to cover evenly.

Lavy and Mashiach-Eizenberg (2009) found that when the statistical meaning is *similar* to the meaning in the spoken language, most of the definitions given by the students in their study were correct but when the statistical meaning is *different* from the meaning in the spoken language, the majority of the students' definitions were influenced by everyday meaning of the terms. When the statistical meaning is the *opposite* of the meaning in the spoken language, as in the example of 'random' above, the majority of the students' definitions included the meaning of the concept as it is used in the spoken language.

Our quantitative study revealed that prospective secondary mathematics teachers (n=134) perceive statistics as difficult to learn (Hannigan et al. 2013). The quantitative study also provides data on the specific statistics concepts on which they performed poorly. The present study is designed to provide insights into *why* prospective teachers perceived statistics as difficult to learn. The literature examined in the previous section identifies language and context, in addition to the unique nature of statistical thinking and reasoning, as factors that may contribute towards the perceived difficulty of statistics. This information, combined with the attitudinal and achievement data arising from the quantitative study, were used to frame and generate the interview questions (see the Appendix) that formed part of this study. Hence, this qualitative study, a follow-up study to the quantitative study, explores the perceptions that prospective teachers have about teaching and learning statistics, with a view to identifying the factors which contribute to the perception of statistics as being difficult to learn.

3. Purpose of Study

This study is an exploration of the attitudes of prospective secondary mathematics teachers towards the teaching and learning of statistics. There is a specific focus on examining the factors contributing to the development of attitudes towards statistics that were identified in the larger quantitative study (Hannigan et al. 2013) and exploring, where it exists, the reasons behind the perception of statistics as being difficult to learn.

4. Methodology

Over the course of a four-year program, a team of mathematics and statistics educators documented the development, understanding and experiences of 134 students in an Undergraduate Degree Program in Physical and Mathematics Education. This is a two-subject teaching degree program. The focus of the quantitative study was on exploring and documenting the conceptual knowledge of and attitudes towards statistics of these prospective mathematics teachers (Hannigan et al. 2013). Following this, all participants in the quantitative study were informed of a follow-up qualitative study and requested to participate. Arising from this, a sample of prospective teachers, at two different stages in the program and representing a range of attitudes and understandings towards statistics as identified in the quantitative study, volunteered to participate, and were interviewed at length. Interviews were designed to explore some of the themes and questions arising from the quantitative study and focused primarily on identifying the factors contributing to the formation of prospective teachers' attitudes, and perceptions of difficulty, towards learning statistics. This paper focuses on the findings from the analysis of these in depth interviews with nine prospective teachers, eight of whom had participated in the previous quantitative study.

4.1 Participants

Participants in the study were prospective secondary mathematics teachers. All participants had studied higher level mathematics in secondary school and their performance on national university entrance examinations placed them in the top 10% of results in mathematics in Ireland. Their secondary school mathematics syllabus had a small statistics component; however, this statistics component was not a required topic of study. Topics of study in secondary school statistics are descriptive statistics, distribution (binomial and normal), populations and samples, sampling distribution of the mean, the role of the normal distribution, standard error of a mean, confidence interval for a mean, and testing of the null hypothesis at a 5% level of significance. There were nine prospective teachers in the study, four of whom were in the second year of a four-year teacher education degree program with the remainder in their final year. In qualitative research, the goal is not to generalize meaning but to describe specific cases in detail (Creswell 1997). Hence, a sample of nine, while seeming small relative to the larger sample sizes found in quantitative research, was sufficient to meet the goal of qualitative research, i.e. to provide in depth insights into the perspectives of participants relating to the perceived difficulty of statistics as a subject to learn. All nine had completed an introductory module in statistics as part of their undergraduate degree. The introductory statistics was taught in a lecture-style setting. Prospective teachers studied the module with a large group (n=200) of science degree students. The module consisted of 34 contact hours – 24 lectures and 10 tutorials over a 12 week period. Examples drew on contexts of relevance to the students and the relevance of statistics to the degree programs of the students (including biology, chemistry, sports science, food science, and biomedical engineering) was emphasised throughout the module. However, this module was not a tailored module for prospective teachers and didn't provide opportunities for the students to engage in the statistical investigation cycle or in the practices of statistics (e.g., implementing a survey or designing an experiment themselves). After completion of the statistics module, the students were expected to be able to classify data according to type and scale of measurement and distinguish between populations and samples; summarize data using graphical and numerical methods; calculate probabilities based on the application of commonly used distributions, e.g. the Normal, Binomial, Poisson and exponential distributions; construct confidence intervals and test hypotheses about population means, proportions and variances; and describe, quantify the strength of and model the relationship between two quantitative variables.

Perspectives of the four second year students were important to the researchers as this cohort of students had most recently completed the statistics module and had not yet been on their teaching practice field placement. In contrast, the five final year students had taken their statistics module two years prior to the interview; these same participants had completed a semester-long teaching practice placement in a secondary school where they had the opportunity to teach statistics.

In the previous quantitative study of 134 prospective secondary mathematics teachers (Hannigan et al. 2013), conceptual knowledge of statistics was measured using the Comprehensive Assessment of Outcomes in Statistics (CAOS) test (delMas et al. 2007). The CAOS assessment is a standard, internationally used assessment of university students' statistical reasoning after a first course in statistics. The CAOS scores for the nine participants in this study are given in Table 1; this can be compared to the mean score for the cohort of 134 students which is also presented. As indicated on the table, participants vary in their conceptual knowledge of statistics. The mean percentage correct was 45% for 134 students; the score for participants in this study ranged from 30% correct to 48% correct.

Attitudes towards statistics were measured using the Survey of Attitudes Towards Statistics (SATS) (Schau et al. 1995). The SATS-36 scale measured six attitudes subscales: Affect - students' feelings concerning statistics; Cognitive Competence – students' attitudes about their intellectual knowledge and skills when applied to statistics; Value – students' attitudes about the usefulness, relevance, and worth of statistics in personal and professional life; Difficulty – students' attitudes about the difficulty of statistics as a subject to learn; Interest – students' level of individual interest in statistics; and Effort – amount of work the student expends to learn statistics. The mean of the item responses for each subscale was obtained to give a measure on a scale of 1 to 7. Higher values indicate more positive attitudes. The SATS-36 score for the participants involved in this study are given in Table 1, as is the mean score for the cohort (n=134) on each of the subscales. As can be seen from Table 1, participant scores represent a range of attitudes across the six attitude subscales. For example, scores on the 'Affect' subscale range from 3.7 (Participant 5) representing negative feelings towards statistics to 5.8 (Participant 2) representing positive feelings towards statistics. Also evident from the table is the low mean score for the cohort of prospective secondary teachers relating to their attitudes about the difficulty of statistics. In fact, the Difficulty subscale was the lowest mean score (mean of 3.7) of all subscales on the SATS scale (as compared to other subscale means which range from 4.8-5.8). A low score on the Difficulty subscale indicates greater perceptions of difficulty and conversely a high score suggests low perceptions of difficulty. Examination of individual interview participants' scores on the difficulty subscale indicate a range in perceptions of difficulty from Participant 5 (P5) who indicated a high perception of difficulty (a mean subscale score of 2) compared to Participant 2 (P2) who had relatively low perceptions of difficulty (a mean subscale score of 4.7).

It is important to note that in the quantitative study (n=134), there was no significant correlation between scores on the six attitude subscales and performance on the CAOS test ($r < 0.2$ for all six subscales) indicating the absence of a relationship between attitude towards and understanding of statistics (see Table 2).

Table 1. Concept knowledge and attitudes of prospective teachers participating in interviews

			CAOS	SATS-36 Attitudes Towards Statistics Subscales					
Participant	**Year**	**Gender**	**(%)**	**Affect**	**Cognitive Competence**	**Value**	**Difficulty**	**Interest**	**Effort**
P1	4	Female	48	4.7	5.2	6.2	3.1	5.8	6.8
P2	4	Male	48	5.8	5.3	6	4.7	4.5	5
P4	4	Female	40	5.2	6	5.8	3.7	4.8	7
P5	4	Female	43	3.7	4	5.7	2	7	7
P6	2	Male	38	4.8	4.7	6.9	3.3	6	6
P7	2	Male	30	4.8	4.7	4.9	4	4	6
P8	2	Male	30	4.8	4.7	4.9	4	4	6
P9	2	Male	38	5	5	5.7	2.7	5	4.8
Mean for participants (SD)			39 (6.99)	4.9 (0.59)	5.0 (0.58)	5.8 (0.66)	3.4 (0.85)	5.1 (1.05)	6.1 (0.85)
Mean for cohort (SD)			**45 (10.5)**	**4.8 (1.08)**	**5.1 (0.87)**	**5.5 (0.78)**	**3.7 (0.77)**	**5 (1.02)**	**5.8 (1.09)**

Table 2. Correlation coefficients for CAOS test results with the six attitude components scores (n = 104)

	Affect	**Cognitive Competence**	**Value**	**Difficulty**	**Interest**	**Effort**
CAOS	0.17	0.19	0.13	0.16	0.01	-0.02

Table 3 provides the responses of each participant to the seven items on the Difficulty subscale of the SATS-36 scale together with the responses to questions on mathematical ability and confidence in mastering introductory statistics. Responses are on a scale from 1 to 7 where 1=strongly disagree and 7=strongly agree for the difficulty items (1=very poor/not all confident to 7=very good/very confident for the questions on mathematical ability and confidence in mastering statistics). An examination of participant responses highlights P5 as having the strongest perceptions of statistics being difficult as compared to P2 whose responses indicate disagreement relating to the difficulty of statistics. Examination of responses on the Difficulty subscale items reveals a convergence of scores on most of the items. Most participants hold neutral responses to the statement ‘Most people have to learn a new way of thinking to do statistics.’ This might suggest that participants do not see statistics as different from mathematics. Most participants disagree with the statement that ‘Statistics formulas are easy to understand’ while most agree with the statement ‘Learning statistics requires a great deal of discipline.’

Table 3. Participant scores on SATS-36 difficulty items and on mathematical ability and confidence in mastering introductory statistics

		Participants							
	Item	**P1**	**P2**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**
Items on the difficulty subscale of the SATS-36 scale	Statistics formulas are easy to understand	2	4	4	1	4	5	3	4
	Statistics is a complicated subject *	5	2	5	7	5	4	6	5
	Statistics is a subject quickly learned by most people	2	3	4	1	2	4	2	5
	Learning statistics requires a great deal of discipline *	5	4	5	7	5	4	5	7
	Statistics involves massive computations *	5	2	4	7	4	4	4	6
	Statistics is highly technical *	3	3	4	2	5	5	6	6
	Most people have to learn a new way of thinking to do statistics *	4	3	4	5	4	4	5	6
	How good at maths are you?	6	5	5	4	5	4	7	6
	How confident are you that you have mastered introductory statistics material?	4	4	3	4	6	4	6	7

** Reverse coded items in subscale*

4.2 Selection and interview protocols

All undergraduate students in the program were informed of the purpose and nature of the research via email and were invited to participate in a 30 minute interview. Nine students volunteered to be interviewed for this study and were interviewed individually. We do not contend that these nine participants are necessarily representative of the population of prospective teachers in this program. The goal of qualitative research is not to generate findings that are representative of the population under study (Creswell 1997). Our intent is not to generalize or aggregate findings across individuals; rather, our goal is to provide insights into the realities and perspectives of these nine individual prospective teachers relating to statistics. However, we can locate these participants, and their responses, relative to the larger group (Tables 1, 3). In doing so, we allow the reader to examine the data emerging from individuals which can then be framed within the context of the larger group.

The interviews were semi-structured and were carried out separately by two interviewers (one for the students in year 4 and one for the students in year 2). We were aware that the presence of participants’ lecturers during the data collection may have influenced the participants' responses. As a result, two interviewers were necessary to ensure that participants were not interviewed by the same person responsible for delivery of their course. Interviews were audiotaped. All participants were assured of the confidentiality of the responses. Ethical approval was obtained for the study from the Faculty Research Ethics committee in the University.

4.3 Data Analysis

Three researchers were involved in the design of interview questions (see Appendix), the conduct of interviews and the analysis of data. Researchers worked together to agree on precise interview foci, informed by the literature and the larger quantitative study, and to formulate interview questions. Following the interviews, researchers worked together to establish analytic frames, emerging themes and strength of evidence for claims. The quality of qualitative research is very dependent on the skills of the individual researchers. Moreover, findings emerging from qualitative analysis may be influenced by the researcher's personal biases (Suter 2012). Hence, all interviews were analysed separately by the three researchers. Each researcher completed a 'first pass' through the data and identified emergent themes from the data. These themes, and associated evidence for the claims, were distributed electronically between researchers. This allowed researchers the opportunity to revisit the data, and their own analyses, in light of the analyses of the other two researchers. Researchers then met, discussed the findings emerging from the data, identified dominant themes (a theme was identified as dominant if it occurred in over 50% of the interviews) and reached agreement around the clustering of themes into categories (Merriam 1988). Only the dominant themes are reported in this paper. Data analysis was informed by Silverman's (2000) constant comparisons and Miles and Huberman's (1994) tactics for generating meaning. The constant comparisons approach "stimulates thought that leads to both descriptive and explanatory categories" (Lincoln and Guba 1985, p. 341). It required we take a piece of data, applied a code to it and provided a description for that code. This coded data was compared with other data in an effort to construct meanings of the possible relations between various pieces of data. In this way, the consistency and accuracy of interpretations emerging from the interview data were scrutinized. The data were then clustered into categories in an effort to identify themes or patterns. The fit between the data and categories was a process of continual refinement while the data were being collected (i.e. across interviews) and retrospectively. These approaches of noting patterns, clustering, checking for plausibility across interviews and making comparisons constituted some of Miles and Huberman's (1994) strategies used to generate meaning.

5. Results

The quantitative study (Hannigan et al. 2013) revealed that prospective mathematics teachers perceive statistics as being a difficult subject to learn. In our follow up interviews, participants were first reminded of the original study and informed of the purpose of the interviews. They were told the following:

> *You might remember last year that you filled out a survey of your attitudes towards statistics. We found that many students found statistics difficult. We are interested in improving students' attitudes towards statistics and would find it useful if you could tell us a little more about why/not you find statistics difficult.*

A series of questions was then posed (see Appendix) in an effort to tease out the factors contributing to attitude towards statistics and the perceived difficulty of the subject. Analysis of the data revealed three dominant themes that contributed to the perceived difficulty of statistics: (i) The uniqueness of statistical thinking and reasoning as compared to mathematics, (ii) the use of context and language in statistics, and (iii) the positive impact of experiences teaching statistics during their teaching practice on their attitudes towards statistics. Only the first two of these themes are examined in this paper as they relate directly to factors that can be explored and addressed in the statistical curricula and experiences we provide in college-level statistics courses.

5.1 Factors influencing attitudes towards statistics

5.1.1 The uniqueness of statistical thinking and reasoning

Two subthemes constituted the theme 'the uniqueness of statistical thinking and reasoning;' both subthemes accounted for some of the perceived difficulty of statistics for prospective secondary teachers. These subthemes were: Statistics was *new* and statistics was *different from mathematics*.

Participants referred to statistics being *new and unfamiliar* to them at the college level, predominantly because they had received little if any exposure to statistics as part of their secondary school mathematics curriculum. Therefore for many of the students encountering statistics for the first time, when enrolled in the statistics module during the second year of their undergraduate studies, both the statistical content and the ways of thinking and reasoning were novel.

When asked if statistics was harder than mathematics, many participants replied that statistics was not more difficult than mathematics; it was just *different* from mathematics. In fact, when asked for recommendations of ways to improve perceptions of statistics, three participants recommended that it be taught discretely from probability, a position advocated by Cobb and Moore (1997), as it is the probability component of statistics modules that they reportedly found difficult (not the statistics). P9 stated "stats is fine but probability is a whole different area. It's so theoretical." It appeared that participants found probability more conceptually challenging that statistics.

Furthermore, participants referred to one of the difficulties with statistics as being that statistics presents a *different way of thinking* as compared to mathematics. They referred, in particular, to their comfort with the focus on there being one correct answer in mathematics and compared this with their uneasiness of dealing with the uncertainty of statistics. Our data indicate that this 'uncertainty' refers to (a) the difficulty in determining whether the answer was correct and (b) having to reason about and interpret statistical output. In mathematics, one participant referred to algebra and being able to "plug the answer back into the equation" to determine if their solution was correct. P9 articulated this difficulty in the quotation which follows.

> "They [mathematics] are things you can practice and you know you are right when you've done it. Whereas in probability and statistics, you're not sure if you're right. And I think that's why people find it difficult especially student maths teachers who are of the general mental idea that 'I like working down through a question and knowing I'm right and that's why I like maths you know?'. So when you're doing probability and statistics

> and you're not sure you're after getting it right, if you're doubting yourself then, what's the fun in that? It's more difficult.'" P9, 2nd year, interview

The uncertainty presented by statistics, as compared to mathematics, impacted the decisions of participants (and their teachers), when they were in secondary school, regarding whether to study statistics for the secondary school exit examinations (commonly referred to in Ireland as the "leaving certificate"). P6 described the tendency, during secondary school exit-examination preparation, for secondary school students to study mathematics topics, as opposed to statistics, due to the accuracy of their solution in mathematics questions being easily determined. Thus, we see from the data (and in the quotation below), that the uncertainty posed by statistics, as opposed to mathematics, also presented a challenge for the in-service secondary teachers, reported on in this study, a challenge which they in turn communicated to their students.

> "we were kind of discouraged from doing it [statistics] because you never knew if you were right. You could think you were right for the whole question and then it could end up that you weren't right at all. So we learned the other ones [mathematics topics] more, we focused on the other ones more. Well at least we will know if we were right or wrong." P6, 2nd year, interview

Some participants referred to statistics as being harder to understand than mathematics and referred to the difficulty posed by having to *interpret* the outcomes from data analytic techniques and processes, what is referred to in the literature as *statistical reasoning*. In the quote below, P4 referred to his experiences of the challenges faced by secondary school students studying statistics. He identified the source of their problems as being the different thought processes needed in statistics, in particular the element of interpretation needed.

> **"**So it is new to them [secondary school students] … they're like 'Whoa we've never done this before why'… you know 'why do we have to do that?' 'I don't know what that means.' You know … it's just the thought processes that they haven't got used to yet and it just yeah I think the interpretation is difficult." P4, 4th year, interview

Arising from responses to interview questions, participants confirmed that interpretation was a primary source of difficulty when studying their college level statistics module. Participants referred to difficulties interpreting distributions, p-values, and confidence intervals with explaining the meaning of statistical output. The following quote is from one participant when asked to identify the most challenging components of his statistics course.

> "the interpretation of graphs …you know you have bell-shaped and … distribution. It was just because it was new I think I was a bit overwhelmed. I found the interpretation and everything to go with it [difficult] … the p-values and the mean, the mode, standard deviation … just from one graph there's so much you can get from it."
> P4, 4th year, interview

5.1.2 The role of context and language in statistics

All participants were asked about their opinion relating to the role played by context in statistics (see Appendix). Opinions about the impact of context were categorized into two contrasting camps: one group considered context as a positive factor in terms of motivating students and the other group considered context as a complicating factor due to the high dependency on language/skills reading when interpreting contexts used in statistics.

All participants, however, seemed keenly aware of context as a differentiating factor between mathematics and statistics. In the transcript below, when asked if he considered statistics more difficult than mathematics, P9 distinguished between mathematical and statistical thinking by referring to the numbers in statistics as being derived from a context.

> "No I wouldn't say that [statistics is more difficult than maths], I wouldn't necessarily. I would say it's [a] slightly different way of thinking behind it but … with your mathematical thinking. There can be some very abstract thoughts. With statistics it's all there for you. They are stats. You do have to manipulate formula and stuff like that but it's not … it is all there for you … it's the statistics *of* something … so like it's laid down, it's concrete you know?" P9, 2nd year, interview

Seven participants held positive opinions about the context presented in statistical activity. Three of these participants said that the context in statistics made it more interesting for themselves as learners.

> "I always enjoyed taking statistics and figuring out what they mean. That comes from an interest in other areas … even in sport. Reading the paper you see the possession territory especially in rugby games … [context] makes it more interesting, relevant and easier as well … if you provide a context … definitely" P6, 2nd year, interview

Participants also referred to the impact of context on the experience of statistics for secondary school students. This was particularly the case for 4th year students who had had the opportunity to teach statistics while on teaching practice placement. They mentioned how the use of context makes statistics more interesting and engaging for students. P3 referred to context acting as an incentive to engage in reasoning by "masking" the mathematics for students.

> "the thing about statistics is you can just use it for anything so it's just a way of representing information. So long as they're [secondary school students] interested in the information they're not going to be as much focusing on the maths side of things. So you can kind of lead them with a carrot there you know. They'll enjoy it. They are trying to find out who scored the most goals or whatever … they're not thinking about maths for thirty minutes." P3, 4th year, interview

The remaining three participants communicated some reservations about the use of context in statistics and referred to experiences or situations where context caused problems for students who had reading difficulties. Thus, we can see that the context poses difficulties when we take into consideration the characteristics of learners. P7, who himself is dyslexic and referred to the relevance of statistics when engaging in reasoning about sports scores, provided valuable insights into how context poses problems for him. Similarly P4 drew on her experiences of a specific students' difficulties understanding the role played by context in statistics questions.

> "there is a lot of English in statistics and being dyslexic it was a huge problem for me … doing problem solving. I was always really good at maths. I always do algebra but then it came to a written question and if I slipped up on a word, then I got the question wrong. I was never good at English … I was never good at reading, I was never good at writing … I was never good at spelling … but I excelled at maths. For a student like me it [statistics] would be a lot more difficult." P7, 2nd year, interview

> "[context] depends I think on the student … I was helping her [a student] out … she just thinks that that's [the introduction to the question] only a bit of information that you don't need. She just thinks that's only an introduction … it just didn't connect … she just didn't connect the two parts of it. I have to constantly tell her read the questions, read the question … she just jumps straight into the actual part with the question mark you know … she has dyslexia but maybe English for her as well is difficult … it's like there is too much information sometimes … too much English. They're like 'this is maths where are the numbers?'" P4, 4th year, interview

The role of language was confirmed by many of the participants as posing difficulty in the learning of statistics.

> "because you're mixing … some people aren't strong at English, some people don't like wordy problems" P9, 2nd year, interview

> "I think the students that are weak at English, I do think they struggle with it … it was more like they were doing a comprehension for English and they didn't understand English properly" P2, 4th year, interview

While the interviewers had guiding interview questions, the interviews were of an open structure so as to allow an interviewer to probe any interesting or unexpected responses. During the interview with P6, a discussion arose regarding why he thought people who were good at mathematics may think statistics is difficult. His response referred back to the role played by language in the teaching and learning of statistics.

"… integrate literacy more I think with stats … so you are kind of seeing things and you have to turn them from words into data … I think people view it as harder because of all the wordiness" P6, 2nd year, interview

Lexical ambiguity was specifically mentioned by one of the students in her discussion of language.

> "There are words that have two different meanings … the meaning in the real life and the meaning in statistics … the word random … they might not recognise that we are talking about statistics here" P4, 4th year interview

Participants' long history and relationship with mathematics posed difficulties when learning statistics, particularly when confronted with text. The tendency of participants to recall strategies that deal with text in mathematics and apply these same strategies to statistical situations contributed to the difficulty of statistics. As Cobb and Moore (1997) identified, mathematics treats context as almost superfluous and the approaches used in mathematics to deal with statistics is the root of some of the difficulties in statistics. This was the case in our study. For example, P3 reported that the tradition in mathematics to identify keywords in questions is quite different from the expectations in statistics curricula and, when combined with the tendency of learners to not associate words with mathematics, poses considerable difficulties for learning statistics.

> "in years gone by you could look at a question and you could pick one word and .. you could just tear into it. Whereas now [with the implementation of a reform statistics/mathematics curriculum in Ireland] … you kind of have to read it and [you] might interpret it wrong"
>
> P3, 4th year, interview

> "[people] don't really associate words and writing with maths. When you see a load of words on pages, you just think 'oh that's not maths'. And you just take it as words and not think about it as numbers." P6, 2nd year, interview

It appears from the quotations above that traditional ways of teaching mathematics present conflict for some learners when approaching the study of statistics, especially within the context of *reform curricula*. Several participants reflected on how new expectations emerging from reform curricula pose challenges particularly in terms of the requirement to demonstrate understanding[1]. P7 reported on a conversation he had with his mentor teacher about the new Irish reform curriculum and how it contrasts with his experience of doing mathematics.

> "She was saying it's [the reform curriculum] all about your comprehension of it. I would have hated that in school. I know I would have hated that. I don't think Project Maths would have complimented that way I did maths." P7, 2nd year, interview

[1] At the time of the study, secondary schools in Ireland were undergoing curriculum reform in mathematics and were transitioning to a new mathematics curriculum called *Project Mathematics* [http://www.projectmaths.ie/].

6. Discussion

This study represents an effort to explore the complex landscape of attitudes towards learning statistics. A prior study of the attitudes of 134 prospective secondary mathematics teachers (Hannigan et al. 2013) revealed positive attitudes towards statistics with participants placing a value on statistics, communicating interest in statistics and confidence in their intellectual knowledge and skills when applied to statistics. However, these same participants perceived statistics as difficult to learn, a finding that supports other international research on teacher attitudes towards statistics (Estrada et al. 2005; Estrada and Batanero 2008; Zientek, Carter, Taylor and Capraro 2011). These perceptions of difficulty are a cause of concern due to the possible impact on prospective teachers' inclination to teach statistics in addition to the potential for these attitudes to be communicated to secondary students, hence continuing the cycle of negative attitudes towards statistics. There is, however, a paucity of research examining the factors that contribute to these perceptions of difficulty. Without this information, teacher educators can do little to counteract these perceptions of difficulty through, for example, the modification of course design and implementation of pedagogical innovations. As a result, this research seeks to contribute to our understandings of the factors contributing to prospective teachers' perceptions of the difficulty of statistics.

Statistics represented a new and challenging way of thinking for participants in this study. This was an interesting finding, as students in teacher education programs in Ireland are generally high academic achievers. In fact, participants' performance in state examinations indicated that they ranked within the top 10% of mathematics students graduating from secondary school in Ireland. Despite their demonstrated expertise in mathematics, statistics presented a challenge for them. As might be expected, due to their demonstrated mathematical abilities, participants rarely referred to problems associated with implementing specific statistical skills or procedures, i.e., statistical thinking. Interestingly, the challenges for them occurred in the area of statistical reasoning. These problems reflected components of what Galotti (1999) defined as reasoning – when transforming given information (e.g. statistical output) and making inferences and drawing conclusions. Participants frequently referred to the challenges they faced when interpreting the meaning associated with statistical processes and with providing justifications. This type of thinking was not similar to the ways of mathematical thinking they had experienced and developed in their secondary school studies and thus posed a significant challenge when they encountered it in their university statistics module. The core statistical ideas participants grappled with were not mathematical in nature and thus represented a new departure in terms of developing understandings. This finding supports the growing recognition that statistics is not a subfield of mathematics; it is a separate discipline (Moore 2004). While part of the reason accounting for this finding had to do with their limited *exposure* to statistics, a large part had to do with statistical *ways of thinking* that are distinctly different from mathematical ways of thinking.

It appears that participants' lack of awareness of this distinction between mathematics and statistics contributed to their perceptions of statistics as being difficult. The statistical ideas and reasoning they were presented with were counterintuitive to them. In fact, the data reveal that in some cases the context presented in statistics problems posed a significant obstacle, in part due to learner characteristics, but also due to a lack of recognition of the important role played by context in the discipline of statistics. This finding is in keeping with research indicating that 'the context in many statistical problems may mislead the students, causing them to rely on their experiences and often faulty intuition to provide an answer" (Ben-Zvi and Garfield 2004, p. 4).

It was evident from analysis of the data that probability presented difficulties for many prospective teachers. They distinguished statistics from probability and for the most part indicated that statistics was their preferred subject of the two. Several went as far as to suggest that probability and statistics be studied separately at secondary level so that negative feelings regarding probability not contaminate attitudes towards statistics. The notion of separating the study of probability and statistics at the college level is supported by Cobb and Moore (1997) who posit two reasons to support their argument that introductory statistics courses should contain no formal probability. Firstly, developing conceptual understanding of statistical inference does not necessitate exposure to formal probability; informal probability, they argue, is sufficient. Secondly, formal probability they also contend "is conceptually the hardest subject in elementary mathematics" (p. 821) and they refer to research carried out by Tversky and Kahneman (1983) and Garfield and Ahlgren (1988) when outlining the complexities of developing probabilistic reasoning and addressing the commonly reported defective probabilistic intuitions held by many. Perhaps these cautions regarding probability, supported by participants in this study, deserve some consideration because, as Moore (1992) accurately pointed out, probability is a field of mathematics whereas statistics is not.

The data emerging from this study also provide insight into prospective teachers' perspectives on the role played by language in the teaching and learning of statistics. There is evidence presented that concurs with the conclusions of Kaplan et al. (2010) that lexical ambiguity presents a significant challenge to the teaching of statistics. Our data suggests that instructors need to pay close attention to their own use of terms which are used both in statistics and in everyday English and address them explicitly in pedagogy and content modules. Our data support the recommendation of Lavy and Mashiach-Eizenberg (2009) that discussions in which the relationship between the terms' meaning in statistics and meaning in everyday use be initiated. Instructors need to repeatedly draw students' attention to the differences between the meanings of these terms in the two domains and explicitly link them so that students will develop statistical meanings of them. This is challenging given that Konold (1995) found that students enter statistics classrooms with strongly held, but incorrect, intuitions which are highly resistant to change. Lavy and Mashiach-Eizenberg (2009) also suggest that in addition to computational exercises given after learning new concepts, a portion of the lesson should be dedicated to understanding the spoken language meaning of terms or concepts and the inter-relations between them.

A unique contribution of this study to the literature on statistics education is the finding that perceptions of prospective teachers regarding the difficulty of *teaching* statistics were open to change; and in the case of these participants, teaching practice placement was the vehicle which brought about change in perceptions. This potential of teaching practice to positively impact perceptions of difficulty is a welcome and somewhat unique finding which merits further investigation with larger cohorts of prospective teachers and in different educational jurisdictions and contexts.

6.1 Implications and recommendations

It is clear from this study that many of the mathematical skills and understandings held by participants were not sufficient to support the development of understanding in statistics. In fact, some of the mathematical ways of thinking and reasoning were in conflict with the types of statistical thinking necessary to succeed in undergraduate level statistics modules. We recommend that undergraduate students of mathematics, in particular prospective teachers, are provided with background regarding the history of the development of statistics and supported in developing an awareness of the distinctive characteristics of statistical reasoning, literacy, and thinking. One theme of an "Introduction to Statistics" course might be examination of the genesis of statistics and an overview of the attributes of statistical reasoning, literacy, and thinking. Such an approach might support learners in understanding that statistics is a different way of thinking and help them develop a sense of the landscape of statistical thinking and reasoning. Such a perspective might counteract the tendency to "equate statistics with mathematics and expect the focus to be on numbers, computations, formulas, and on right answers" (Ben-Zvi and Garfield 2004, p. 4). It may also provide opportunities for prospective teachers to become aware of the role played by context in statistics, to become comfortable with the messiness of data, and develop an awareness of the extensive use of writing skills required in the teaching and learning of statistics. Furthermore, developing awareness that statistical thinking and reasoning constitutes a different way of thinking, and emphasizing that it may take some time to develop this type of statistical literacy, may move the focus from statistics being *difficult* to statistics being *different*. Then, when a student first struggles with a concept in statistics they may be less likely to "doubt themselves" (as was the case with P9) and more likely to recognize it as a part of the learning process. Emphasis should also be placed on terminology and the meaning of statistical terms. Efforts to highlight the cases where statistical meanings differ from their real world meanings may also support learners in developing statistical understandings.

There are limitations associated with this research. As with all research carried out within the qualitative paradigm, the knowledge arising from the research might not generalize to other people or other settings. Two factors influence the generalizability: the context and the sample size. The context of this study is unique to these prospective teachers, with these specific experiences within this geographical location. The sample size is also small, focusing on nine prospective teachers. As this is one of the few studies exploring prospective teachers' perceptions of the difficulty of statistics to learn, many of the unanswered questions arising from this study may serve as the focus of future research. Will these findings extend to other populations of prospective secondary mathematics teachers? Do different experiences of statistics at secondary school impact perceptions of difficulty of statistics at college level? Does the role of language and terminology in statistics pose similar difficulties in other languages?

The findings arising from this study emphasize the need for a greater presence of statistics in mathematics teacher education programs, in part, to provide time to develop statistical ways of thinking and reasoning. As the larger quantitative study (Hannigan et al. 2013) indicates, statistical reasoning poses a greater challenge than statistical thinking for prospective mathematics teachers. The CAOS test used in the quantitative study (Hannigan et al. 2013) assesses statistical reasoning and prospective mathematics teachers scored poorly on this instrument. These findings support the recommendation, arising from other studies (Estrada and Batanero 2008), of the necessity for more than one statistics course in undergraduate degrees. Statistics courses for prospective teachers should focus on the development of not only content and pedagogical content knowledge; a problem-solving approach needs to drive the design of statistics curricula. Statistical activity needs to reflect the practices of statisticians and be situated within an investigative cycle reflecting statistical activity as it happens in the discipline of statistics. This process of enculturation (Schoenfeld 1992; Resnick 1988) involves students engaging in the practices of the community of statisticians and developing their points of view. In these ways, prospective teachers can begin to develop the habits of mind necessary to teach statistics in authentic and meaningful ways. Developing the habits of mind associated with statistical thinking is as important as developing the knowledge and understandings that underpin statistical activity.

Appendix

1. Is there any one specific experience that you recall which may have influenced your perception of the difficulty of statistics?
2. Do you find statistics more difficult than mathematics? Why/not?
3. Has your perception of the difficulty of statistics changed from second level to third level?
4. What areas of statistics do you find the most difficult e.g. producing data, describing data, drawing conclusions from data?
5. Does the context in statistics i.e. a description in English of the background make statistics easier? more difficult? more interesting? more relevant?
6. Do you find questions which ask you to interpret your answers in context easy?
7. Does the language/symbols used make statistics difficult?
8. Do you have any suggestions regarding how we might improve perceptions of students regarding the difficulty of statistics?

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