Letter to the Editor

Further comments and conceptualizations regarding the mean

Thoughts on the paper "*What does the mean mean*?" by Nicholas N. Watier, Claude Lamontagne, and Sylvain Chartier in the July 2011 issue of JSE. I commend the authors for offering an interesting progression of conceptualizations of the mean and I offer these observations:

While the term "socialist" is attention-grabbing, students may not necessarily assume that socialism implies everyone gets the same amount, especially if people have unequal needs or contributions. In any case, there may be less ambiguity in the classroom to refer to it as the "redistribution" value (e.g., see Lesser, 2007, p. 12) and perhaps even less if an instructor refers to the "fair share value" (e.g., Franklin, Kader, Mewborn, Moreno, Peck, Perry, and Scheaffer, 2007, p. 30) or "leveling value".

The "algebraic form of the least squares" conceptualization claims that students would have to know how to differentiate a single variable quadratic polynomial. It should be noted that the necessary minimization can be done without calculus at all, but simply with the traditional (high school or college) algebra tool of completing the square. Using this tool, equation (3) can be expressed as $n(c - \bar{x})^2$ plus terms not involving c, and so it is clear (without calculus) that this expression in c is minimized when $c = \bar{x}$. Another issue, however, is that instructors should not assume that students will find it natural to focus on the sum of the squares of deviations in the first place rather than, say, the sum of the absolute deviations. (As an aside, this also applies to regression: see Sorto, White, and Lesser, 2011; Lesser, 1999; Puritz, 1981.) In light of this, instructors may want to consider giving students opportunities to generate and explore possible ideas, such as the simulation applets in Lane (2011).

Care should be taken in distinguishing the mean from the midrange (half the sum of the minimum and maximum values) and the median. It may be confusing for students if an instructor uses two-number datasets (in which case, the mean, median, and midrange are all the same) or includes words such as "middle" and "midpoint" in discussion. For example, Russell and Mokros (1990) note that students who are "midpointers" may struggle to find and interpret the mean of a nonsymmetric distribution where the fulcrum balance point is not obvious.

It may be helpful for some students to see that a mean can be conceptualized as a proportion. The mean of n Bernoulli trials (i.e., the mean of n numbers that are each 0 or 1) is the proportion of those trials that are successes. This simple connection allows us to apply the Central Limit Theorem to find the sampling distribution of a proportion (e.g., COMAP, 2009, p. 272).

One further conceptualization of a mean to consider is the estimation of a signal amidst a noisy process or environment. This is a very different idea because it is the mean of a process rather than of a population (Konold and Pollatsek, 2002).

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