Using GAISE and NCTM Standards as Frameworks for Teaching Probability and Statistics to Pre-Service Elementary and Middle School Mathematics Teachers

Mary Louise Metz
Indiana University of Pennsylvania

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Abstract

Statistics education has become an increasingly important component of the mathematics education of today's citizens. In part to address the call for a more statistically literate citizenship, The *Guidelines for Assessment and Instruction in Statistics Education (GAISE)* were developed in 2005 by the American Statistical Association. These guidelines provide a framework for statistics education towards the end of enabling students to achieve statistical literacy, both for their personal lives and in their careers. In order to achieve statistical literacy by adulthood, statistics education must begin at the elementary school level. However, many elementary school teachers have not had the opportunity to become statistically literate themselves. In addition, they are not equipped pedagogically to provide effective instruction in statistics. This article will discuss statistical concepts that have been identified as necessary for statistical literacy and describe how an undergraduate course in Probability and Statistics for pre-service elementary and middle school teachers was revised and implemented using the *GAISE* framework, in conjunction with the NCTM Standards for Data Analysis and Probability. The aims of the revised course were to deepen pre-service elementary and middle school teachers' conceptual knowledge of statistics; to provide them with opportunities to engage in, design, and implement pedagogical strategies for teaching statistics concepts to children; and, to help them make connections between the statistical concepts they are learning and the statistical concepts they will someday teach to elementary and middle school students.

1. Introduction

1.1 Significance of Statistical Education for Today's Students

Until a few decades ago, being able to understand and apply mathematical concepts in a variety of contexts was left to those who could 'do math.' In today's world, however, the enormous amount of data available affects the decisions one makes politically, as a consumer, in one's career, and in everyday life. Therefore today's students must learn to reason quantitatively. This requires learning and applying a set of skills necessary to become informed citizens, intelligent consumers, productive employees, and healthy and happy individuals (NCTM, 2000, p. 48; ASA, 2007, p.1). Statistical reasoning skills, which comprise a large portion of quantitative reasoning skills, are also a necessity of all U. S. citizens if we are to survive in a competitive, global market. "An investment in statistical literacy is an investment in our nation's economic future, as well as in the well-being of individuals (ASA, 2007, p. 2)."

1.2 Knowledge Needed to Be Statistically Literate

Several organizations have outlined the knowledge needed by today's school students in order to become statistically literate.

In 2000, the National Council of Teachers of Mathematics (NCTM) released the *Principles and Standards for School Mathematics* (PSSM) which outlined standards and expectations for 5 content areas, including Data Analysis and Probability, spanning from pre-kindergarten through grade 12. The Data Analysis and Probability Standards include enabling students to:

1.) formulate questions that can be addressed with data and collect, organize and display relevant data to answer them;
2.) select and use appropriate statistical methods to analyze data;
3.) develop and evaluate inferences and predictions that are based on data; and
4.) understand and apply basic concepts of probability.

Within each of the four standards are expectations for students at various grade band levels. For example, for the standard, "develop and evaluate inferences and predictions that are based on data," students at the grades 3 – 5 level are expected to "propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions" while at the grades 9 – 12 level students are expected to "understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference" (NCTM, 2000). The NCTM Standards for Data Analysis and Probability are summarized in Appendix A.

The College Board has also specified "Standards for College Success" (College Board, 2006) in order to prepare all students for success, not only in college, but also in the workplace and in civic life. Unlike the NCTM Standards, the College Board Standards are specified within mathematics courses, beginning at the middle school level and continuing through precalculus. Each course includes concepts related to data analysis and probability that build in depth and complexity from one course to another. For example, a beginning middle school student should be able to organize and summarize categorical and numerical data using summary statistics and a

variety of graphical displays while a precalculus student is expected to assess association of bivariate data in tables and scatterplots and use the correlation coefficient to measure linear association. Even though the College Board Standards are specified by course rather than grade band, they are consistent with the NCTM Standards for grades 6-8 and 9-12. The College Board Standards for College Success that address statistics and probability concepts are summarized in Appendix B.

Finally, in 2007, the American Statistical Association published *Guidelines for Assessment and Instruction in Statistics Education for Pre K-12 Education* (GAISE) that outlined a curriculum framework for achieving statistical literacy. The framework consists of two dimensions – components of statistical problem solving and developmental levels of statistical understanding. The four components of statistical problem solving are consistent with the NCTM Standards for Data Analysis and Probability. These components include:

1. formulating questions
2. collecting data
3. analyzing data
4. interpreting results

The three developmental levels of statistical understanding, Level A, Level B, and Level C, outline a progression of statistical understanding students must experience in order to become statistically literate.

Similar to the PSSM, the GAISE framework makes salient the importance of giving students experiences and opportunities in the early elementary grades in order to be successful at learning more complicated statistical concepts when they reach middle and high school. An additional component of the GAISE framework is the focus at every level on understanding variability and its impact on the collection, analysis, and interpretation of data. The GAISE framework is summarized in Appendix C.

1.3 Knowledge Needed by Teachers to Teach for Statistical Literacy

If the knowledge needed by today's students to become tomorrow's statistically literate adults involves years of opportunities learning and experiencing a plethora of concepts as noted above, then the wealth of knowledge needed by those who carry the responsibility for teaching those concepts is overwhelming. Unfortunately, the Conference Board of the Mathematical Sciences (CBMS) noted in their report on the mathematical preparation of teachers, "of all the mathematical topics now appearing in middle grades curricula, teachers are least prepared to teach statistics and probability (CBMS, 2001, p. 114)."

In addition, a growing body of research in mathematics education points to the fact that the knowledge needed to teach mathematics effectively is a complex and somewhat underspecified body of knowledge that goes beyond just knowing the content one is teaching (Ball, Thames and Phelps, 2008; RAND, 2003 ; USDE, 2008; CBMS, 2001). This knowledge is substantial, yet quite different from that required by students pursuing other mathematics-related professions. Teachers of mathematics must have a deep understanding of the content they teach and understand how that content connects to other important mathematical concepts that come prior

to and beyond the level they are teaching. Not only must they recognize correct and incorrect answers - they must also be able to analyze student thinking that produces mathematical errors and misconceptions and know how to best address the sources of confusion. Mathematics teachers need to understand the meanings of algorithms and procedures and know why they make sense. They must be able to represent and recognize mathematical concepts in multiple ways and make sense of multiple strategies used by students to solve problems.

The substantial and overwhelming body of knowledge needed by today's teachers of mathematics presents a daunting challenge to those whose work involves preparing pre-service teachers to provide the best possible learning opportunities in mathematics. In recent years, however, several organizations have begun specifying the knowledge needed by pre-service teachers in order to effectively teach mathematics at the elementary and middle school levels. The National Council for Accreditation of Teacher Education (NCATE) has outlined standards and indicators for the preparation of elementary and middle school mathematics "specialists" (NCATE/NCTM, 2003) which include the following concepts related to statistics: design investigations, collect data through random sampling or random assignment to treatments, and use a variety of ways to display the data and interpret data representations including both categorical and quantitative data; draw conclusions involving uncertainty; use hands-on and computer-based simulation for estimating probabilities and gathering data to make inferences and decisions; and use appropriate statistical methods and technological tools to analyze data and describe shape, spread, and center. In addition to the knowledge of content future teachers are expected to know, they must have the pedagogical knowledge to "evaluate instructional strategies and classroom organizational models, ways to represent mathematical concepts and procedures, instructional materials and resources, ways to promote discourse, and means of assessing student understanding" (NCATE/NCTM, 2003, p.3). The NCATE/NCTM Standards and Indicators for Data Analysis and Probability are shown in Appendix D.

Finally, the Conference Board of the Mathematics Sciences specified content that should be addressed in courses for pre-service middle school teachers in the area of data analysis and probability (CBMS, 2001). They suggest coursework should provide prospective teachers opportunities to design simple investigations and collect data through random sampling or random assignment to treatments in order to answer specific questions; explore and interpret data by observing patterns and departures from patterns in data displays, particularly patterns related to spread and variability; anticipate patterns by studying, through theory and simulation, those produced by simple probability; and draw conclusions with measures of uncertainty by applying basic concepts of probability models. In addition, the CBMS notes that mathematics courses for pre-service teachers should be designed to not only strengthen mathematical understanding to the extent that it can be taught to others, but also to know when students have understood and what to do if students have not understood.

Though somewhat general in scope, the knowledge of statistics and probability needed by pre-service teachers as described by NCATE and CBMS parallels that of the NCTM Standards and the GAISE framework. This allows for a natural alignment when considering the content pre-service teachers need to know along with the related concepts they will someday be teaching.

2. The Statistics Course for Pre-service Elementary and Middle School Teachers

2.1 The Statistics Course for Pre-service Teachers Prior to Revision[1]

The 3 credit undergraduate[2] course at the university, *Probability and Statistics for Elementary and Middle School Teachers*, had previously been developed for pre-service elementary and middle school elementary education teachers who were pursuing a concentration in mathematics. (The Elementary Mathematics Concentration program at the university is a 15 credit program for elementary education majors wishing to gain a deeper understanding of mathematics and how it can be conceptualized for elementary and middle school students. The courses in the program are designed specifically for elementary education majors to increase their knowledge of both mathematics content and mathematics pedagogy by embracing a 'hands on' teaching approach. This particular course is one of seven courses from which pre-service teachers may select.) The development of the initial course was heavily influenced by the *Quantitative Literacy* series (Gnanadesikan, Scheaffer, and Swift, 1987; Landwehr, Swift, and Watkins, 1987; Landwehr & Watkins, 1987; Newman, Obremski, and Scheaffer, 1987) beginning with the Exploring Data unit. The overall objectives of the course were to demonstrate an understanding of data analysis and probability concepts as tools for decision-making, to explore technology applications in probability and statistics, and to investigate probability and statistics concepts and activities appropriate for K-8 students. An abbreviated, general, topical outline of the course is shown in Table 1.

[1] For the remainder of the article, when referring to the prospective teachers enrolled in the course the term 'pre-service teachers' will be used. The word 'student' will refer to those in K-8 classrooms.

[2] The course was and still is offered as a dual level course for graduate students as well. This article focuses only on the content of the undergraduate course.

Table 1. Course Outline Prior to Revision

Course Outline Prior to Course Revision

I. Quantitative Literacy Descriptive Statistics Activities
- A. Statistical studies, surveys and experiments
- B. Graphing techniques for one-variable data
- C. Measures of central tendency
- D. Measures of dispersion
- E. Two-variable statistics
- F. Descriptive statistics activities for K-8 students.

II. Quantitative Literacy Probability Activities
- A. Theoretical and experimental probability through experiments
- B. Geometric probability
- C. Expected value
- D. Probability activities for K-8 students

III. Technology for Probability and Statistics
- A. Graphing calculator
- B. Spreadsheet
- C. Software and web resources

IV. Quantitative Literacy Inferential Statistics Activities
- A. Normal and standard normal distributions
- B. Sampling distributions
- C. Statistical significance and p-values
- D. t-tests
- E. Chi-square test

2.2 The Revised Course

The course was revised by examining the previous course outline and using the *GAISE* framework and NCTM Standards to connect and expand on the topics. The revised course outline is shown in Table 2.

Table 2. Course Outline After Revision

Course Outline after Revision

I. Quantitative Literacy through Descriptive Statistics
- A. Introduction
  1. *Guidelines for Assessment and Instruction in Statistics Education* problem solving process and levels of understanding
  2. NCTM Standards for Data Analysis and Probability
- B. Statistical studies and connections to the real world
  1. formulating questions and collecting data
  2. types of variability
  3. connections to the elementary and middle school classroom
- C. Displaying one-variable data
  1. graphs according to types of data
  2. identifying variability in graphs
  3. analyzing data displayed in graphical displays
  4. using technology to display data
  5. interpreting results of data displayed graphically
  6. connections to the elementary and middle school classroom
- D. Describing data numerically - measures of central tendency (mean, median, mode) and dispersion (range, IQR, standard deviation)
  1. analyzing data using numerical summaries according to types of data
  2. identifying variability in numerical summaries
  3. using technology to numerically summarize data
  4. interpreting results of data summarized numerically
  5. connections to the elementary and middle school classroom
- E. Two-variable data
  1. formulating questions and collecting data
  2. graphically displaying, analyzing, and identifying variability in two-variable data
  3. determining and interpreting correlation and linear regression
  4. using technology to graph and interpret two-variable data
  5. connections to the elementary and middle school classroom

II. Quantitative Literacy through Probability
- A. Formulating questions and collecting data
  1. theoretical and experimental probability and simulations
  2. expected value and fair games
  3. sources of variability
  4. using technology to collect and analyze data from probability experiments
  5. connections to the elementary and middle school classroom
- B. Probability distributions
  1. from graphical displays to probability distributions
  2. the normal and standard normal distributions and properties
  3. analyzing data and interpreting results from data displayed in a normal distribution
  4. sampling distributions of means and proportions
     a. generating a sampling distribution
     b. properties of sampling distributions
     c. Law of Large Numbers and the Central Limit Theorem
  5. using technology to analyze and interpret results from normal, standard normal, and sampling distributions
- C. Inferential statistics*
  1. p-values, statistical significance and the role of variability
  2. the statistical problem solving process and tests of significance

*OPTIONAL CONTENT IF TIME PERMITS

2.2.1 Integrating the GAISE Statistical Problem Solving Process

The primary organizing feature of the revised course was the integration of the four components of statistical problem solving, the first dimension of the framework, throughout the course. The statistical problem solving process was used to introduce and discuss statistical concepts by beginning with a question to be addressed, collecting data to address the question, analyzing the data, and interpreting the results in the context of the question. All statistical concepts were introduced, reviewed, or discussed within one or more of the components and in the context of the question that was formulated. This process was utilized both as the pre-service teachers were learning or reviewing a statistical concept and as related statistical concepts appropriate for elementary and middle school students were explored. Therefore, the statistical problem solving process made it possible to make a concrete connection between the concept being learned or reviewed and the related concept the pre-service teacher would someday be teaching.

A second modification made to the course was being explicit about discussing the role of variability when dealing with data, a concept that has often proven difficult and that is often not addressed in the depth that would be useful in terms of developing statistical literacy. In the revised course, as the statistical problem solving process progressed for various statistical concepts, a discussion of the nature of variability occurred within the context of the question being answered, both for concepts that pre-service teachers were learning and for concepts that they might be expected to teach at the elementary and middle school levels.

2.2.2 Drawing on the NCTM Standards for Data Analysis and Probability

Although the topics that were addressed in the revised course did not change from those of the original course, the order and ways in which they were addressed changed substantially. Rather than treating the "Descriptive Statistics Activities for K-8 students" and "Probability Activities for K-8 students" as separate topics, they were integrated with the topics the pre-service teachers were learning. (NOTE: Technology for Probability and Statistics was also integrated within each topic and sub-topic as appropriate.) The NCTM Standards for Data Analysis and Statistics provided a structure for examining and connecting the various statistical concepts throughout the course. Most concepts were initially presented and discussed with the pre-service teachers at the grades 9-12 level expectations in order to deepen their own knowledge of those concepts. (Except for the concepts falling under the topic, inferential statistics, these expectations are in line with topics covered in a typical college level introductory statistics course.) Connections were then made to the same standard but at the grades PreK-2, 3-5, and/or 6-8 expectation level. This allowed pre-service teachers to see how the statistical knowledge children learned in elementary school was connected to and built towards knowledge at the middle and high school levels. It also provided opportunities for the pre-service teachers to reflect on the fact that they need a much deeper understanding and knowledge of concepts that that required by the students they will teach.

2.2.3 Using GAISE Framework to Illustrate a Coherent Structure for Teaching Statistical Concepts

The second dimension of the GAISE structure for teaching statistics, the levels of understanding, provided another avenue for making connections between statistical concepts being learned and those that might be taught at the elementary or middle school level. These levels were discussed when transitioning from what the pre-service teachers were learning, typically at level B or C, to what they might someday teach at level A or B, similar to what was done with the NCTM Standards and Expectations. These two frameworks served to demonstrate to the pre-service teachers that the teaching and learning of statistical concepts should be a coherent continuum of concepts, beginning in the early elementary grades, and continuing beyond the high school level.

The following section describes excerpts from a sequence of lessons that illustrate the connection between developing a deep understanding of statistical concepts and teaching statistics at the elementary level and middle school levels.

3. Illustrating Connections – Descriptive Statistics

3.1 Descriptive Statistics Example - *Typical Person Activity*

Note: The activity spanned several lessons. In addition, other data sets and examples were incorporated and explored within each of the components. The following excerpts describe only those associated with the *Typical Person Activity*.

3.1.1 Component 1 of the Statistical Problem Solving Process: Formulating the Question

The *Typical Person Activity*, adapted from an activity used by a colleague, was used to begin class on the first day. Incorporating the first component of the statistical problem solving process, the following question was posed, "How would we describe the typical person taking this class?" A discussion followed as to what data we could gather to help answer the question and a variety of characteristics were identified by pre-service teachers in the class that would help answer the question. The characteristics included hair color, favorite subject, birth month, height, and distance from home to class.

3.1.2 Component 2 of the Statistical Problem Solving Process: Collecting the Data

Using the second component of the problem solving process, the data were then collected via a survey of the class. Once the data were compiled, the four components of the GAISE statistical problem solving process were introduced and discussed. The nature of variability was discussed and potential sources of variability and bias in the data that had been collected were identified. The pre-service teachers noted things such as the following:

- for hair color it was not specified as to whether current hair color or natural hair color should be used.
- for height, some responses included fractions of an inch while others were given to the nearest whole inch.

- for distance from home, some responses were to the nearest mile and some to the nearest 5 miles or 10 miles. In addition, some responded using the distance from their campus home to class while others responded using the distance from their home town to class.

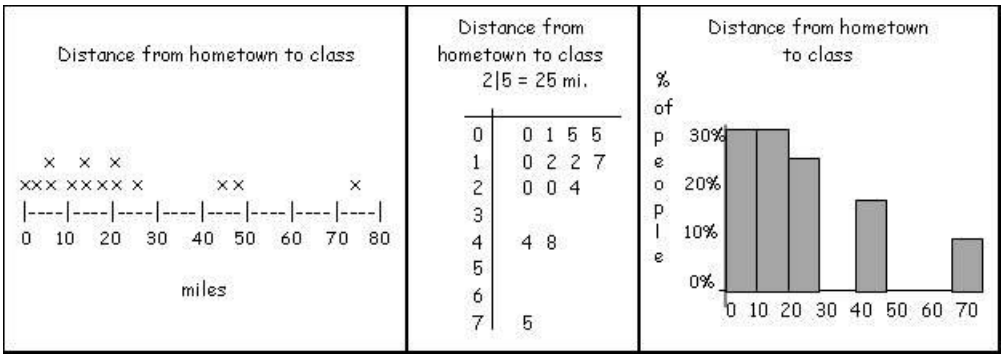
The group decided to record current hair color, height to the nearest inch, and distance from home to class to the nearest mile and from one's hometown. The survey was completed a second time using the new criteria.

The concepts of samples and populations were then discussed with the pre-service teachers being asked to identify the sample and population for the survey. Some incorrectly identified the population as "students at this university" and "elementary education majors concentrating in mathematics" which provided an opportunity to discuss the importance of knowing both the sample and the population when reading or hearing about results of studies. Simple random sampling techniques were introduced and applied to several data sets using random digit tables, a graphing calculator, and a spreadsheet. Other sampling techniques, including convenience sampling, were discussed and the pre-service teachers correctly identified the sample for the class survey as a convenience sample. Implications of convenience sampling and bias were also discussed and compared to sampling variability occurring from taking different samples.

3.1.3 Component 3 of the Statistical Problem Solving Process: Analyzing the Data

Graphical analysis of data was introduced which addressed the third component of the statistical problem solving process. Beginning with hair color, pre-service teachers immediately suggested constructing a bar graph and concluded more people in the class have brown hair than any other hair color. This was followed by a discussion of which characteristics in the survey could be analyzed with a bar graph. Categorical and quantitative variables were then defined and the characteristics were sorted based upon the type of data. Appropriate graphs for categorical data were reviewed briefly since the pre-service teachers were familiar with these techniques. Graphical techniques for quantitative data were presented, some of which were new to many of the pre-service teachers. Using quantitative data collected from the survey, dot plots, stem plots, and histograms were defined constructed and analyzed. For example, distance from home to class resulted in graphs similar to those in Figures 1, 2, and 3. When defining and constructing histograms, the difference between histograms and bar graphs were stressed by focusing on the fact that in bar graphs, the categories can be placed in any order without changing the results the graph displays. Similar graphs were constructed for height with a stem plot allowing for the introduction of split stems.

Figure 1. Line Plot **Figure 2.** Stem Plot **Figure 3.** Histogram



The concepts of shape, center and spread of the data were then introduced and the results of the graphs were interpreted in terms of what they told us about the typical person in the class. For example, the pre-service teachers stated the majority of people in the class "live between 20 and 24 miles from class" and "there is a range of 75 miles in the distances people live from class." The three graphs in figures 1, 2, and 3 were identified as being skewed to the right. When discussing the center of the data, many of the pre-service teachers believed the center to be at 37.5 miles since it was halfway between 0 and 75. The question of center was left unanswered until numerical analyses of the data were discussed.

Numerical analyses began by constructing and interpreting frequency and relative frequency distributions for hair color and favorite subject, the categorical variables. Figure 4 illustrates the distribution similar to that constructed by the pre-service teachers. The mode was also identified for the two variables.

Figure 4. Relative Frequency Distribution

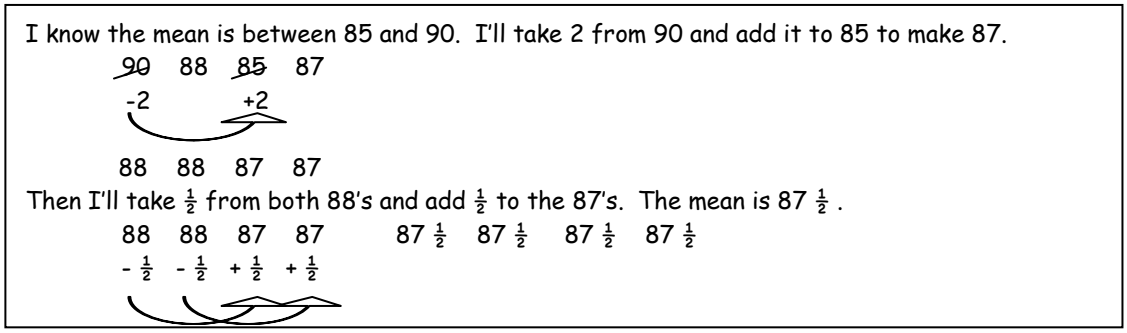
Hair Color of People in this Class

Color	Frequency	Relative Frequency
Blonde	3	.214
Brown	7	.500
Red	1	.071
Black	2	.143
Gray	1	.071

For the quantitative data, pre-service teachers were asked to describe the meaning of mean and median. Most responses consisted of the typical definitions, "for mean you add up all of the numbers and divide by how many numbers there were" and "the median is the number in the middle." The median was first explored using data sets as well as data displayed on dot plots and stem plots. The mean was then explored using the concepts of "leveling off" and as a "balance point." The exploration for the mean as "leveling off" began by leveling off Unifix

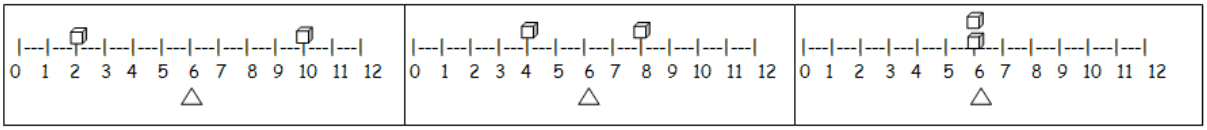
cubes. (Unifix cubes are multi-colored plastic cubes, 1 in. by 1 in. by 1 in., that can be connected and taken apart.) It then progressed to a more abstract representation such as finding the mean test grade for a student with test grades of 90, 88, 85, and 87 using the concept of leveling off. A typical student response is shown in Figure 5.

Figure 5. Typical Student Response to Representing the Concept of Mean as "Leveling Off"



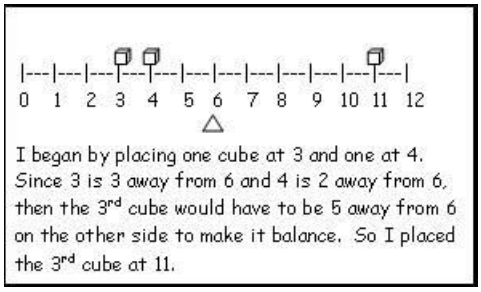
The concept of the mean as a balance point was more difficult for the pre-service teachers to comprehend. The exploration began by placing 2 Unifix Cubes on a ruler with a fulcrum as a base so that the ruler balanced at 6. A variety of responses were shared and discussed as shown in Figure 6. The main point of the first exploration was for the pre-service teachers to understand each of the cubes was the same distance from 6.

Figure 6. Student Response to Demonstrating the Concept of Mean as a Balance Point



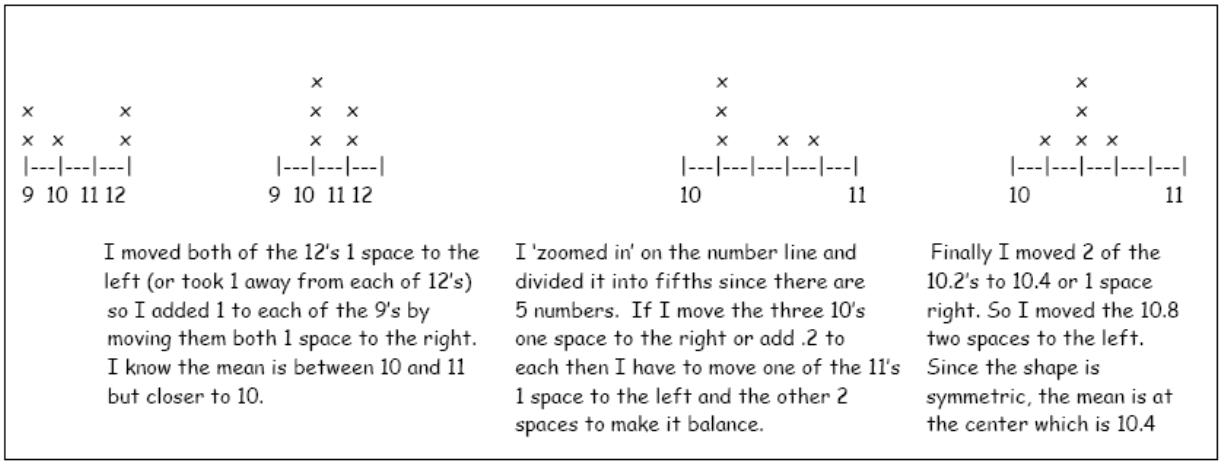
Pre-service teachers were then asked to use 3 Unifix cubes to balance the ruler at 6. (They were not permitted to place all 3 cubes on the 6.) Some used a guess and check method while others resorted to using their knowledge of the formula to choose the 3 numbers and then position their cubes on those numbers. The pre-services teachers were then prompted to use what they had discovered in the first exploration to choose the position for the 3 cubes and to record their thought process. Figure 7 illustrates a typical approach used by the pre-service teachers.

Figure 7. Typical Approach to Representing the Mean of 3 Numbers as a Balance Point



The exploration then progressed to asking the pre-service teachers to place 3 cubes at various places on the ruler and to find the balance point. Moving to a more abstract level, they then explored finding the mean for a set of data displayed on a dot plot, including data whose mean was not a whole number. Figure 8 is a sample response to the task: *Find the mean quiz score for the set of scores 9, 9, 10, 12, and 12 using the concept of the mean as a balance point.*

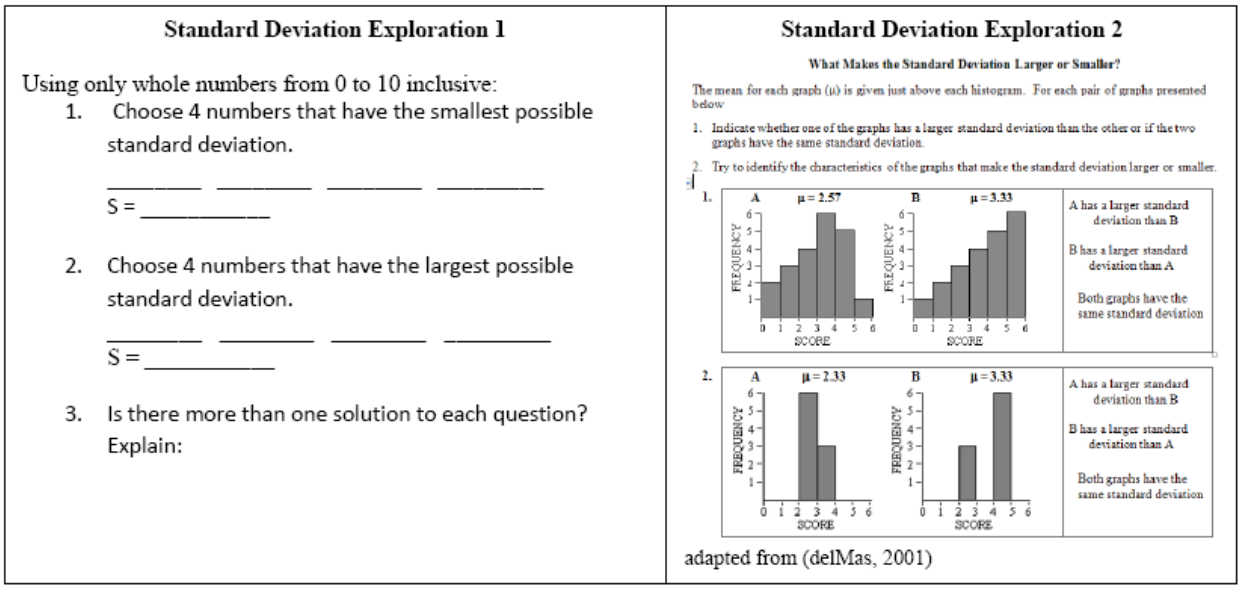
Figure 8. Student Response to Finding the Mean of 3 Numbers Using the Concept of Balance Point



After exploring measures of center, measures of dispersion or spread were introduced. Standard deviation was a term familiar to the pre-service teachers but they did not have an understanding of what it was or what it meant. The definition was given and then an example was used to demonstrate how standard deviation is calculated with an emphasis on the fact that the deviations are squared to account for positive and negative deviations and the mean of the squared deviations is calculated resulting in what is called variance. Since the deviations were squared, the square root of the variance is taken to determine the standard deviation. Explorations of standard deviation focused on identifying data sets and graphical displays having large and small

standard deviations and stating the reasons why the standard deviation was large or small. Figure 9 illustrates parts of each exploration. Discussion included the fact that data that is skewed or contains outliers results in larger standard deviations. Standard deviation for the quantitative variables in the survey was then calculated and discussed.

Figure 9. Exploring Standard Deviation



Standard Deviation Exploration 1

Using only whole numbers from 0 to 10 inclusive:

1. Choose 4 numbers that have the smallest possible standard deviation.

   _____ _____ _____ _____

   S = _____

2. Choose 4 numbers that have the largest possible standard deviation.

   _____ _____ _____ _____

   S = _____

3. Is there more than one solution to each question? Explain:

Standard Deviation Exploration 2

What Makes the Standard Deviation Larger or Smaller?

The mean for each graph (μ) is given just above each histogram. For each pair of graphs presented below

1. Indicate whether one of the graphs has a larger standard deviation than the other or if the two graphs have the same standard deviation.
2. Try to identify the characteristics of the graphs that make the standard deviation larger or smaller.

1. A μ = 2.57 B μ = 3.33 — FREQUENCY / SCORE — A has a larger standard deviation than B; B has a larger standard deviation than A; Both graphs have the same standard deviation

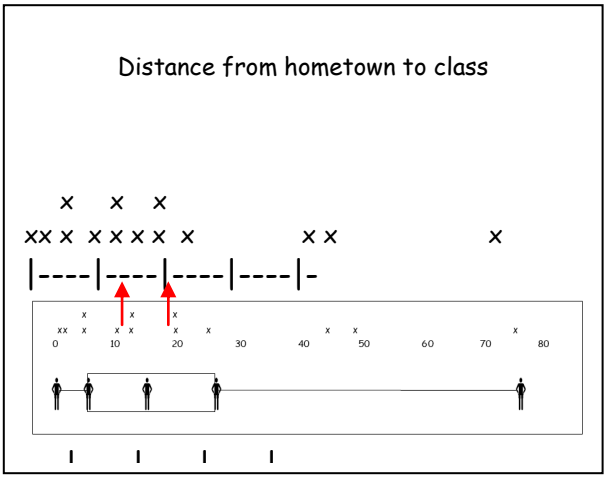
2. A μ = 2.33 B μ = 3.33 — FREQUENCY / SCORE — A has a larger standard deviation than B; B has a larger standard deviation than A; Both graphs have the same standard deviation

adapted from (delMas, 2001)

The final discussion of numerical analysis focused on the 5 number summary and construction and interpretation of boxplots. This was done using the distance from home data set and the dot plot previously constructed. A human boxplot of the distance from home data was constructed and interpreted. (See section 3.1.5 for a detailed description of how the human boxplot was constructed.)

A discussion of the impact of the shape of the data on measures of spread and dispersion concluded the part of the lesson on analyzing data by revisiting the data on distance from class. The pre-service teachers were asked to locate the median distance on the dot plot previously constructed. (See Figure 10) They were then asked to predict where they thought the mean would be and to explain why. Although some reverted to their previous beliefs that the mean would be at 37.5 because it was the halfway point of the number line or that it would be close to the median, the majority correctly conjectured that the mean would be to the right of the median because to "balance out" the data the values of 44, 48 and 75, the lower values would need to move more to the right of the median. The mean was then calculated as 20.93 miles and located on the dot plot. This was used as one of the examples to illustrate the fact that the mean and standard deviation are not resistant to outliers and skewed data.

Figure 10. Locating Mean and Median on a Skewed Dot Plot



3.1.4 Component 4 of the Statistical Problem Solving Process: Interpreting the Results

To conclude the lesson, pre-service teachers were first asked to describe the typical person in the class using the characteristics that had been analyzed and to see if anyone in the class possessed all of the characteristics. The typical person was described as someone with brown hair, born in June, whose favorite subject was math, lived 14.5 miles from class and was 54.14 inches tall. When no one could actually be identified as the 'typical person,' a discussion was held about how statistics can be used to describe a group as a whole but cannot tell you about the individuals in the group.

The question was then posed, "Would the typical person in this class be representative of the typical student who has taken the course before or who will take it in the future? Why or why not?" The pre-service teachers' responses varied. Some concluded that all those taking the class would be pre-service teachers majoring in elementary education with a concentration in mathematics but that the other characteristics would vary. The ideas of natural and sampling variability were revisited. Finally, the question, "Would the typical person in this class be representative of the typical student at the university? Why or why not?" was posed. The pre-service teachers correctly stated the typical student in the class would not be representative of students at the university citing reasons such as, "everyone in the class is an elementary education major – everyone at the university is not" and "we are all sophomores, juniors, and seniors so freshmen are not represented." The concepts of appropriately making inferences from samples to populations and random sampling were also revisited.

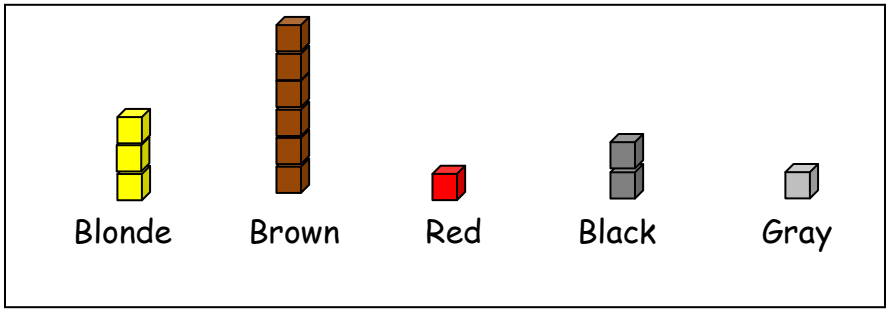
3.1.5 Connections to the elementary and middle school classroom

The GAISE framework was revisited by examining the three levels of understanding and how the related statistical concepts from the activity might be addressed at the elementary and middle school levels. For example a discussion of how formulating questions varies at the different

levels of understanding was held. Examples from the GAISE Introduction were used as the focus of the discussion. (ASA, 2007, p. 16)

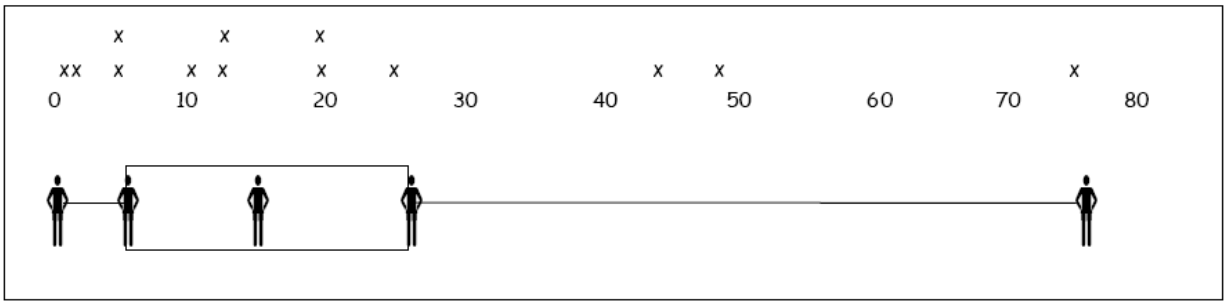
Graphical analysis appropriate for different levels was also examined. For example, at Level A, students might construct glyphs, picture graphs, pictographs, bar graphs, dot plots, etc. The pre-service teachers engaged in constructing glyphs displaying hair color, eye color, glasses, and smiles for those who brought a snack to class, etc. A connection was also made to the bar graph constructed at the beginning of the *Typical Person Activity.* The pre-service teachers each selected a Unifix cube representing their hair color and cubes of like color were stacked into a bar graph as shown in Figure 11. The cubes were then disconnected and formed into a circle thus displaying a pie graph.

Figure 11. Bar Graph Constructed with Unifix Cubes



At Level B students should have opportunities to construct and analyze contingency tables for two categorical variables. The pre-service teachers engaged in constructing a "human dot plot" and a "human boxplot" of the distance from home. A number line was drawn on the whiteboard and pre-service teachers stood at the number representing their distance from home. For the numbers corresponding to more than one person, the pre-service teachers stood in front of each other thus constructing the human dot plot. The dot plot was also drawn on the board. The minimum and maximum were identified and each of those people was given a piece of string. Since no one had the median distance, one pre-service teacher stood at the median. This person was given a piece of string and asked to extend her arms holding the string. People at the first and third quartiles were identified and the remaining pre-service teachers were asked to sit down. A box of string was formed around the persons at the first and third quartiles and the string from the people at the minimum and maximum value was tied to the box. (See Figure 12.)

Figure 12. A Human Box Plot



The NCTM Standards, as well as Pennsylvania's mathematics standards dealing with data analysis, across grade levels were then examined and discussed in terms of the statistical concepts addressed in the *Typical Person* activity as well as the graphing activities in which the pre-service teachers engaged. NCTM Standard 1, *formulate questions that can be addressed with data and collect, organize and display relevant data to answer them,* and Standard 2, *select and use appropriate statistical methods to analyze data,* were identified as being addressed in the activity and pre-service teachers discussed which parts of the activities addressed standards at the different grade bands. For example, the construction of glyphs and Unifix bar graphs addressed Standard 1 at the PreK-2 level, while differentiating between categorical and quantitative data addressed the grades 3-5 level and constructing and analyzing histograms and box plots addressed the grades 6-8 level. The beginning and conclusion of the activity focused on the grades 9-12 level since much of the discussion included the concepts of sample, population, random sampling, and variability.

Finally, resources for teaching the concepts were examined including materials and journals from the National Council of Teachers of Mathematics and the suggested activities in the GAISE document.

3.1.6 Assessing understanding

As a form of assessing pre-service teachers' understanding of descriptive statistics and connections to the elementary and middle school classroom, two major assignments were given. The first assignment required pre-service teachers to use the problem solving process in designing and carrying out a statistical study. The study required pre-service teachers to formulate a question of interest to them that could be answered by collecting data from either an observational study or an experiment. They were required to identify the sample and population for their study as well as the method used to select the sample. After the data were collected, the pre-service teachers analyzed the data using appropriate graphical displays and numerical summaries. They then interpreted their results in terms of the question they asked using their graphs and numerical summaries as evidence. They also were required to identify potential sources of variability in their study.

The second assignment required the pre-service teachers to teach a mini-lesson to their peers that developed a concept related to descriptive statistics at the elementary or middle school level. The lesson was required to follow the statistical problem solving process. Both state and national standards for data analysis were to be identified as was the GAISE level of understanding. Pre-service teachers were also required to discuss the prior experiences children would need to be successful in the lesson and what statistical concepts in the future were built upon the concept in their lesson.

4. Assessing the Effectiveness of the Revised Course

In order to determine the effectiveness of the revised course, two sets of artifacts were examined –one group of pre-service teachers' grades in the course and evaluations of the course prior to it being revised and a second group of pre-service teachers' grades in the course and evaluations of the revised course.

The grades in the course were not substantially different from each other. For the pre-service teachers taking the course prior to it being revised, all 12 received A's or B's. Twelve of the 14 pre-service teachers taking the course after revision received A's or B's and 2 received C's. These results are not surprising since the pre-service teachers have elected to take the course. In addition, the university requires education majors to maintain a 3.0 average which would further motivate pre-service teachers to do well.

There was a slight difference in the evaluations done by the pre-service teachers at the conclusion of the course. Three questions in particular show slight increases for the revised course:

Question	**Pre-Service Teacher Responses in Course Prior to Revision**	**Pre-Service Teacher Responses in Revised Course**
1. I learned valuable skills in this course.	92% agree or strongly agree	100% agree or strongly agree
2. As a result of this course my knowledge and skills have increased.	91% agree or strongly agree	100% agree or strongly agree
3. In this course critical thinking was encouraged.	92% agree or strongly agree	100% agree or strongly agree

Open responses on the course evaluations also showed some differences. While several pre-service teachers in both courses commented that they enjoyed "hands on learning" and "actually getting to do the activities in class," several students in the revised course commented "we used examples applicable to the elementary classroom" and "we got to practice things we will someday have to teach."

While the course evaluations provide support for the revised course, they cannot be used to definitely say the revised course was effective. Many other factors could have contributed to the differences in the course evaluations. Therefore, an assessment is planned for the next time the revised course is taught using the *Learning Mathematics for Teaching* items developed by Ball et al. (2008) to assess pre-service teachers understanding of statistical concepts prior to and after their participation in the revised course.

5. Conclusion

Statistical literacy is no longer a skill relegated to the few. It is essential knowledge required by all that must be developed beginning at an early age, continuing throughout one's school years. Therefore, it is crucial for teachers at all levels to be statistically literate themselves and to possess the pedagogical tools necessary to provide quality learning experiences that develop and deepen their students' statistical understanding. Integrating content knowledge with pedagogical knowledge has been shown to be vital in preparing tomorrow's teachers for the classroom. The course described in this article is an attempt to do just that by using the GAISE document as a framework. By integrating the learning of statistical concepts with the teaching of statistical concepts, it is hoped pre-service teachers who experience the course will not only deepen their own understanding of statistics but will also develop an understanding of the various ways the children they will soon be teaching learn statistics.

Appendix A

NCTM Data Analysis and Probability Standards			
1. formulate questions that can be addressed with data and collect, organize and display relevant data to answer them			
Pre-K–2 Expectations: In prekindergarten through grade 2 all students should–	*Grades 3–5 Expectations: In grades 3–5 all students should–*	*Grades 6–8 Expectations: In grades 6–8 all students should–*	*Grades 9–12 Expectations: In grades 9–12 all students should–*
• pose questions and gather data about themselves and their surroundings; • sort and classify objects according to their attributes and organize data about the objects; • represent data using concrete objects, pictures, and graphs.	• design investigations to address a question and consider how data-collection methods affect the nature of the data set; • collect data using observations, surveys, and experiments; • represent data using tables and graphs such as line plots, bar graphs, and line graphs; • recognize the differences in representing categorical and numerical data.	• formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population; • select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots.	• understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each; • know the characteristics of well-designed studies, including the role of randomization in surveys and experiments; • understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable; • understand histograms, parallel box plots, and scatterplots and use them to display data; • compute basic statistics and understand the distinction between a statistic and a parameter.
2. select and use appropriate statistical methods to analyze data			
• describe parts of the data and the set of data as a whole to determine what the data show.	•describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed; • use measures of center, focusing on the median, and understand what each does and does not indicate about the data set; • compare different representations of the same data and evaluate how well each representation shows important aspects of the data.	• find, use, and interpret measures of center and spread, including mean and interquartile range; • discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots.	• for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics; • for bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools; • display and discuss bivariate data where at least one variable is categorical; • recognize how linear transformations of univariate data affect shape, center, and spread; • identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.
3. develop and evaluate inferences and predictions that are based on data			
• discuss events related to students' experiences as likely or unlikely.	• propose and justify conclusions and predictions that are based on data and design studies	• use observations about differences between two or more samples to make conjectures about the	• use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions;

	to further investigate the conclusions or predictions.	populations from which the samples were taken; • make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit; • use conjectures to formulate new questions and plan new studies to answer them.	• understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference; • evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions; • understand how basic statistical techniques are used to monitor process characteristics in the workplace.
4. understand and apply basic concepts of probability			
	• describe events as likely or unlikely and discuss the degree of likelihood using such words as certain, equally likely, and impossible; • predict the probability of outcomes of simple experiments and test the predictions; • understand that the measure of the likelihood of an event can be represented by a number from 0 to 1.	• understand and use appropriate terminology to describe complementary and mutually exclusive events; • use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations; • compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.	• understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases; • use simulations to construct empirical probability distributions; • compute and interpret the expected value of random variables in simple cases; • understand the concepts of conditional probability and independent events; • understand how to compute the probability of a compound event.

compiled from *Principles and Standards for School Mathematics,* National Council of Teachers of Mathematics, 2000

Appendix B

COLLEGE BOARD STANDARDS FOR SUCCESS
MIDDLE LEVEL MATHEMATICS I **Standard MI.4** *Univariate Data Analysis* Objectives: MI.4.1 Student formulates a question about one small population or about a comparison between two small populations that can be answered through data collection and analysis, designs related data investigations, and collects data. MI.4.2 Student organizes and summarizes categorical and numerical data using summary statistics and a variety of graphical displays. MI.4.3 Student interprets results and communicates conclusions regarding a formulated question using appropriate symbols, notation, and terminology. Student identifies flaws in faulty or misleading presentations of data found in the media. **Standard MI.5***Experimental and Theoretical Probability* Objective: MI.5.1 Student estimates probabilities in experiments with common objects, including comparing experimental and theoretical probabilities and examining experimental probabilities in the long run.
MIDDLE LEVEL MATHEMATICS II **Standard MII.4** *Bivariate Data* Objectives: MII.4.1 Student formulates questions about a small population that can be answered through collection and analysis of bivariate data, designs related data investigations, and collects data. MII.4.2 Student organizes and summarizes bivariate data, examining data on the two attributes separately and together, and classifies each attribute as a categorical variable or numerical variable. Student uses summary statistics and a variety of graphical displays to represent the data. MII.4.3 Student interprets results and communicates conclusions from a bivariate data analysis to answer the formulated question using appropriate symbols, notation, and terminology. Student identifies flaws in faulty or misleading presentations of bivariate data found in the media. **Standard MII.5** *Probabilities in One-Stage Experiments* Objective: MII.5.1 Student determines the sample space for one-stage experiments and determines, where possible, the theoretical probabilities for events defined on the sample space. Student describes and applies the addition rule for probabilities.
ALGEBRA I **Standard AI.4** *Surveys and Random Sampling* Objectives: AI.4.1 Student formulates questions that can be addressed through collection and analysis of survey data. Student explains the importance of random selection of members from the population, and designs and executes surveys. Student uses the results of a survey to communicate an answer appropriate to the question of interest. Student distinguishes between sampling error and measurement error. Student evaluates survey results reported in the media. AI.4.2 Student understands that results may vary from sample to population and from sample to sample. Student analyzes, summarizes, and compares results from random and nonrandom samples and from a census, using summary numbers and a variety of graphical displays to communicate findings.
GEOMETRY **Standard G.4** *Two-Stage Experiments, Conditional Probability, and Independence* Objectives: G.4.1 Student determines the sample space for two-stage experiments, and employs the multiplication rule for counting. Student distinguishes between independent and dependent compound events and computes their probabilities using representations for such events and using the multiplication rule for probability. G.4.2 Student develops, uses, and interprets simulations to estimate probabilities for events where theoretical values are difficult or impossible to compute.
ALLGEBRA II **Standard AII.4** *Experiments, Surveys, and Observational Studies*

Objectives:
AII.4.1 Student identifies problems that can be addressed through collection and analysis of experimental data, designs and implements simple comparative experiments, and draws appropriate conclusions from the collected data.
AII.4.2 Student distinguishes among surveys, observational studies, and designed experiments and relates each type of investigation to the research questions it is best suited to address. Student recognizes that an observed association between a response variable and an explanatory variable does not necessarily imply that the two variables are causally linked. Student recognizes the importance of random selection in surveys and random assignment in experimental studies. Student communicates the purposes, methods, and results of a statistical study, and evaluates studies reported in the media.
Standard AII.5 *Permutations, Combinations, and Probability Distributions*
Objectives:
AII.5.1 Student solves ordering, counting, and related probability problems. Student recognizes a binomial probability setting and computes the probability distribution for a binomial count.
AII.5.2 Student identifies data settings in which the normal distribution may be useful. Student describes characteristics of the normal distribution and uses the empirical rule to solve problems.

PRECALCULUS
Standard PC.6 *Bivariate Data and Trend-Line Models*
Objectives:
PC.6.1 Student assesses association in tables and scatterplots of bivariate numerical data and uses the correlation coefficient to measure linear association. Student develops models for trends in bivariate data using both median-fit lines and least-squares regression lines.
PC.6.2 Student examines the influence of outliers on the correlation and on models for trend.
PC.6.3 Student examines the effects of transformations on measures of center, spread, association, and trend and develops basic techniques for modeling and more-advanced data analytic techniques.

compiled from *Standards for College Success: Mathematics and Statistics*, The College Board, 2006

Appendix C

GUIDELINES FOR ASSESSMENT AND INSTRUCTION IN STATISTICS EDUCATION FRAMEWORK SUMMARY			
Problem Solving Components	**Level A**	**Level B**	**Level C**
I. Formulate the question	• Teachers pose questions of interest • Questions restricted to the classroom	• Students begin to pose their own questions of interest • Questions not restricted to the classroom	• Students pose their own questions of interest • Questions seek generalization
II. Collect the data	• Census of classroom • Simple experiment	• Sample surveys - begin to use random selection • Comparative experiment - begin to use random allocation	• Sampling designs with random selection • Experimental designs with randomization
III. Analyze Data	• Use particular properties of distributions in the context of a specific example	• Learn to use particular properties of distributions as tools of analysis	• Understand and use distributions in analysis as a global concept
IV. Interpret Results	• Students do not look beyond the data • No generalization beyond the classroom • Note difference between two individuals with different conditions • Observe association in displays	• Students acknowledge that looking beyond the data is feasible • Acknowledge that a sample may or may not be representative of the larger population • Note the difference between two groups with different conditions • Aware of distinction between observational study and experiment • Note differences in strength of association • Basic interpretation of models for association • Aware of the distinction between association and cause and effect	• Students are able to look beyond the data in some contexts • Generalize from sample to population • Aware of the effect of randomization on the results of experiments • Understand the difference between observational studies and experiments • Interpret measures of strength of association • Interpret models of association • Distinguish between conclusions from association studies and experiments

compiled from *Guidelines for Assessment and Instruction in Statistics Education*, American Statistical Association, 2007

Appendix D

NCATE/NCTM Program Standards (2003)
Programs for Initial Preparation of Mathematics Teachers

Standards for Elementary Mathematics Specialists

Standard 12: Knowledge of Data Analysis, Statistics, and Probability
Candidates demonstrate an understanding of concepts and practices related to data analysis, statistics, and probability.
Indicators
12.1 Design investigations that can be addressed by creating data sets and collecting, organizing, and displaying relevant data.
12.2 Use appropriate statistical methods and technological tools to analyze data and describe shape, spread, and center.
12.3 Apply the basic concepts of probability.
12.4 Demonstrate knowledge of the historical development of probability and statistics including contributions from diverse cultures.

Standards for Middle Level Mathematics Teachers

Standard 14: Knowledge of Data Analysis, Statistics, and Probability
Candidates demonstrate an understanding of concepts and practices related to data analysis, statistics, and probability.
Indicators
14.1 Design investigations, collect data through random sampling or random assignment to treatments, and use a variety of ways to display the data and interpret data representations.
14.2 Draw conclusions involving uncertainty by using hands-on and computer-based simulation for estimating probabilities and gathering data to make inferences and decisions.
14.3 Identify misuses of statistics and invalid conclusions from probability.
14.4 Use appropriate statistical methods and technological tools to analyze data and describe shape, spread, and center.
14.5 Investigate, interpret, and construct representations for conditional probability, geometric probability, and for bivariate data.
14.6 Demonstrate knowledge of the historical development of probability and statistics including contributions from diverse cultures

compiled from *NCATE/NCTM Program Standards: Programs for Initial Preparation of Mathematics Teachers,* NCATE/NCTM, 2003

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Mary Louise Metz, Ph.D.
Mathematics Department, 236 Stright Hall
210 S. Tenth St.
Indiana University of PA
Indiana, PA 15705
Phone: (724) 357-4759
Fax: (724) 357-7908
mlmetz@iup.edu