



Enhancing Students' Inferential Reasoning: From Hands-On To "Movies"

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Abstract

Computer simulations and animations for developing statistical concepts are often not understood by beginners. Hands-on physical simulations that morph into computer simulations are teaching approaches that can build students' concepts. In this paper we review the literature on visual and verbal cognitive processing and on the efficacy of animations in promoting learning. We describe an instructional sequence, from hands-on to animations, developed for 14 year-old students. The instruction focused on developing students' understanding of sampling variability and using samples to make inferences about populations. The learning trajectory from hands-on to animations is analyzed from the perspective of multimedia learning theories while the learning outcomes of about 100 students are explored, including images and reasoning processes used when comparing two box plots. The findings suggest that carefully designed learning trajectories can stimulate students to gain access to inferential concepts and reasoning processes. The role of verbal, visual, and sensory cues in developing students' reasoning is discussed and important questions for further research on these elements are identified.

1. Introduction

This paper forms part of a large program of work on the staged development of the big ideas of statistical inference over a period of years. It was prompted by the authors' need to make inferential ideas accessible to New Zealand high school students in response to the learning goals of the new high school statistics curriculum in New Zealand. Much of this paper's discussion is also relevant, however, to adult education and introductory statistics courses at colleges and universities. Papers on this work that have already appeared include [Wild, Pfannkuch, Regan and Horton \(2011\)](#) proposing conceptual pathways to inference for novices and [Pfannkuch, Regan, Wild, and Horton \(2010\)](#) addressing verbalizations and story telling about data. The current paper deals with some of the work done on translating our conceptual pathways into learning trajectories for students. Computer animations perform a pivotal role in our learning trajectories. It is well known that computer animations are no panacea and that often students simply do not see the things they are meant to see ([Wild 2007](#)). This issue led us into a careful review of the literature on cognition as it relates to animations. The particular learning trajectories described were devised for and trialed on students aged about 14.

The paper is organized as follows. Section 2 presents the problem that was occurring in New Zealand classrooms when students and teachers compared two box plots. Section 3 sketches how we went about solving this problem and the new problems that arose as a consequence. Section 4 discusses some of the research literature that should inform designers of animations. The research component of this paper relates to action research involving students aged approximately 14. Section 5 presents our research plan while Section 6 reports on analyses of our evaluation data. Section 6.1 focuses on the reasoning domain of providing evidence for making a claim when comparing two box plots. In relation to the literature, we describe and analyze the research design and its implementation of a learning trajectory for a Year 10 class (approximate age 14). Section 6.2 deals with pre- and post-test learning outcomes of students from four classes (13 to 16 year-olds) and pre- and post-interviews of 14 students. Lastly, in Section 7 we discuss our findings and pose further questions about developing students' inferential reasoning using technology such as dynamic animations together with broader cognitive questions.

2. The Problem

In a classroom of Year 11 (15 year-old) students a teacher was comparing male and female university students' Verbal IQ, which she referred to as IQ (Fig. 1).

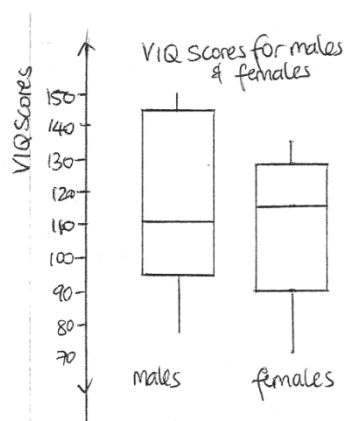


Figure 1. Comparison of male and female university students' verbal IQ scores

She said (all quotations verbatim): "I've got some conflicting information, the median – females are more clever, but when I look at the whole graph, the whole graph's a bit higher for males ... so I'm not ready to say, yes, males have a higher IQ than females." Since the situation appeared to be inconclusive the teacher wrote down: "Based on these data values we are not certain that males have a higher IQ." However, in response to a student who queried this statement with, "but couldn't you say, from the graph, that males do have a little bit higher IQ than females," she added: "there is some evidence to suggest that males have a higher IQ for these Uni. students."

Her first written statement drew a conclusion about populations whereas her second statement drew a conclusion about the samples. Furthermore, the student's focus appears to be on the fact that the male box is higher up the scale than the female box. Such a situation is typical in New Zealand classrooms. Students and teachers are expected to draw a conclusion with evidence in a national assessment standard yet they do not know what features of the graphs will provide evidence for their conclusions and they do not know what they are drawing a conclusion about – the data in hand or the populations from which the data were sampled. Only the latter type of reasoning will lead students towards statistical inference and assist them to be enculturated into statistical practice by learning how and why statisticians make decisions based on data. The situation observed in the Year 11 classroom also led us to question why the focus of the school curriculum was on descriptive statistics and why the concepts underpinning statistical inference were not being laid down in the junior secondary school in preparation for their introduction to formal statistical inference such as confidence intervals and t-tests in the last year of school. Some research evidence (e.g., [Chance, delMas, & Garfield 2004](#)) pointed to the fact that a limited conceptual base was a factor in students' inability to understand inferential ideas such as the sampling distribution in introductory undergraduate courses. Other researchers have also explored how to promote students' inferential reasoning (e.g., [Pratt & Ainley 2008](#)).

Since 2003 members of our group have been concerned about how students and teachers were drawing inferences from the comparison of box plots and the justifications that they were giving for their conclusions ([Pfannkuch 2006](#)). Through exploratory research in junior secondary classrooms we took gradual steps to understand the problem, explore possibilities, and reflect on the outcomes. [Pfannkuch \(2007\)](#) believed that lack of appreciation of sampling variability was limiting the students' informal inferential reasoning. Therefore, in the next two years she experimented with developing Year 10 (14 year-olds) students' concepts of sample, population,

and sampling variability using two web applications ([Pfannkuch 2008](#)). These seemed to stimulate in students some awareness of variability in both categorical and numerical data, of the effect of sample size, and of a connection between sample and population. Dynamic visual imagery, tracking the history of the variability of medians and proportions, and the discourse of the teacher were considered important in stimulating these concepts. But the problem of judging whether A tended to have bigger values than B still remained as the students and teacher continued to draw conclusions based on the difference between the medians without considering sampling variability ([Pfannkuch 2011](#)). The problem was that even though they were aware of sampling variability, samples and populations, they had no rational basis such as a “test” on which to make a judgment.

3. Resolutions and New Problems

Our group set about developing conceptual pathways including guidelines about how to make a call on group differences incorporating visual animations. Some appreciation of the nature of these animations can be gained from a 1.5 minute movie clip at:



click on camera or below link – you need a [Flash player plugin](#) to play this video.

<http://www.amstat.org/publications/jse/v19n2/09.USCOTS.excerpt.swf>. These are described in [Wild et al. \(2011\)](#). Although we sketch some of the main principles in the paragraphs that follow, the discussion will be much easier to understand if you look at the animations webpage: www.censusatschool.org.nz/2009/informal-inference/WPRH/. Further information is available in the screen capture video of Chris [Wild's 2009 USCOTS Plenary address](#) at http://www.causeweb.org/uscots/uscots09/program/uscots09_wild.php.

Some important principles behind the development of the “tests” were that the approach should:

- Allow students to make a decision or call about whether one group tends to have bigger values than another group by providing a pathway of increasingly more complex heuristics that seek to deepen students' understanding of sampling variability (see [Fig. 2](#)).
- Work from a minimal set of inferential ideas that can gradually be built upon.
- Connect to more formal statistical inferential methods.
- Encourage students not to take their eyes off their plots (see [Fig. 3](#)).

For a more complete discussion about how well these heuristics for making a call work, their rationale and justifications, and on further issues and technical aspects associated with our approach, see [Wild et al. \(2011\)](#).

The major aim behind our development of the visual animations is to ensure that whenever students see a box plot they will visualize a vibrating box plot ([Fig. 3](#) and the dynamic images referred to above), recall the words “what I see is not really the way it is back in the populations” and remember to take sampling variability into account before making a call. The desire is for this embedded visual imagery in their minds to be associated with concepts of sample,

population, sampling variability and sample size effect and with a heuristic that is appropriate for their age or developmental level for making a call (Fig. 2).

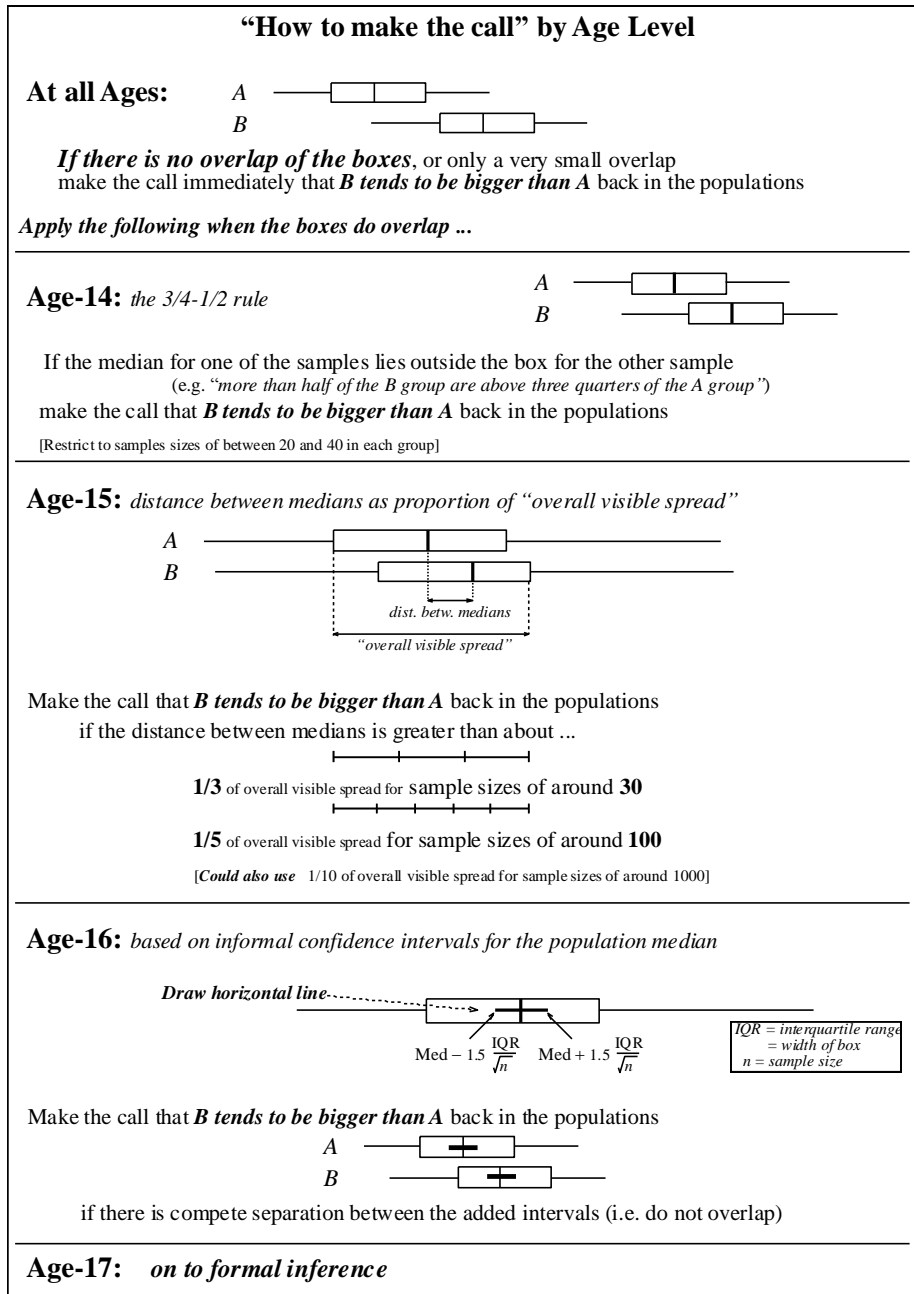


Figure 2. How to make a call at each age or developmental level

We had developed plausible conceptual pathways whereby teachers and students could make inferences about populations from samples and visualize sampling variability but now we were faced with a new set of problems.

- How could we scaffold students' conceptions to understand the animations and make sense of them?
- What learning trajectories could assist students to re-invent, for themselves, the "tests" we had developed?

We were aware that the animations would make sense to an expert but may not to a novice ([Lovett 2010](#)), were underpinned by a multitude of concepts such as sample, population, sample size, and sampling variability, and used a conceptually demanding abstraction, the box plot ([Bakker 2004](#)). The understanding of the "tests" ([Fig. 2](#)) and the animations were cognitively demanding. Our hope was that the animations would be memorable enough that when students saw a boxplot they would remember a vibrating box plot, which would act as a catalyst to retrieve a connected set of ideas about inference including the informal decision rules for making a claim.

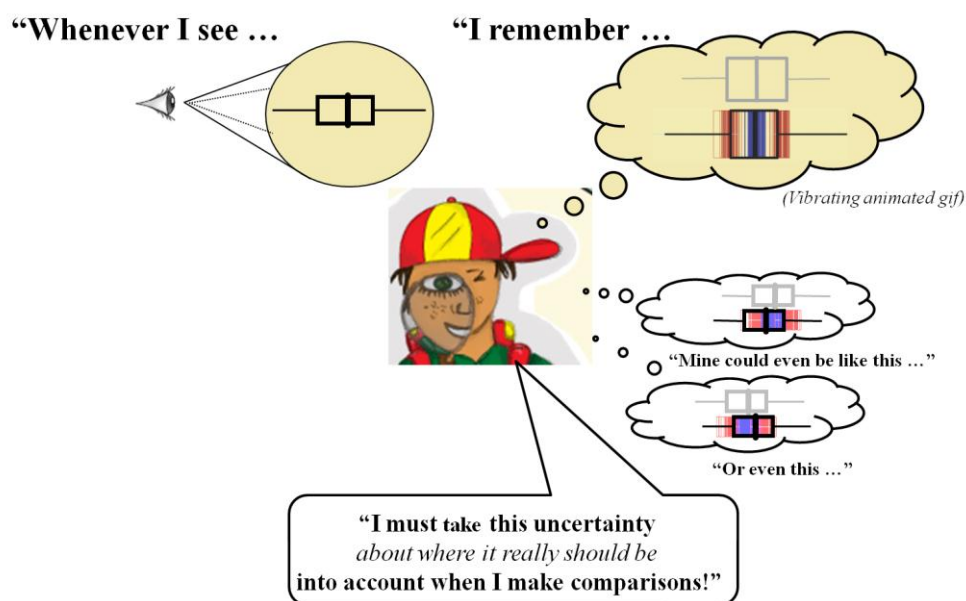


Figure 3. The desired habit of mind

From our previous research and the literature we recognized that students do not see what experts see and that prior knowledge such as hands-on learning experiences are essential for understanding abstract dynamic imagery ([delMas 1997](#); [Rossman 2008](#)). Also essential for concept development through imagery is the concurrent development of language and discourse that communicates the essence of what is being captured in the visuals. (See [Pfannkuch et al. \(2010\)](#) for a discussion on the language and verbalization issues we considered.)

Our two-year research project, which involved a team of two statisticians, two statistics education researchers, and nine secondary teachers, was about how to resolve these new problems. The resolution of these problems needed to take into account: the current technology constraints of the New Zealand learning environment, where many secondary classrooms have access to only one computer and a data projector; the time constraints imposed by the school

program; and the national assessment constraints, which occur at Years 11, 12, and 13. Although we developed learning trajectories for Years 10, 11, and 12, this paper will describe the teaching and learning trajectories that were developed for Year 10 students (14 year-olds) through a case study of one class.

The focus of the paper is on building the concepts of sampling variability for inference in comparison situations. This is only an extremely small part of the many ideas, such as posing statistical questions, how to capture relevant measures and variables, how to select samples, how to obtain good quality data, how to design experiments, and how to reason from scatter plots, which need to be developed for exploring data and doing investigations.

Before we describe the designed learning route we used and the resultant learning outcomes we discuss the literature related to the use of animations and how students' understanding of animations can be facilitated.

4. Literature Review

Technological advances are facilitating a change to the way we think and communicate as a visual culture starts to dominate over a print culture ([Arcavi 2003](#)). Such a shift has profound implications for education and in particular for statistics. A single visual image can contain more than “a thousand words” because it is two or three-dimensional and has a non-linear or even dynamic organization as opposed to the logical sequential exposition of the printed word. Often concepts underpinning the sequential manipulation of symbols remain obscure to many students. These concepts are now becoming accessible through visual representations, which allow a new way of engaging with abstractions. There is evidence that student understanding can be enhanced by the addition of visual representations and that encouraging students to generate mental images improves their learning ([Clark & Paivio 1991](#)). [Ware \(2008\)](#) predicts that pictorial forms of teaching will continue to grow and complement verbal forms of teaching. Two theories in particular, the *dual coding theory* and *cognitive load theory* are being used to support research into ways to enhance students' learning and reasoning from dynamic visualizations.

4.1 Dual Coding Theory

Dual coding theory divides cognition into two processing systems, verbal and visual ([Clark & Paivio 1991](#)). Within and between these verbal and visual systems are three separate levels of processing – representational, associative, and referential. In the verbal system the mental *representations* for words such as book, learn, statistics, and anxiety are verbal codes that denote concrete objects and events as well as abstract notions. The visual system includes the mental *representations* for shapes, sounds, actions, sensations, and other non-linguistic artifacts. In what Clark and Paivio call the imaginal structure, a shape such as a triangle can be mentally rotated and transformed, which is not possible with verbal representations. The *associative* connections, according to the dual coding theory, join words to other related words in the verbal system, and images to other images in the visual system. For example, the words PPDAC (problem, plan, data, analysis, conclusion) may be linked in a statistics lesson as an associative chain. The image of a dot plot of heights of a sample of Year 10 students may be linked to an image of a population distribution of heights of Year 10 students.

The links between the verbal and visual systems are called *referential* connections. The word population might invoke an image of all people in New Zealand or an image of a population distribution of a particular attribute such as height. The dual coding theory predicts that learning is enhanced when information is coded in both systems and connected between them. [Rieber, Tzeng, and Tribble \(2004\)](#) showed that the integration of information is most likely to occur if the learner has corresponding pictorial and verbal representations in the working memory at the same time. However, if time is not provided for the learner to reflect on the principles being modeled then referential processing might not occur. This theory seems to fit well with [Bakker and Gravemeijer \(2004\)](#) who noted that students' conceptual growth in understanding distribution was predicated on the images (individual cases to clumps to distributional notions) and the classroom discourse. The classroom discourse allowed time for the students together with the teacher to actively reflect on and draw out the main ideas underpinning their lesson experiences.

Although [Clark and Paivio \(1991\)](#) divide cognition into two systems, verbal and visual, [Radford \(2009\)](#) has proposed that "sensuous cognition" needs to be taken into account in the learning process, particularly for novices. He believes thinking is facilitated *in* and *through* speech (language), body, gestures, symbols and tools. To Radford, gestures and actions are genuine constituents of thinking and may be a source for pre-linguistic conceptual formation and abstract thinking. He describes how learners processed and understood information from a mathematics graph task using gestures and actual actions with signs and artifacts. After they conceptualized the information the gestures diminished and the verbal interactions became dominant as the students discussed the graph. Such research lends support to the ideas that tactile, kinesthetic, hands-on tasks are invaluable for novices ([delMas 1997](#); [Lovett 2010](#); [Rossman 2008](#)). Hence we believe that verbal, visual, and sensory cognitions need to be addressed when designing learning trajectories to scaffold students' statistical thinking.

4.2 Cognitive Load Theory

Another learning theory that has caught the attention of researchers into learning with animations is the cognitive load theory originally proposed by [Sweller \(1988\)](#). Since learners' working memory capacity is extremely limited and a major bottleneck in cognition, we transcend such limitations of the mind by providing external visual aids ([Ware 2008](#)). However, to process new information takes a great deal of attention and learners' attentional capacity is limited ([Lovett 2010](#)). The three sources of cognitive load are intrinsic, extraneous, and germane.

- *Intrinsic load* is determined by the complexity of the application domain and by the learners' prior knowledge (the approach is to increase prior knowledge before learning from computer simulations).
- *Extraneous load* is the mental effort imposed by the way information is presented externally (the approach is to reduce this).
- *Germane load* is the result of mental activities that are directly relevant to learning. These activities contribute to learning and are relevant to the construction and automation of knowledge in the long-term memory (the approach is to increase this).

Because germane load is the most important factor in learning there have been studies on how to reduce the extraneous load of visual animations and increase prior knowledge before using them. Initially there was a general belief that dynamic media tools had enormous potential for

instruction but research showed that animated displays were no better than static displays for learning ([Hegarty 2004](#)). Researchers are now finding out the *conditions* for animations to be effective in learning, that is, how to reduce the extraneous load.

4.3 Processing Visual Information

The design of dynamic visualizations requires not only an appreciation of the cognitive mechanisms that underlie complex thought ([Chandler 2004](#)) but also an understanding of how visual information is processed ([Ware 2008](#); [Canham & Hegarty 2010](#)). In particular, for graphics comprehension there is a constant interaction between bottom-up perceptual processes of encoding information, which drives pattern building, and top-down inferring processes that are based on prior knowledge. Prior knowledge affects which parts of the graphic are fixated on and encoded, which then influences inferences. Some eye-tracking studies ([Mayer 2010](#)) demonstrated how the comprehension of graphics and learning outcomes are significantly affected by the display format. Improved student performance was noted and linked to eye fixations for designs following the signaling, prior knowledge, and modality principles but not for the pacing principle. The signaling principle states that visual cues such as color should be used to highlight the relevant features to attend to, and fading should be used for irrelevant features. The prior knowledge principle states that the more relevant prior knowledge students have before viewing an instructional graphic the better they will perform. The modality principle states that only two modes, visual and spoken, should be used to avoid overload. Accordingly, [Mayer and Moreno \(2002\)](#) maintain that the visual and spoken must occur together, that extraneous words, sounds, and videos should be excluded, and that spoken words should be personalized and be in a conversational style. That is, avoid overloading a single channel and process information in parallel not sequentially. [Hegarty \(2004\)](#) noticed that the pace of dynamic displays could affect comprehension. She proposed that learners should be able to speed up or slow down a display to match their comprehension speed and view or review different parts of a display in any sequence. This pacing principle was not corroborated in the eye-tracking studies but nevertheless should be a consideration for learners.

Other research has suggested further ways to support learners dealing with complexity such as giving them an opportunity to process static information before viewing dynamic visualizations. This gives learners time to identify and become familiar with relevant structures in representations. That is, animations should offer the learner a small number of changes against a familiar expected background. Once an animation finishes it is no longer available to the viewer. Therefore, static visual displays of the animation allow learners to re-inspect parts of the display. The eye-tracking research ([Mayer 2010](#)) indicates that viewers re-inspect parts of graphics many times in the process of comprehension.

Research is starting to elucidate the *conditions* for animations to be effective in learning. Conditions include reducing extraneous cognitive load and increasing prior knowledge before learning from visual animations and recognizing the importance of promoting visual and sensory cognition alongside verbal cognition. What is missing from the literature is evidence about improved learning outcomes for some of the methods we used for our visual displays. For example, we used both color and motion cues to foster understanding of variability. We pondered many questions, such as: Do motion cues improve the learning of concepts since we know that

motion attracts attention ([Ware 2008](#))? Does motion affect “memorability”? Does using body movements together with visual and auditory stimuli result in cognitive overload due to competition with one of the other channels or is it a separate channel? Is it possible to work in these three modes simultaneously? [Greer \(2009, p. 701\)](#) commenting on an earlier version of our tools remarked: “it is notable that the sample values are not represented numerically which may well be very significant since numerical values could cue computation, whereas the visual counterpart invites comprehension. Consider also how the process unfolds in time, and leaves a history ... that could stimulate episodic memory of the process that gave rise to it.” Further questions arise such as: Do numeric cues hinder concept formation? Do visual representations of numeric values assist the building of concepts? How does the tracking feature that leaves a history of the variability influence learning outcomes? Is it possible to design visual and auditory experiences for memorable moments that will act as catalysts, links, triggers, or pathways to recall other sets of memories that facilitate reconstruction of the original ideas associated with memorable moments?

Statistical graphs, static and dynamic, are visual thinking tools. They are more than illustrative images; they are tools for reasoning and thinking. Visualization processes are key components of that reasoning through “deeply engaging with the conceptual and not the merely perceptual” ([Arcavi 2003, p. 235](#)). Since access to statistical concepts is strongly related to representational infrastructure, there is evidence that with *good design* technology may allow students to engage with concepts previously considered too advanced for them ([Sacristan, Calder, Rojano, Santos-Trigo, Friedlander, & Meissner 2010](#)). Technology also seems to facilitate transitions in students’ thinking such as from the concrete to the abstract ([Shaughnessy 2007](#)). Since the way information is presented matters in the learning process, an instructional route to a desired goal needs to be designed.

The *hypothetical learning trajectory* (HLT) is a construct that underpins good task designs by characterizing and identifying an instructional route to develop students’ thinking and reasoning processes ([Simon 1995](#)). The generation of an HLT is based on the prior knowledge of the students, identifies learning processes and tasks that will assist concept formation, and aims to guide and scaffold student learning towards the goal of what we want students to learn. There are several learning theories aligned with HLTs such as the *scaffolding and abstraction theory*, which suggests starting with the concrete situation and moving towards abstraction, and the *webbing and situated abstraction theory*, which involves abstracting within, not away from, the situation. Both of these theories involve giving students support towards new ideas and concepts in an abstraction process. Even though “research into DT [digital technology]-based learning trajectories is still in its infancy” ([Sacristan et al. 2010, p. 220](#)), our conjecture is that the devising of learning trajectories that include conceptually accessible visualizations together with new verbalizations (cf. [Pfannkuch et al. 2010](#)), without the need for mathematical manipulations, will allow students to understand and use inferential reasoning successfully.

5. Research Plan

The research reported in this paper is part of a large two-year project. The research method is a mixed methods approach of pre- and post-tests, interviews and design research principles ([Roth 2005](#)) for a teaching experiment in a classroom. Such research engages researchers in improving

education and provides results that can be readily used by practitioners ([Bakker 2004](#)). There are three main features of design research ([Cobb, Confrey, diSessa, Lehrer, & Schauble 2003](#); [Edelson 2002](#)). The first feature is the aim to develop theories about both learning and the instructional design that supports that learning. The second feature is the interventionist nature of the methodology whereby instructional materials are designed in an attempt to engineer and support a *new type of learning and reasoning*. The third feature is the iterative nature of the research whereby attention to evidence about learning and reasoning results in revision of learning trajectories and trialing of new designs. In the preparation and design stage the research project team, consisting of nine teachers, two statisticians, and two researchers, worked together to develop the teaching and learning materials to use in the teaching experiments. The learning trajectories were evaluated and critiqued in a series of six meetings before, during, and after implementation with follow-up discussions continuing over many days.

The research, conducted over two years, went through two developmental cycles. In both years four classes participated from Decile 1 to 9 schools (Decile 1 is the lowest socio-economic level while 10 is the highest). Classes were: First year – two Year 10s, one Year 11, one Year 12; Second year – two Year 9s, one Year 10, one Year 11. The students participating in the research were selected because their teacher was in the research project team, their teacher agreed to conduct the research, and the school was teaching the statistics unit during the data collection period. The intention was to have only Year 10 and 11 classes, as box plots are not part of the Year 9 curriculum, but this proved not to be possible. In the implementation teachers were free to adapt and modify the designed resources and proposed learning trajectory to suit their students and approach to teaching. The main data collected were: pre- and post-tests, videos of four classes implementing the teaching unit (selected on the basis of researcher availability when the statistics unit was taught), pre- and post-interviews of some students from these video taped classes, and teacher reflections.

In the domain of making a call or decision about whether one group tends to have bigger values than another group we were particularly interested in the following:

1. How can students be stimulated to start developing concepts about statistical inference?
2. What type and level of informal inferential reasoning can students achieve?

6. Analyses

The first part focuses on the implementation in a Year 10 class, the second part on the learning outcomes of students.

6.1 Analysis Part One: Class Implementation

The data used were drawn from the implementation in a Year 10 class (14 year-olds) in the first year. This teacher had a researcher with her in the classroom. Reflective discussion followed each lesson, which allowed adjustments to be made to the hypothetical learning trajectory. The 26 students in the class were average in ability and were from a mid-size (1300 students), multicultural (about 35% New Zealand European, 45% Pacific Island and Maori, and 20% Other), Decile 5 socio-economic inner city girls' secondary school.

The retrospective analysis of the implementation is focused on three lessons (lessons 12 to 14 in a 15-lesson teaching sequence) where students learned about sampling variability for samples of about size 30 and what “calls” or claims they could make about populations from samples when

comparing box plots (All resource material is available at: www.censusatschool.org.nz/making-the-call-year-10/). Prior work in the teaching sequence focused on: posing different types of questions, describing summary and comparative distributions, learning about taking samples from populations, constructing box plots from dot plots, and conducting investigations involving the comparison of groups using dot plots and box plots. Students' prior knowledge is discussed and then the type of learning experiences the students had in three main phases in lessons 12 to 14 is described and *analyzed in relation to the literature* discussed in Section 4 to understand why this particular approach might assist students to develop sampling variability concepts. A fourth phase implemented only in higher level classes is briefly described.

6.1.1 Prior knowledge

The prior knowledge that students developed before the lessons, described in Sections 6.1.2 to 6.1.5, had many strands. Students were familiar with the CensusAtSchool survey data and how they were measured. They had participated in the actual survey and had good general knowledge of the context. Because the idea of a population is abstract we decided to give a concrete representation of a population of students from which the students physically drew a sample. In a bag were 616 students' datacards from a fictitious college, Karekare College. The data recorded on the cards for the students came from the CensusAtSchool 2009 database. A further rationale for the use of the population bag (Fig. 4(a)) was that it gave a single image or conception of a population. While it would be desirable for the focus to be on the CensusAtSchool database as a population from which samples were drawn, it would bring complications. The students would know that the database itself was a sample drawn from a population and this could lead to confusion. Use of the population bag circumvents this issue.

Questions then arise about why one should sample and whether samples are able to give information about populations. We believed that we should first create a *need* to sample. Hence students were given the question: "What is the typical time it takes for Karekare College students to get from home to school?" Using the datacards themselves as symbols for the data points (Fig. 4(b)), the students created plots on their desks and did not say anything until they ran out of space. (Another class in a different year level decided to find the mean and after some time stopped and said there must be an easier way.) The class then had a discussion on an easier way to find the typical time and conceived the idea of taking a sample relating the idea to practicality and costs in real situations. No mention was made of random sample in order to keep the focus on a few central concepts.

The next question was: "How big a sample?" We considered just asking students to grab a handful of datacards and then to decide on the basis of observation of plots of other samples from the same population what would be a reasonable size. However, the teachers decided not to address this issue at this level. They simply asked students to take a sample of size 30. Students drew dot plots and box plots, compared each group's plots considering what was similar, what was different, what messages were consistent from the samples about the population, Karekare College, and whether a sample of 30 was a reasonable size. Therefore, before the first phase of the lessons described in the next section, a foundation for inferential reasoning was set up, as students had prior knowledge and ideas about population, sample, sample size, sampling variability, and that different samples give similar messages about the population. With so many

concepts underpinning inference we came to a consensus about what would be too much information that may overwhelm students (e.g., the intricacies about how to take a random sample or deciding on a suitable sample size for inference) and what were essential ideas that students were capable of grasping.

6.1.2 Analysis of phase one of lessons

The first phase of the lessons was preparation for building the concept of sampling variability and making a claim about whether group A tended to have bigger values than group B. Each pair of students had a population bag ([Fig. 4\(a\)](#)) from which they selected samples of size 30 to explore the following two questions: Do the heights of boys at Karekare College tend to be greater than the heights of girls at Karekare College? Do Karekare College students who walk to school tend to get there faster than Karekare College students who take the bus? For example, the students were asked for the first question to take a sample of size 30 from the girls' cards and a sample of size 30 from the boys' cards. By physically taking 30 girls' datacards and 30 boys' datacards we hoped to build imagery of comparing two groups; that is, comparing two distributions.

When students plotted the data they did a quick dot plot and then recorded the box part only on pre-prepared graph outlines. The whiskers were excluded as this information is extraneous to building inferential concepts and as Pfannkuch ([2008](#), [2011](#)) noted, diverts attention on to anomalies in the data. The students used blue for the median and red for the box part (not all teachers followed this color code in the implementation). This application of the signaling principle ([Mayer 2010](#)) focused students' attention on the relevant structures in the representations in order to reduce cognitive load. The same color cues were used in the animations. Altogether 14 different samples were taken for each question.

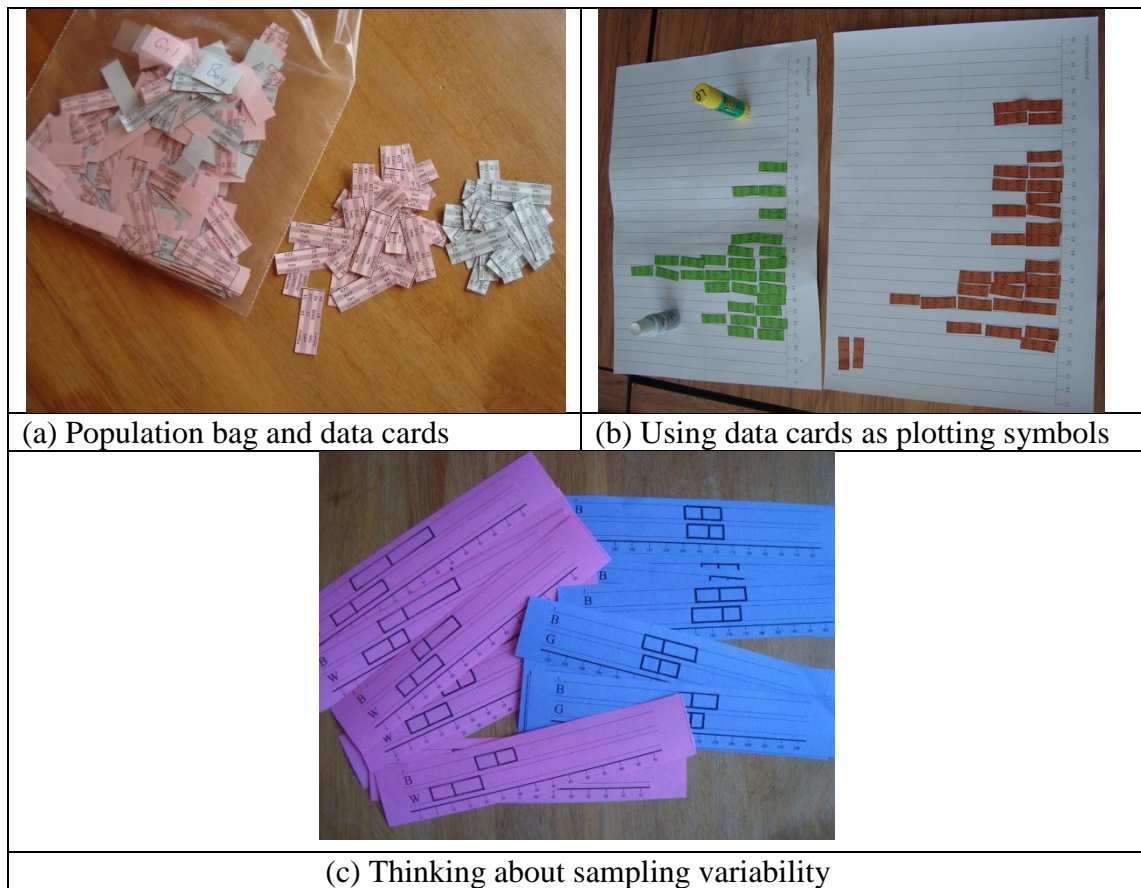


Figure 4. Hands-on simulations towards making a call

6.1.3 Analysis of phase two of lessons

Each group of students was given a copy of all the graphs from the class samples ([Fig. 4\(c\)](#)). The students were simply asked to sort the samples for the heights question and sort the samples for the time-to-school question. The students looked for patterns among the box plots for the two questions. After some time they were directed to sort on shift and median properties of the representations. According to [Bodemer, Ploetzner, Feuerlein, and Spada \(2004\)](#), leaving students to generate hypotheses about relationships on their own is very hard and as [Canham and Hegarty \(2010\)](#) found they will not pay attention to salient features. [Bodemer et al. \(2004\)](#) suggest that learners' interactions with learning materials should be structured so that hypotheses are formulated only on one relevant aspect of the visualization at a time, which the students did in this study by first focusing on the distributional shift and then on which median was bigger.

After the students sorted their samples for each question the teacher and class actively reflected on the process. They described and abstracted the patterns and criteria for making a claim back in the two populations. This allowed students an opportunity to extract principles ([Bakker & Gravemeijer 2004](#)) and in terms of the dual coding theory allowed time for referential processing

to occur ([Clark & Paivio 1991](#)). The students noticed that in the samples for the heights the boxes were close together, whereas in the samples for time-to-school the boxes were apart. They named these two situations about the relative location of the boxes Situation One and Situation Two, respectively. They also noticed that in Situation Two the 14 suggested messages about the direction of the two medians back in the populations were consistent, allowing them to determine the larger of the two population medians. This was not the case in Situation One. Through recognizing and reasoning from the patterns in the two situations they “discovered” collectively the criteria for making a call when two box plots are compared:

- Teacher: So in our first situation we've got the boxes; they're all overlapping some of them are going this way and some of them are going the other way. The medians are very close together and the medians are also within the overlap of the boxes. In the second situation how is it different? What's different about the overlap here? Is there no difference between the overlap on these boxes and these boxes?
- Student: They're not overlapped so much.
- Teacher: They're not overlapped so much. No, they're not. Okay do they all overlap?
- Student: No.
- Teacher: No, so when they do have an overlap they don't overlap much and otherwise they don't overlap at all. What can you tell us about the medians in this one?
- Student: They're not overlapped.
- Teacher: They're not in the overlap.

Visually and verbally the students and teacher described the differences in the two situations in terms of shift, overlap, and the location of the medians. Students and teacher started to develop the criteria for making a claim. Collectively they used hand gestures to describe the two situations, close and apart, with vibrations, which according to [Radford \(2009\)](#) is a precursor to verbal conceptualization.

6.1.4 Analysis of phase three of lessons

The third phase involved using “movies” or animations to reinforce the message from the two situations. Situation One occurs when the boxes indicate little or even no distributional shift. There is a great deal of overlap of the boxes, and the medians are within the overlap and can swap positions relative to one another. That is, one median might be higher in one sample, but the other median higher in the next sample. In this situation the ideas that are consolidated are that when there is a large overlap and the medians are within the overlap, the suggested message across many samples is inconsistent about the pattern back in the populations. For Situation Two, the boxes indicate a large distributional shift. The overlap of the boxes is small or even non-existent, and at least one of the medians is outside the overlap. In this situation the location of the boxes and the position of the medians relative to one another stays consistent across many samples and, therefore, the suggested message about the pattern back in the populations is consistent.

To reinforce the messages of the two situations about what is happening back in the two populations students viewed animations of a large number of samples for the two questions (see: www.censusatschool.org.nz/2009/informal-inference/teachers/workshop2/heights_2samp_dots_30.pdf and www.censusatschool.org.nz/2009/informal-inference/teachers/workshop2/times_2samp_dots_30.pdf). (Note: Repeatedly clicking on or holding down the down arrow advances the animation.) The population bag was linked to the database, and instead of the student drawing samples, the computer did. Originally the animations operated in a gif format but we changed this to a pdf format, which

allowed the teacher to control the pace of the animations (Hegarty 2004) and time to integrate verbalization and visual imagery. By controlling the pace, the comparisons can be initially shown one at a time, giving students time to make sense of what they are seeing, and once comprehended the animation can be sped up. The animations start with two population distributions, which are then faded out with the slogan “unfortunately we do not get to see this” (Fig. 5). De Koning, Tabbers, Rikers and Paas (2010) mention fading out is a good design principle in order to draw attention to other features.

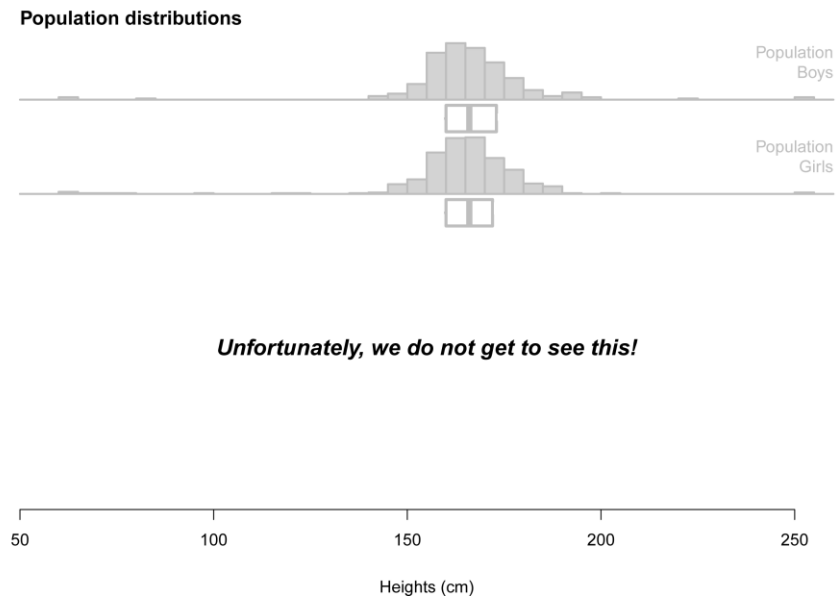


Figure 5. Fading the population into the background

For each set of box plot comparisons perceptual attention was drawn to the medians, a thick black line. Students raised either their left or right hand depending on which median was higher. In Situation One they were swapping their raised hand constantly (e.g., Fig. 6) whereas for Situation Two the same hand remained raised (Fig. 7), thus reinforcing physically inconsistent and consistent messages, respectively, about what was happening back in the two populations. The deliberate use of gestures or body movements was to encourage further sensory perceptions, perceptual attention, and engagement. Since dynamic visual imagery is ephemeral, static wall displays (Fig. 8, cf. photo at bottom of Anthony Harradine’s (2008) page at <http://www.censusonline.net/bears.html>) of the multiple samples (Fig. 4(c)) were constantly visible and used along with gestures to remind the students of the two situations. As Hegarty (2004) stated, static displays are necessary to allow learners to revisit ideas.

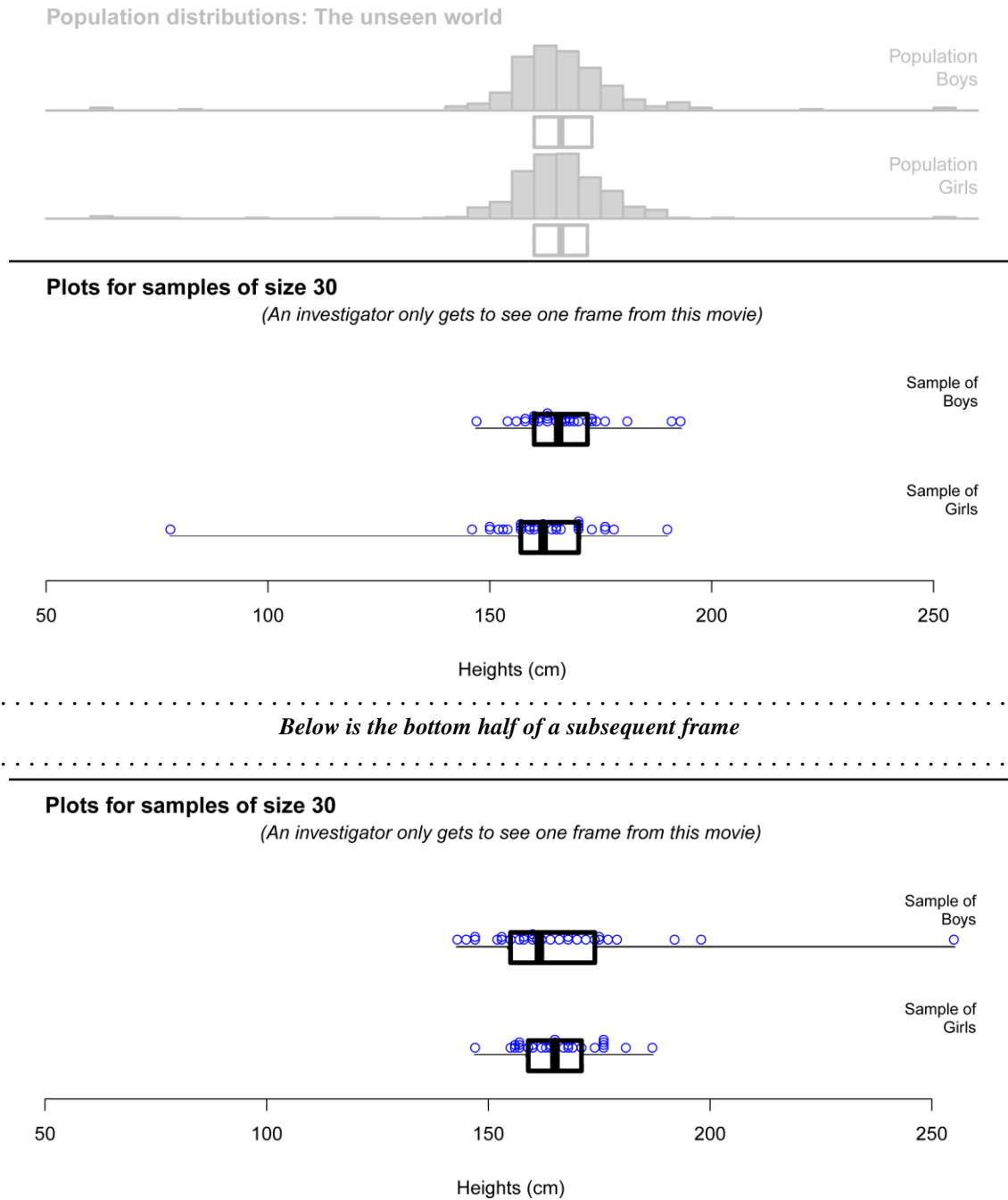
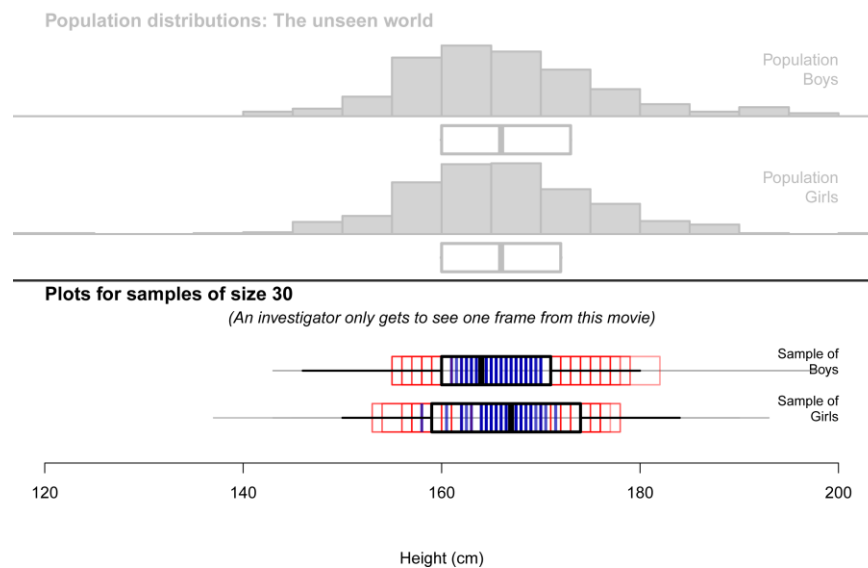


Figure 6. Samples telling opposite stories

**Figure 7.** Students in class raising hands**Figure 8.** Teacher wall display

6.1.5 Phase Four (Not in class described, only with Year 11 and 12)

Phase Four, which time did not allow with the class described, was to engage the students' attention on the animations that track the history of the variability for these two situations (see: www.censusatschool.org.nz/2009/informal-inference/workshops/heights_2samp_mem_30.pdf and www.censusatschool.org.nz/2009/informal-inference/workshops/times_2samp_mem_30.pdf). First the animations were projected onto a whiteboard and a student recorded with a blue pen the position of the median as random samples were drawn. In this way a student could physically experience the extent of the variability in the median and the rest of the students' attention were drawn to focus on the variability in the median. Second, the animations were shown to assist students to appreciate fully and develop the visual imagery for the extent of sampling variability with samples of size 30 (Fig. 9). Other learning experiences included the sample size effect. In an interview with a Year 12 student, three weeks after the teaching of the unit, the visual imagery of sample size effect was still present (Fig. 10).

**Figure 9.** Tracking the history of sampling variability

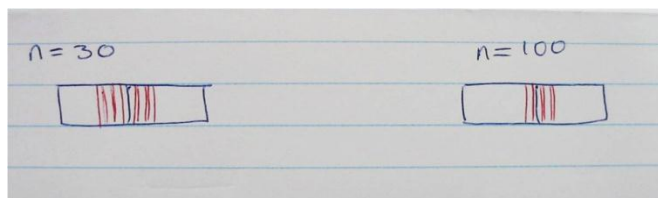


Figure 10. Visual imagery retained by Year 12 student

The analysis of the learning trajectory, according to the literature reviewed, seems to confirm that the sequence of instruction provides effective learning conditions. The trajectory may be conducive to learning but the question remains about the resultant learning outcomes of the students, which are now addressed in part two of the analysis.

6.2 Analysis Part Two: Student Learning Outcomes

This brief précis of the analysis addresses: the pre- and post-interviews of 14 students from the first and second years of the project; the pre- and post-test results of four classes who participated in the second year of the project (two year 9s, 10 and 11) from Decile 1, 4, 5, and 8 schools. Only the test results in the second year are given because at the end of the first year we improved and clarified the types of reasoning and verbalizations for communicating the messages we were seeing when comparing box plots. We also modified and clarified the assessment framework.

The pre- and post-interviews, based on the pre- and post-tests, were analyzed qualitatively focusing on the following domains: reasoning about samples, populations, and sampling variability. For the pre- and post-tests, assessment frameworks were developed for five domains of reasoning, namely, making a call, shape, spread, unusual patterns, and context. In this paper, however, we focus only on the domain of making a call. A researcher and an independent person coded the data separately and then came to a consensus on the final codes and scores.

In the pre-interviews students' initial conceptions of a sample were typically about a product sample: "like those shopping stores that give you out free samples but you only get a little bit and it's kind of a sample" or a part of a whole, similar to [Watson's \(2006\)](#) findings. Their conceptions of population were mainly centered on the number of data values, akin to thinking they were being asked: "What is the population of New Zealand?" Typically students thought a random sample was taken to get a variety of different measurements and a sample of size 30 was insufficient to make a statement about all New Zealand students. Worrying but classic, one student said that a sample was taken to find out the average. When asked to elaborate, she said, "well you know how when you get a whole set of data you can find out the highest and the lowest and the upper and lower quartiles. And the average, because the average is normally used to generalize the heights." Generalize, however, to her meant just for the data collected, as she did not think you could make a statement about all boys' heights in New Zealand. Despite some probing there was no evidence any of the students understood the relationship between samples and populations. They believed they were reasoning about sample distributions not reasoning about population distributions using samples.

In the post-interviews most students showed a better understanding of the relationship between samples and populations. Previously the term population was not part of their verbalizations but now it was: “[a sample] will give you an idea of what the population will look like, but it might not always be the same.” They were clearer about how populations were defined in statistics and that they were reasoning about *all* New Zealand Year 11 students (Fig. 11(b)). They were also now fairly confident for situations similar to Figure 11(b) that they could make a call about the populations using a sample of size 30. Teachers in the project reported that the population bags acted as a useful prop to remind students that they were reasoning about the populations.

With regards to sampling variability all students knew prior to the teaching intervention that another sample would produce slightly different plots. What they did not know was the extent of the sampling variability or that the relative position of the medians in a situation such as Figure 11(a) could swap around from sample to sample. In the post-tests and interviews for this situation some students were able to verbalize “another sample could show the medians were the other way around” or show with their hands how the box plots would jiggle (Fig. 11 (c)). In fact, these images seemed to endure. The teacher and researcher reported that the Year 11 students who had been introduced to the topic in the previous year immediately raised their hands to show the two situations when comparing box plots (Figs. 11(c, d)).

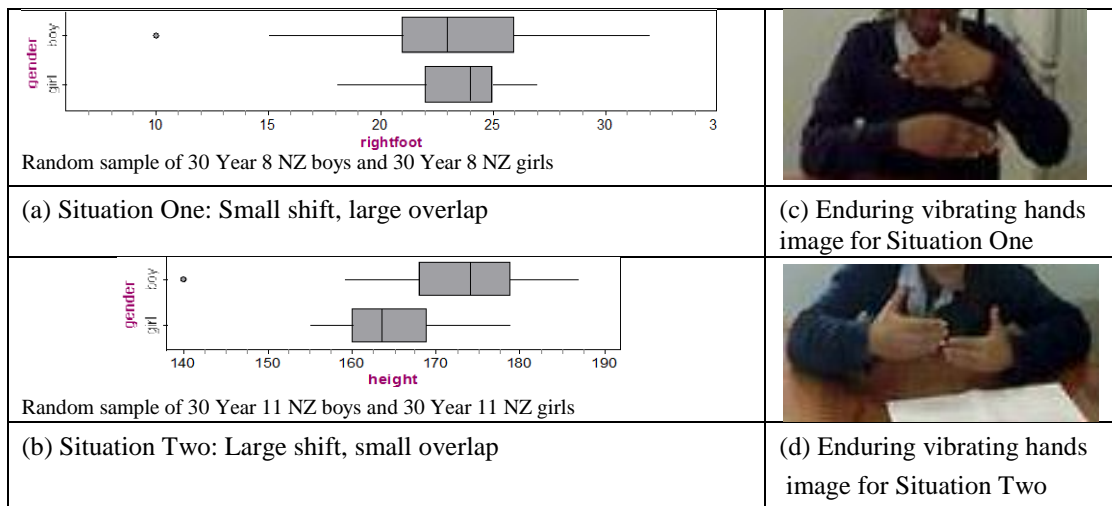


Figure 11. Box plot examples

In the domain of making a call, a pre- and post-test assessment framework was developed based on the student responses. The framework was a six-level hierarchy with qualitative descriptors for each level of reasoning (Fig. 12). A student response was scored from 0 to 11 using this framework. For example, a student who made a call on the medians and partially verbalised one element of evidence was given a score of 5, while a student who fully verbalized three or four elements of evidence was given a score of 9. An example of a relevant evidence (RE) response, score 9, for the second item in the post-test (see Appendix) from a Year 11 student is:

Yes, I would make the same claim as Matt (Yr 9 NZ girls rate themselves better at dancing than Yr 9 NZ boys). This is because in the overall visual spread the medians are more than 1/3 apart with the girls' median being higher. This means that if I was able to take another sample the medians may move a little

but would not swap (the girls would stay higher). There is also no overlap and the girls' middle 50% is clearly shifted more to the right.

Since the student fully verbalized the four elements of evidence of shift, overlap, decision guideline and sampling variability she scored 9 rather than 8 in the RE category. To obtain a score of 10 or 11, the student would need to state that she was fairly confident that another sample would give the same message and she was going to conclude from these samples that back in the populations Year 9 NZ girls tended to rate themselves better at dancing than Year 9 NZ boys, but she could not be 100% certain about this conclusion. Note that Year 11 had a different decision guideline (Fig. 2) to Year 10 who would state that the median of the girls' rating for dancing was outside the box of the boys.

Category	Score	Descriptor
Idiosyncratic (I):	0	No response or makes a statement not based on the data or any feature of the data.
Irrelevant evidence (IE):	2	Makes a call on any feature that appears bigger (e.g., maximum, box length, Upper Quartile).
Transitional (T):	4	Compares centres, the medians or central 50 percent.
Towards relevant evidence (TRE):	6	Makes correct call and fully verbalises one element of evidence and partially verbalises some other elements (shift, overlap, decision guideline and sampling variability) for justifying decision.
Relevant evidence (RE):	8	Makes correct call and fully verbalises at least two elements of evidence.
Full evidence (FE):	10	Fluent response with four elements of evidence and mentions samples, populations, and level of confidence where appropriate.

Figure 12. Categories for providing evidence for making a call

In both the pre- and post-tests (see www.censusatschool.org.nz/2009/documents/ProjectPreTestFINAL2010.pdf and www.censusatschool.org.nz/2009/documents/ProjectPosttestFINAL2010.pdf) three items assessed making a call. The items, different in context, were similar in both tests. The first two items gave a claim and required students to discuss whether they would make the same claim and why (see Appendix for these pre- and post-test items). The third item was set in an investigative context, gave background on the data, and gave box plots, dot plots, and a table of summary statistics. Students were required to make their own claim and provide evidence for their conclusion. Each item was scored out of 11 and then the average of these three scores determined a student's level of reasoning for each test. Using a paired comparison *t*-test, there was extremely strong evidence that students had improved their average reasoning score for making a call ($\bar{x}_{diff} = 2.82$, 95% C.I. = [2.52, 3.12], P-value ≈ 0). It should be noted that the Year 10 and 11 students improved their reasoning scores slightly more than the Year 9s, which was not unexpected as box plots are usually introduced to students in Year 10 and the required verbalizations demand a greater level of literacy.

To convey more insight into these improved scores, the comparison of pre- and post-test average scores for making a call is presented in Table 1. Note that no student had an average score in the FE category.

Table 1. Making a call pre- and post-test (n=91)

		Post-test Average					
		I	IE	T	TRE	RE	Total
Pre-test Average	I	0	4	9	2	0	15
	IE	0	5	22	16	5	48
	T	0	1	5	7	13	26
	TRE	0	0	0	0	2	2
	RE	0	0	0	0	0	0
Total		0	10	36	25	20	91

[Notation: I=Idiosyncratic, IE=Irrelevant Evidence, T=Transitional, TRE=Towards Relevant Evidence, RE=Relevant Evidence]

In the pre-test about 70% of the students were unable to use relevant features of the box plot distributions to justify their decision for either making (e.g., [Fig. 11\(b\)](#)) or not making a call (e.g., [Fig. 11\(a\)](#)). The rest were mainly at the transitional level, as they justified their decision using only the medians or central 50%. We conjecture that this *focus on the center* of the distributions to compare groups is a *critical juncture* in students' perception and conception of reasoning from plots. In the post-test about 50% of the students had gone beyond the focus on the center to include attention to other features by partially or fully verbalizing relevant elements of evidence for justifying their decision. Another 40% of students had reached the transitional level where they were now focused on central features of the distributions. About 12% of the students did not improve their category level from pre- to post-test in this domain since five remained at the irrelevant evidence level, five at the transitional level, and one student moved from the transitional to irrelevant evidence level. None of the students gave idiosyncratic responses, which means that all students were giving responses based on features of the data.

Post-interviews confirmed students were beginning to grasp the underlying concepts behind using samples to make inferences about populations. As could be expected there were gaps and conflicts in their reasoning such as wanting to draw multiple samples to be more confident about their call and wanting to give an answer rather than saying they could not make a call or it was "too close to call" in situations similar to Figure 11(a). Our research seems to show that Year 9 to 11 students (even those with low literacy and numeracy levels, as was the case in the Decile 1 school) can start to verbalize and understand statistical inferential reasoning.

7. Discussion

Learning statistics, with its reliance on visual graphic representations to assist in unlocking the stories in the data, can be transformed through the creation of new technological tools. Technology allows the underpinning theory and conceptualizations behind statistics to become accessible to learners and allows teachers to build concepts. In the devising of the learning trajectories we conjectured that the creation of the visualizations together with new verbalizations, which are strongly linked to the box plot displays, would allow students to access inferential concepts without the need for mathematical manipulations. From our analysis of pre- and post-tests and interviews of the students in this study we believe that these students were beginning to reason inferentially and to verbalize the concepts underpinning statistical inference.

Perhaps these students' understanding was enhanced through encouraging them to generate mental images in conjunction with verbalizations, as [Clark and Paivio \(1991\)](#) suggest.

From the perspective of cognitive load theory the devised learning trajectory attended to students' minimal prior knowledge base, the intrinsic load, and carefully scaffolded the learners through the tactile hands-on tasks ([delMas 1997](#)) to the point where they could understand the animations. Extraneous load was reduced through using the box part only for pattern searching, the color-coding principles, and, most importantly, using visual representations of numeric values. We believe that numeric cues would have added to the cognitive load and hindered concept formation. Hence, we conjecture that the students were able to start constructing new knowledge, the germane load.

The students engaged with concepts that previously were considered too advanced for them, a finding that [Sacristan et al. \(2010\)](#) state has occurred in some mathematics education studies. There are several reasons why Year 10 students (US grade 9) were able to engage in inferential reasoning after a very short experience. First, the learning trajectory has elements of effective instructional design as espoused by [Clark and Paivio \(1991\)](#), [Mayer \(2009\)](#), and [Lovett \(2010\)](#). In particular the germane load seemed to be sufficient for the construction and development of some inferential reasoning knowledge. Second, the learning trajectory took account of students' prior knowledge and scaffolded them from hands-on activities to animations. Also, the learning trajectory seems to be aligned with the webbing and situated abstraction learning theory of [Noss and Hoyle \(1996\)](#), (as cited in [Sacristan et al. 2010](#)) where support is given to students to develop new meanings, expressing generality is a central component, and abstraction is within, not away from, the situation. Third, we believe that our method of keeping the abstraction within the situation – by planting conceptual imagery underpinning inference *within* the box plot display – is a pivotal innovation that facilitates understanding. Students appear to perceive no longer the box plot as a picture but engage with the conceptual infrastructure of the box plot for reasoning and thinking. Such an engagement is akin to [Arcavi's \(2003\)](#) thoughts on visualization processing.

With respect to the learning trajectory, however, two modifications may be appropriate. First, the animations showed the two populations, then the samples. Some teachers preferred to show the samples first on the basis that in reality we never know about the actual populations and that the learning focus was on building concepts about how to make a call on two samples. The animations were now aligned with the hands-on simulations in terms of unknown population distributions in the population bag of datacards. All the students could see were the plots of the samples they had drawn. These teachers then revealed the populations so that the students could see the relationship between the populations and the resultant samples. Second, drawing multiple samples to demonstrate sampling variability led some students to feel not confident about making a call when comparing two groups using one sample – to be sure they wanted to take another sample. Such a problem arose because students noted that when they took a sample that most students in the class made the same call but some others said it was too close to call or vice versa. The learning trajectory could be modified to include more discussion about “right answers” using one sample.

In this study student awareness of sampling variability and criteria for making a call was via dot plots and box plots. Our decision to use box plots was based on problems observed in classrooms and in national assessment, which required students to pose and answer comparative questions. Also, box plots afforded straightforward visual imagery and decision criteria to be developed. Box plots and dot plots should not be the only way for students to experience sampling variability. Histograms, bar graphs (see the animations at www.censusatschool.org.nz/2009/informal-inference/WPRH/ and Section 3.5 of [Wild et al. 2011](#)), scatter plots, and two-way tables should be used to enrich and further develop students' appreciation of sampling variability and inference. Therefore, the approach we used to begin more formally to develop inferential concepts may not be the best or only method. Time and national assessment constraints, however, are the reality in the school curriculum and teachers need to think about how to immerse their students in a coherent set of statistical experiences that includes inferential ideas. Shifting the emphasis in both teaching and assessment from constructing plots, which can now be computer generated, to exploring and reasoning from data is one way of finding more time for developing students' thinking.

The dual-coding and cognitive overload theories used by researchers (e.g., [Mayer 2010](#); [Verhoven, Schnotz, & Paas 2009](#)) to determine the effectiveness of animations emphasize combining visual imagery and verbalizations. However, we conjecture that for learning inferential reasoning in statistics at school level a triple-coding theory should be considered: verbal, visual, and sensory (e.g. physical movement, song cues, smell). We wonder if sensory elements can operate in parallel with the visual and verbal channels or compete with them. We conjecture that visual, sensory, and verbal cues could either singly or combined stimulate episodic memory and facilitate the reconstruction of the concepts underpinning inference. Even though one of [Mayer's \(2009\)](#) principles for multimedia learning eschews extraneous sounds and other similar distractions, we think that the color signaling principle that focuses perceptions on salient features could be extended to motion cues ([Ware 2008](#)) such as the vibrating box plot, to relevant, underlying, sound-signaling cues such as song clips (as used in Chris Wild's 2009 USCOTS Plenary – URL given previously) and even to smell cues with future technology. Determining the viability of any of these conjectures will require some focused research in this area.

In the domain of making a call we have shown that students of very mixed abilities and backgrounds can be stimulated to start developing concepts about statistical inference through well-designed learning trajectories. From the pre- and post-tests and interviews there is evidence that students can start to understand, demonstrate, and verbalize notions such as sample, population, and sampling variability. Students engaged in box plot comparisons to the extent they can partially or fully verbalize notions of shift, overlap, what may occur if another sample were taken and the decision criterion to justify the claim they make. This is a much improved situation when considering the original problem described at the beginning of this paper. Furthermore, the students did improve their average reasoning scores ([Table 1](#)). The enduring images of vibrating box plots as reported by the teacher and researcher from one class one year later suggests that it may be possible to design learning experiences that spring into consciousness from memory when required. Some questions though still remain: How can learning trajectories be designed to include memorable moments that encapsulate and trigger a network of concepts when required? How do the animations play a role in learning about

inference? Do the animations facilitate and/or reinforce understanding? Do the animations act as cues for future reconstruction of concepts? Do physical actions compete with other demands such as verbal and visual comprehension? Do song cues improve memorability? Again such questions need in-depth research.

Teaching statistics using a reasoning approach focused on concepts and learning from the data, rather than a procedural approach focused on how to construct plots and find the “meanmedianmode” ([Friel, O’Connor & Mamer 2006](#)), is *new* to teachers in New Zealand and will take time to perfect and disseminate. Technology can now be used to carry out the procedures of statistics, allowing students to learn how to think statistically, to reason from plots, and to make evidence-based decisions under uncertainty. That is, technology can help students become enculturated into the art of statistical investigation and decision-making.

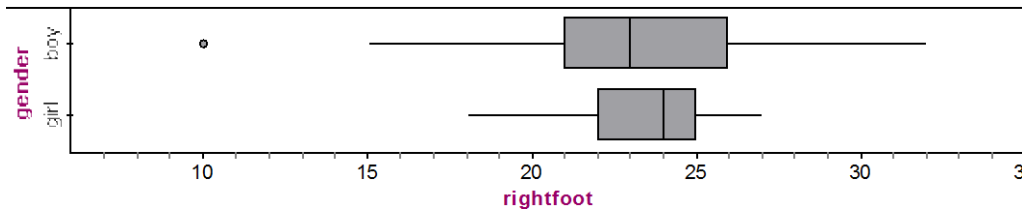
Appendix

Two pre-test items for making a call

Emma is interested in comparing the right foot lengths (in cm) of Year 8 NZ boys and girls. She takes a *random sample* of 30 Year 8 NZ boys and a *random sample* of 30 Year 8 NZ girls. She can find out their right foot lengths without shoes on in cm.

She plots the right foot lengths from her samples correctly.

Emma's graph



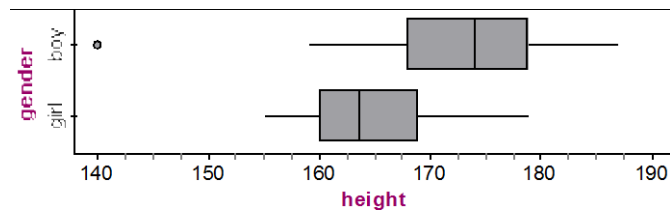
Emma looks at her graph and claims that the right foot lengths of Year 8 NZ girls tend to be bigger than the right foot lengths of Year 8 NZ boys.

1. Would you make the same claim as Emma? Why?

Emma is interested in comparing the heights (in cm) of Year 11 NZ boys and girls. She takes a *random sample* of 30 Year 11 NZ boys and a *random sample* of 30 Year 11 NZ girls. She can find out their height without shoes on in cm.

She plots the heights from her samples correctly.

Emma's graph



Emma looks at her graph and claims that the heights of Year 11 NZ boys tend to be greater than the heights of Year 11 NZ girls.

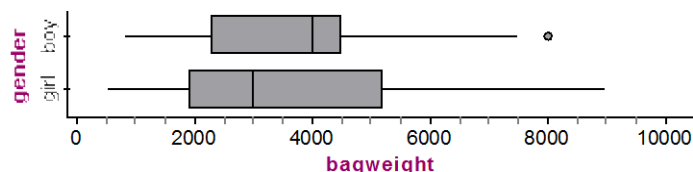
2. Would you make the same claim as Emma? Why?

Two post-test items for making a call

Matt is interested in comparing the schoolbag weights (in g) of Year 11 NZ boys and girls. He takes a *random sample* of 30 Year 11 NZ boys and a *random sample* of 30 Year 11 NZ girls. He can find out their school bag weights to the nearest 100g.

He plots the weights from his samples correctly.

Matt's graph



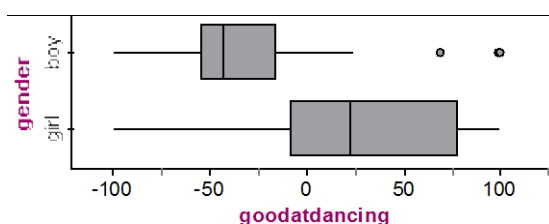
Matt looks at his graph and claims that the schoolbag weights of Year 11 NZ boys tend to be heavier than the schoolbag weights of Year 11 NZ girls.

1. Would you make the same claim as Matt? Why?

Matt is interested in comparing how Year 9 NZ boys and girls rate themselves at dancing. He takes a *random sample* of 30 Year 9 NZ boys and a *random sample* of 30 Year 9 NZ girls. He can find out how good they rate themselves at dancing on a scale of -100 (no good) to +100 (very good).

He plots the results from his samples correctly.

Matt's graph



Matt looks at his graph and claims that Year 9 NZ girls tend to rate themselves better at dancing than Year 9 NZ boys.

2. Would you make the same claim as Matt? Why?

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