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How Much Math Do Students Need to Succeed in Business and Economics Statistics? An Ordered Probit Analysis

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Abstract

Because statistical analysis requires the ability to use mathematics, students typically are required to take one or more prerequisite math courses prior to enrolling in the business statistics course. Despite these math prerequisites, however, many students find it difficult to learn business statistics.

In this study, we use an ordered probit model to analyze the impact of alternative prerequisite math course sequences on the grade performance of 1,684 business and economics statistics students at a large Midwestern university. In addition, we show how imposing a minimum grade requirement of C- for the math prerequisite course would influence student success in the business statistics course. Although several studies have examined the impact of different math *skills*, our study is the first to provide a detailed analysis of the impact of different prerequisite math course sequences on student performance in business statistics. We demonstrate that, other things the same, taking more math credit hours, taking math courses that emphasize calculus, and imposing a minimum grade of C- on the prerequisite math course have significant positive impacts on student grade performance in the business and economics statistics course.

1. Introduction

Virtually all accredited business schools require their students to take one or more courses in both mathematics and business statistics. In addition, most introductory business statistics courses require one or more math courses to provide the necessary mathematical foundation for statistics. However, despite these prerequisite math courses, many students do poorly in their business and economics statistics (hereafter, business statistics) course. It has even been alleged that "… Business Statistics is the most hated, most unpopular course in the business program." [1] Potential reasons cited for poor student performance include statistics anxiety, inadequate statistics instruction, inadequate math preparation before matriculation and inadequate math prerequisites prior to taking the statistics course. [2]

In this study, we focus on the importance of math prerequisites for student performance in the business statistics course. Specifically, we use an ordered probit model to examine the relationship between alternative math course sequences and the grades earned by students the *first* time they complete the business statistics course. [3] We then show how imposing a minimum grade requirement of C- for the prerequisite math course would be expected to affect student performance in business statistics.

Several studies have previously examined the impacts of mathematics skills and topics on student performance in business statistics. [4] To our knowledge, however, this is the first study to examine the effect of alternative prerequisite math course sequences on student performance. [5] It is also the first study to demonstrate the effect on student success in business statistics of imposing a minimum grade requirement for the prerequisite math course.

2. Student Performance in Business Statistics: An Overview

Table 1 shows the percentage distributions for grades F through A for virtually all students who completed their business statistics course for the first time from Fall 2001 through Summer 2006. [6] About 11 percent of these students failed the course and another 14 percent earned a D. Including students who enrolled and then withdrew from the course before the semester ended, more than 30 percent of our students earned a D, F or W the first time they enrolled in the business statistics course during this five-year period. [7]

Table 1. Grade Distribution for Students Enrolled in Business Statistics:

Fall 2001 – Summer 2006

Grade	Percent of Total Grades	Cumulative Percent (F to A)
A- To A	21.10 %	100 %
B- to B+	28.64	78.91
C- to C+	25.16	50.28
D- to D+	13.88	25.12
F	11.24	11.24
Number of Students	2162	

As we noted above, one factor that might account for the poor student performance in their first business statistics course is inadequate prerequisite math courses. Several changes in math prerequisites at our university over the past five years have provided us with a unique opportunity to examine the extent to which different prerequisite math course sequences and math topic coverage can influence student performance in the business statistics course.

3. "Different Strokes for Different Folks": Alternative Math Course Sequences for Business Statistics

Undergraduate pre-business students at our university are required to complete a designated math course and ECON 221: Business Statistics before they can enroll in the College of Business and take specialized upper division business courses. [8] The mathematics course required for admission to the College is also the prerequisite math course for the business statistics course.

Pre-business students have satisfied the math requirement for business statistics in a variety of ways. Prior to the Fall 2004 semester, MATHS 132: Brief Calculus for Business (three credit hours) was the prerequisite math course and the vast majority of pre-business students completed the nine-credit-hour math sequence: MATHS 108: Intermediate Algebra (three credit hours), MATHS 131: Finite Mathematics for Business (three credit hours) and MATHS 132. [9]

During the 2003/04 academic year, the Business College's Undergraduate Curriculum Committee decided to replace the nine-credit-hour MATHS 108/131/132 sequence with a less rigorous seven-credit-hour sequence: MATHS 108 and a new four-credit-hour course, MATHS 135: Mathematics for Business. This change, which became effective in the Fall 2004 semester, created a transition problem for students who had taken MATHS 131 but had not completed MATHS 132 by the end of the Spring 2004 semester. [10] These students were permitted to enroll in ECON 221 in the Fall 2004 and Spring 2005 semesters even though they had only completed a six-credit-hour math sequence: MATHS 108/131.

Finally, to accommodate students who intended to change their academic majors from other disciplines or wanted to either minor or have a double-major in business or economics, the College of Business accepted two other math sequences as *substitutes* for the official math prerequisite sequence. These two alternative math sequences consisted of the nine-credit-hour sequence: MATHS 108 (or MATHS 111: Pre-Calculus Algebra), MATHS 112: Pre-Calculus Trigonometry and MATHS 161: Applied Calculus I and the thirteen-credit-hour sequence: MATHS 108, MATHS 111, MATHS 112 and MATHS 165: Calculus I (four credit hours). [11] These alternative math sequences were accepted as substitutes because they were considered to be more rigorous than the prerequisite math sequence required by the College of Business and published in the university catalog.

Table 2 shows the various math sequences that were either officially required or accepted as substitutes during our study. This table also indicates the number of students in our study that used these sequences as prerequisites for the business statistics course.

Table 2. MATHS Course Sequences for Business Statistics Students

Prerequisite Sequences			Acceptable Alternative Sequences	
Prior to Fall 2004	Fall 2004 - Spring 2005 [12]	Spring 2005 – Summer 2006	Throughout the Entire Period	
108	108	108	108 (or 111)	108
131	**131**	**135**	112	111
132			**161**	112
				165
Number of Students in Our Study Who Used Each Sequence				
929	277	241	179	58

The topics covered in these math sequences are shown in Table 3A and 3B. As Table 3A indicates, the two preliminary algebra classes (MATHS 108 and 111) differ in terms of the topics covered. Because MATHS 108 is a prerequisite for MATHS 111, their common topics are covered in greater detail in MATHS 111, which includes numerous additional topics as well.

Table 3. Alternative Math Courses and Topics

A. Preliminary Math Courses

Topic	MATHS		
	108	111	112
Factoring	X		
Quadratic equations	X	X	
Inequalities	X	X	
Relations and functions	X		
Rational exponents	X		
Systems of linear systems	X	X	
Exponential and logarithmic functions	X	X	
Determinants		X	
Mathematical induction		X	
Binomial theorem		X	
Permutations, combinations and progressions		X	
Trigonometric functions, identities and equations			X
Graphs of trigonometric and inverse trigonometric functions			X
Solution of right and general triangles			X
Polar coordinates			X
Complex numbers			X

B. Alternative Prerequisite/Acceptable Math Courses for Business Statistics

Topic	MATHS				
	131	132	135	161	165
Mathematics of finance	X		X		
Probability	X		X		
Matrix algebra	X				
Linear programming	X				
Differential calculus		X	X	X	X
Integral calculus		X		X	X

Table 3B shows that the alternative required or acceptable prerequisite math courses for business statistics differ considerably in terms of their math coverage. Specifically, MATHS 131 did not cover calculus at all, while MATHS 132 provided an introduction to both differential and integral calculus. [13] Its replacement, MATHS 135, included only an introduction to differential calculus when the decision was made to pool selected topics from the six-credit hour MATHS 131/132 sequence into a single four-credit hour course.

Finally, MATHS 161 and 165 differ from MATHS 132 and 135 in two significant ways. First, their prerequisites include MATHS 108/111 and/or 112, instead of MATHS 108 alone. Because MATHS 111 and MATHS 112 are required for math and science majors, the math content in MATHS 161 and 165 is more rigorous than that in MATHS 131/132 or MATHS 135. Second, the MATHS 165 course sequence involves thirteen credit-hours instead of the seven or nine credit-hours for the MATHS 135 or 132 sequences, respectively. [14]

4. The Model

Our study focuses on the effect of the different prerequisite math course sequences on the specific grade (from A to F) that a student received when completing the business statistics course for the first time. Our model is

$$STAT_i = \beta_0 + \beta_1\, D131_i + \beta_2\, D135_i + \beta_3\, D161_i + \beta_4 D165_i + \beta_5\, MATH_i + \beta_6\, D131M_i + \beta_7\, D135M_i$$

$$+ \beta_8\, D161M_i + \beta_9\, D165M_i + \beta_{10}\, GPA_i + \beta_{11}\, ACC_i + \beta_{12}\, ECON_i + \beta_{13}\, Summer_i + \varepsilon_i$$ [15].

The dependent variable (STAT) is an index for the grade (A- to A = 4; B- to B+ = 3; ...; F=0) that students received the first time they completed the business statistics course. The first set of independent variables consists of four 0, 1 dummy variables (D131, D135, D161 and D165) that designate the specific prerequisite or acceptable alternative math sequence taken *instead of the MATHS 132 sequence* and MATH, which is the numerical grade (A = 4, A- = 3.7, ..., F = 0) that students received in the last course of their specific math sequence.

We do not use a dummy variable for MATHS 132 because its sequence is the base against which the impacts of the other math sequences are measured. We chose this sequence as the base because it had been the prerequisite math sequence for two decades prior to Fall 2004. We wanted to assess the effect on student grade performance in the business statistics course of the recent changes to less rigorous required prerequisite math courses. Furthermore, we wanted to compare the effect on student grade performance in the business statistics course of the more rigorous alternative substitute math sequences relative to that of the previous long-standing prerequisite math sequence. We were interested in determining the types of changes in the prerequisite math sequences that would yield better student performance.

The second set of independent variables consists of four interaction terms (D131M, D135M, D161M and D165M), each of which equals the product of the respective 0,1 math sequence dummy variable and the MATH score described above. These four interaction terms are included to examine whether the math grade received (e.g., A) had different effects on the statistics course grade depending on which math sequence was taken *instead of the MATHS 132 sequence*.

The final set of independent variables consists of selected student characteristics often used in studies of student performance. These are the student's cumulative grade point average (GPA) immediately prior to first-time completion of the business statistics course, the numerical grades (A = 4, A- = 3.7, ..., F = 0) received in the principles of accounting (ACC) and microeconomics (ECON) courses taken prior to first-time completion of the business statistics course, and a 0, 1 dummy variable (Summer) that indicates whether the business statistics course was completed during the summer instead of the fall or spring semesters. [16] Table A1 in the Appendix provides a more detailed definition of these variables.

The estimated coefficients for D131 and D135 measure the impact on student performance in business statistics for students who took the MATHS 131 or MATHS 135 sequences compared to those who took the MATHS 132 sequence. They are expected to be negative because these prerequisite sequences are less rigorous and have fewer credit-hours. [17]

The estimated coefficients for D161 and D165 are expected to be positive because they measure the influence on student performance in business statistics for those who took the more rigorous math sequences compared to students who took the MATHS 132 sequence. The estimated coefficient for MATH is also expected to be positive because it measures the impact of math knowledge gained in the final prerequisite math course on student performance in business statistics regardless of which math sequence was taken.

We have no a priori expectations for the signs of the estimated coefficients for the math interaction terms (D131M, etc.). These coefficients measure the *difference* in the impact on student performance in business statistics for the grade received in that math course relative to the same grade received in MATHS 132. For example, if the estimated coefficient for a specific math course *dummy* variable is positive, a student who earns a B in that course would be expected to earn a higher grade in business statistics than a similar student who had taken MATHS 132. If the estimated coefficient on the math course *interaction* term is also positive, the expected statistics grade would be even higher. The opposite occurs if the estimated coefficient for the specific math course interaction term is negative. We have no reason to expect, a priori, that the signs of the estimated interaction coefficients will necessarily be positive or negative.

The estimated coefficients for GPA, ACC, ECON and Summer are expected to be positive. Previous studies have found that higher grade point averages and better grades in accounting and microeconomics courses have positive influences on student performance. [18] Finally, studies have also shown that grades are higher for classes taken in the summer than in either the fall or spring semesters. [19]

The university student records office provided us with an extensive data set for all students (2,493 in total) who had enrolled in business statistics from Fall 2001 through Summer 2006. We used data for 1,684 students who received a grade of A through F the first time they took the business statistics course after having taken their math, accounting, and microeconomics courses at this university.

5. The Ordered Probit Model Estimation

The dependent variable, STAT, takes on the values 0, 1, ..., 4. Because its range of values is limited, STAT is an example of what is called a *limited dependent variable*. Moreover, its values are *ordered* in a meaningful way—a STAT value of 4 (an A- to A grade) represents a better understanding of business statistics than does a STAT value of 3 (a B- to B+ grade), etc. As Kennedy describes this situation, "For some polychotomous dependent variables there is a natural order. [S]tudents' letter grades for an economics course, [for example,] may be generated by their instructor's assessment of their 'level of understanding' of the course material." [20]

The ordered probit model is often used to analyze problems with ordered limited dependent variables. [21] In the ordered probit model, "[e]stimation is undertaken by maximum likelihood, with β [the parameter vector of explanatory variables] being estimated in conjunction with estimation of the unknown boundary values [called "limit points"] defining the ranges of the [STAT] index." [22, 23]

Table 4 shows our ordered probit model estimation results. The signs of the estimated coefficients are generally consistent with our expectations. The coefficients for D165, MATH, GPA, ACC, ECON and Summer are positive and statistically significant at the .01 level. Those for D161 and D161M are both positive and jointly significant at the .05 level. In contrast, the coefficients for D131 and D131M and for D135 and D135M have mixed positive-negative signs and are not statistically significant, either alone or paired, at usual significance levels. Finally, the estimated coefficient for D165M is negative and statistically significant at the .01 level.

Table 4. Ordered Probit Estimated Business Statistics Grade Equation
Dependent Variable: STAT
Included observations: 1684

Variable	Coefficient [24]	Std. Error	z-Statistic	Prob.
D131	0.067	0.221	0.302	0.763
D135	-0.304	0.256	-1.188	0.235
D161	0.056**	0.276	0.203	0.839
D165	1.742*	0.478	3.642	0.000
MATH	0.275*	0.044	6.300	0.000
D131M	-0.053	0.083	-0.634	0.526
D135M	0.095	0.100	0.941	0.347
D161M	0.071**	0.100	0.715	0.475
D165M	-0.550*	0.179	-3.077	0.002
GPA	1.057*	0.095	11.067	0.000
ACC	0.196*	0.039	4.970	0.000
ECON	0.106*	0.041	2.584	0.010
SUMMER	0.639*	0.087	7.343	0.000

Limit Points

LIMIT_1:C(14)	3.072	0.192	15.982	0.000
LIMIT_2:C(15)	3.782	0.195	19.428	0.000
LIMIT_3:C(16)	4.717	0.203	23.256	0.000
LIMIT_4:C(17)	5.889	0.216	27.269	0.000

Pseudo R-squared	0.178	Akaike info criterion	2.559
Schwarz criterion	2.614	Log likelihood	-2137.907
Hannan-Quinn criter.	2.580	Restr. log likelihood	-2600.528
LR statistic	925.242	Avg. log likelihood	-1.270
Prob(LR statistic)	0.000		

However, while informative, the estimation results presented in Table 4 represent only the first, and not necessarily the best, way to view the outcome of the probit analysis. Wooldridge warns that, "... the most difficult aspect of ... probit models is presenting and interpreting the results." [25] There are two main reasons for this "difficulty". First, "there is no universally-accepted goodness-of-fit measure ... for probit ... models." [26] Second, the outcomes of chief interest from the model are not the estimated coefficients per se. Instead, they are the *conditional response probabilities* that indicate how the explanatory variables affect the *probability* of achieving a specific value of the dependent variable.

With regard to the first difficulty, several measures are often used to determine how well the model performs. Two commonly used measures are the Pseudo R-squared and the percent of index values correctly predicted. [27] A third suggested measure is the sum of the fraction of each index value correctly predicted. If this sum exceeds unity, the model is considered to have predictive power. [28] For our model, the Pseudo R-squared (an analog to the R-squared from linear regression models) is 0.178.

Information of the model's correct and incorrect predictions for values of the dependent variable, STAT, are displayed in Table 5. The model has an overall correct prediction rate of about 44 percent. In comparison, a default (constant probability) model (not shown in Table 5) has a correct prediction rate of about 30 percent. [29] Our model also has a 20 percentage point correction rate for the incorrect predictions made by the default model. For the third suggested goodness-of-fit measure, the sum of the fraction of each index value predicted correctly is 1.96, which is well above the unity threshold.

Table 5. Model Predictions for Values of the Dependent Variable STAT

Estimates from Probit Model					
Dep. Value	Obs.	Correct	Incorrect	% Correct	% Incorrect
0	176	63	113	35.795	64.205
1	231	0	231	0.000	100.000
2	418	196	222	46.890	53.110
3	501	267	234	53.293	46.707
4	358	214	144	59.777	40.223
Total	1684	740	944	43.943	56.057

Second, as noted above, the greatest challenge in presenting and interpreting probit results is that the outcomes of chief interest from our model are the *conditional response probabilities* [e.g., P(STAT = 4 | the individual explanatory variables), P(STAT = 3 | the individual explanatory variables), etc.] that indicate how the explanatory variables affect the probability of achieving a specific grade (A to F) in the business statistics course. There are two alternative ways to present and interpret the relevant response probabilities derived from the model. Both of these are presented in the following section.

6. Student Grade Performance in Business Statistics across Different Math Sequences

The conditional response probabilities are calculated using the model's estimated coefficients and limit points. [30] We used our ordered probit model estimates to derive the response probabilities for grades from A to F in the business statistics course conditional on the math sequence and the grade earned in the last math course in the sequence. [31] These probabilities are shown in Table 6. The math sequence is identified by the last course in the sequence.

The business statistics grades are shown in the first column grouped into grade categories consistent with the STAT grade index: A- to A, B- to B+, etc. The predicted probabilities for each statistics grade category conditional on taking the specific math sequence are listed in the second column (Overall). The third through the sixth columns show the predicted probabilities for the business statistics grades by the math grade earned (again, grouped into grade categories) in the prerequisite math course.

Comparisons of the predicted probabilities for the business statistics grades across the alternative math sequences and math grades show the impact of the math coverage (especially, the extent of calculus covered), the number of math credit-hours, and the math knowledge gained (the math grade) on student performance. Whether we view the Overall column or the specific math grade columns across the alternative math courses, we see that students who took math sequences with more rigorous math coverage, more time devoted to calculus and more credit hours generally had higher probabilities of earning higher grades and lower probabilities of earning lower grades in business statistics.

Table 6. Predicted Probabilities of Statistics Grades for Alternative Math Courses and Grades

	MATHS 131	Grade in MATHS 131			
Stat Grades	Overall	A- to A	B- to B+	C- to C+	D- to D+
A- to A	15.54 %	41.13 %	17.07 %	7.17 %	2.15 %
B- to B+	29.67	37.95	35.96	25.90	13.66
C- to C+	27.28	15.70	28.22	32.05	27.30
D- to D+	15.08	3.97	12.05	19.58	24.73
F	12.44	1.24	6.70	15.30	32.16
Number	277	45	101	89	42

	MATHS 135	Grade in MATHS 135			
Stat Grades	Overall	A- to A	B- to B+	C- to C+	D- to D+
A- To A	18.34 %	54.61%	24.95 %	8.24 %	2.15 %
B- to B+	29.57	31.53	37.94	27.40	13.45
C- to C+	25.92	10.37	22.55	31.21	26.52
D- to D+	14.24	2.58	8.90	18.36	24.42
F	11.93	0.91	4.67	14.80	33.46
Number	241	31	68	119	23

	MATHS 132	Grade in MATHS 132			
Stat Grades	Overall	A- to A	B- to B+	C- to C+	D- to D+
A- To A	20.04 %	48.22 %	22.96 %	8.43 %	2.81 %
B- to B+	29.70	35.02	36.50	27.13	16.42
C- to C+	25.24	12.61	25.11	31.35	29.22
D- to D+	13.79	3.10	10.12	18.56	24.03
F	11.23	1.05	5.31	14.53	27.52
Number	929	194	272	305	158

	MATHS 161	Grade in MATHS 161			
Stat Grades	Overall	A- to A	B- to B+	C- to C+	D- to D+
A- To A	36.85 %	64.25 %	40.32 %	19.77 %	4.44 %
B- to B+	32.13	27.08	38.38	34.37	19.00
C- to C+	17.89	6.95	16.00	26.16	28.77
D- to D+	7.60	1.34	4.03	12.00	22.41
F	5.52	0.38	1.27	7.70	25.39
Number	179	47	62	50	20

	MATHS 165	Grade in MATHS 165			
Stat Grades	Overall	A- to A	B- to B+	C- to C+	D- to D+
A- To A	30.71 %	40.65 %	32.63 %	27.44 %	23.01 %
B- to B+	37.74	39.84	36.71	37.04	42.72
C- to C+	20.96	15.30	20.00	22.78	24.89
D- to D+	7.25	3.36	7.15	8.55	7.17
F	3.34	0.85	3.51	4.18	2.20
Number	58	7	23	23	5

For example, examination of the Overall column in Table 6 shows that the probability of earning an A or A- grade in business statistics doubled from 15.54 percent, for students who took the MATHS 131 sequence, to more than 30 percent for those who took either the MATHS 161 or 165 sequence. In contrast, the predicted probability of earning an F in the business statistics course declined by nearly three-quarters, from 12.44 percent for students who took the MATHS 131 sequence, to 3.34 percent for those who took the MATHS 165 sequence. These substantial differences in predicted probabilities demonstrate the overall importance of math content and math credit hours on student success in business statistics without regard to the grade earned in the math course.

Moreover, the probabilities of better (worse) student performance in the business statistics course generally declined (rose) as the grade earned in the math class declined for each math sequence. Once again, these changes in the probability of student success in the business statistics course are substantial. For example, the probability of earning an A in business statistics declined from 41.13 percent to 2.15 percent as the grade earned by a student in MATHS 131 fell from A to D. Similarly, for students who took MATHS 161, the probability of earning an A in business statistics declined from 64.25 percent to 4.44 percent as the grade earned in the math class fell from A to D. In contrast, the probabilities of earning an F in business statistics generally rose substantially as the grade earned in the specified math course declined from A to D. The changes in the probabilities shown for students who took MATHS 165 are similar but considerably less dramatic. This result reflects the impact of the size of the negative estimated coefficient for the MATHS 165 interaction term.

Finally, we can examine the joint impact that both the prerequisite math sequence and the grade earned in the math class had on the probabilities of student success in the business statistics course. For example, the probability of earning an A in the business statistics course rose from 17.07 percent for students who earned a B in MATHS 131, to 32.63 percent for those who earned a B in MATHS 165. Similarly, the probability of earning an A in the business statistics course doubled from 2.15 percent for students who earned a D in MATHS 131 to 4.44 percent for those who earned a D in MATHS 161. [32] Thus, both students who do well and those who do poorly in their prerequisite math course sequences have higher probabilities of doing well in their business statistics course if they take more rigorous math courses with more credit-hours of instruction.

The second approach to viewing the model's results is to examine the *marginal* (or *partial*) impact on the probabilities of earning business statistics grades of A through F for a one-unit change in a specific explanatory variable. These marginal effects for selected variables are presented in Table 7. [33]

Table 7. Marginal Impacts of a One-Unit Change in Selected Explanatory Variables on the Probabilities of the Business Statistics Grade [34]

Stat Grade	GPA	D131	D135	D161	D165
A- to A	22.82 %	-4.50 %	-1.70 %	16.81 %	10.67 %
B- to B+	19.26	-.03	-.13	2.43	8.04
C- to C+	-16.47	2.04	.68	-7.35	-4.28
D- to D+	-15.82	1.29	.45	-6.19	-6.54
F	-9.79	1.21	.70	-5.71	-7.89

As shown in column 2 of the Table, a student with a one-point higher GPA had a 22.82 percentage point increase in the probability of earning an A and a 9.79 percentage point decrease in the probability of earning an F. While we should not be surprised that better students in general are likely to be among the better students in their individual courses, it is evident from Table 7 that the student characteristics that result in higher GPAs play an important role in their grade performance in business statistics. [35]

The third through the sixth columns in Table 7 show the changes in the probabilities of earning the designated grades in business statistics for students who took the listed math sequences instead of the MATHS 132 sequence. [36] The results in the 3rd and 4th column indicate, as expected, that taking the MATHS 131 or 135 sequences reduced the probabilities of earning higher grades (A or B) and increased the probabilities of earning lower grades (C to F) in the business statistics course compared to students who took the MATHS 132 sequence. However, this impact was relatively small. For example, MATHS 131 students had a 4.53 percentage point decline in the probabilities of earning an A or B in business statistics when compared to MATHS 132 students, while MATHS 135 students had only a 1.83 percentage point decline for the same comparison.

The results in the 5th and 6th columns in the table indicate that, as expected, taking the MATHS 161 or 165 sequences increased the probabilities of earning higher grades (A or B) and reduced the probabilities of earning lower grades (C to F) in the business statistics course compared to students who took the MATHS 132 sequence. Moreover, these changes in probabilities were substantial. For example, MATHS 161 students had a 19.24 percentage point increase in the probabilities of earning an A or B in business statistics when compared to MATHS 132 students, while MATHS 165 students had 18.71 percentage point increase for the same comparison.

At least three important conclusions arise from the results displayed in Tables 6 and 7. First, more rigorous math coverage, especially more calculus, and more time spent on math in general (more credit hours of math) contribute substantially to higher probabilities of better student performance in business statistics. For example, as shown in Table 6, students who earned a C in MATHS 161 or 165 had nearly identical probabilities of earning B or C in business statistics as did those who earned a B grade in MATH 131, 135 or 132.

Second, students who had a better grasp of the material covered (earned higher grades) in their prerequisite math courses, regardless of which sequence they took, and, of course, who had higher GPAs had greater probabilities of earning better grades in the business statistics course. [37]

Finally, Table 6 shows that students who earned a D in their math course had extremely high predicted probabilities of poor student performance (grades D or F) in business statistics, ranging from 56.89 percent, for students who took MATHS 131, to 47.8 percent, for students who took MATHS 161. These probabilities are approximately double those for students who earned a C in their prerequisite math course. [38]

This last result suggests that imposing a minimum grade requirement of C- for the prerequisite (or the acceptable substitute) math class will substantially increase student performance in the business statistics course. This result could occur either if students retake the prerequisite math course, achieve a better understanding of the material and earn a higher grade or if they decide to abandon pursuit of a business or economics degree for other, perhaps easier, degree programs. In our sample, 248 out of 1, 684 students earned a D grade in their math course. Thus, other things unchanged, about 15% of the students in our sample would have had to repeat the prerequisite math course or change their intended fields of study. However, we believe that, had the minimum C- grade requirement for the prerequisite math course actually been imposed, the proportion of students adversely affected would have been much lower. Simply knowing that this minimum math grade requirement exists provides a strong incentive for pre-business students to devote more time and effort to mastering the material covered in their prerequisite math course. [39]

7. Caveats and Lacunae

The results of our ordered probit analysis suggest that students exposed to more rigorous math coverage, especially more calculus, and more time spent on math in general (more credit hours of math) will perform better in their business and economics statistics course. While this claim may appear obvious, this study is the first that provides extensive statistical support for it. However, like all such studies, there are some caveats associated with the conclusions that should be discussed. Also, there are some aspects of the study that are incomplete and, thus, perhaps, call for additional research at some future time. In other words, what alternative conclusions might be drawn from our study and what did we leave out?

First, students who took the more rigorous math courses (MATHS 161 or 165) may simply be better math students (and, perhaps, better students in general) than those who took the pre-business required math prerequisite courses (MATHS 131, 132 or 135). To investigate this issue, we initially included the student’s math SAT scores in our analysis as one of the independent variables. However, its estimated coefficient was never close to statistical significance. [40] Certainly, some students who took the more rigorous math courses prior to taking the business statistics course are among the better students in the university. They include actuarial science and math majors who are interested in pursuing either double majors or minors in economics or business. However, they also include students who desired to change their academic programs to business, perhaps because they believe that it is somewhat easier than their original degree program. Unfortunately, our data set did not include information on the students’ initial or final majors.

Second, even casual examination of business and economics statistics texts shows that the discussion and use of calculus is virtually non-existent. Why, then, would enrolling in more rigorous math courses that include calculus topics result in better student performance in business statistics? Faced with the same question in their study of student performance in principles of economics courses, Hoag and Benedict suggested that "exposure to mathematical ideas appears to be more important in improving (student) performance" than even the math grade received. [41] They recommend that students should be required "to take more mathematics before taking economics on the grounds that students gain some maturity that helps them perform better in economics." [42] If math maturity is an important influence on student performance in business statistics, it explains why students who took the MATHS 161 or 165 sequences had higher probabilities of success.

Finally, a large number of studies have examined the impact of instructor-specific characteristics (e.g., individual teaching styles, gender, type of tests, choice of textbooks, use of on-line resources, etc.) on student performance in statistics and other courses. We did not include these variables because we were unable to link the students in our study with their respective instructors. [43] However, we did examine the course syllabi, exams, and statistics grade distributions for faculty who taught this course during the period of our study and found no major differences among them in their course coverage or grade distributions.

8. Summary and Aftermath

In this study, we demonstrate that prerequisite math course sequences that contain more credit hours, more rigorous coverage and more emphasis on calculus substantially increase the probabilities that a student will earn better grades and reduce the probabilities of earning a D or F in business statistics. We also show that imposing a minimum grade requirement of C- in the prerequisite math course will have similar effects on student performance.

We hope that these results will encourage business schools to re-examine the performance of their students in their business and economics statistics courses and to re-assess the effectiveness of the prerequisite math courses that their students are required to take. At our university, our preliminary research contributed to two important recent changes in the prerequisite math course sequence required for business statistics and admission to the College of Business. First, the College introduced the requirement that students must earn a minimum grade of C in MATHS 135 starting with the Fall 2006 semester. [44] Second, the College adopted a new prerequisite math sequence to replace the MATHS 135 sequence beginning with the Fall 2008 semester. This new sequence consists of MATHS 108, 111 and a new four-credit-hour course, MATHS 136: Mathematics for Business, which includes three additional weeks of calculus applications for business compared to the MATHS 135 course that it replaced. Pre-business students will now be required to take MATHS 111—which is also taken by math and science students and generally taught by tenure-track faculty. The minimum requirement of a C grade has been retained for MATHS 136. We anticipate that future student performance in business statistics under the new prerequisite math sequence will more closely resemble that associated with the MATHS 161 sequence than those previously observed for either the MATHS 135 or 132 sequences.

Appendix

Table A1: Variables and Definitions Used in Ordered Probit Model

Dependent Variable:

STAT: An ordered index (F = 0; D- to D+ = 1; ...; A- to A = 4) for the grade that the student received the *first* time that he/she completed ECON 221.

Independent Variables:

ACC: The numeric grade (F = 0, D- = .7, …, A- = 3.7, A = 4) that the student received in the principles of accounting course taken *prior* to *first* completion of ECON 221.

ECON: The numeric grade (F = 0, D- = .7, …, A- = 3.7, A = 4) that the student received in the principles of microeconomics course taken *prior* to *first* completion of ECON 221.

MATH: The numeric grade (F = 0, D- = .7, …, A- = 3.7, A = 4) that the student received in the relevant prerequisite math class.

GPA: The student's cumulative GPA in the semester *prior* to *first* completion of ECON 221.

D131: A dummy variable identifying whether the student took MATHS 131 as the prerequisite math course for ECON 221 (Yes = 1; 0 otherwise).

D135: A dummy variable identifying whether the student took MATHS 135 as the prerequisite math course for ECON 221 (Yes = 1; 0 otherwise).

D161: A dummy variable identifying whether the student took MATHS 161 as the prerequisite math course for ECON 221 (Yes = 1; 0 otherwise).

D165: A dummy variable identifying whether the student took MATHS 165 as the prerequisite math course for ECON 221 (Yes = 1; 0 otherwise).

D131M: An interaction term for MATHS 131 (D131M = D131 x MATH).

D135M: An interaction term for MATHS 135 (D135M = D135 x MATH).

D161M: An interaction term for MATHS 161 (D161M = D161 x MATH).

D165M: An interaction term for MATHS 165 (D165M = D165 x MATH).

Summer: A seasonal dummy variable for the term in which ECON 221 was first taken (summer = 1; 0 otherwise).

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Endnotes

[1] Norris and Hudson (1999), p. 233.

[2] For discussion of statistics anxiety, see Onwuegbuzie and Wilson (2003). For a small sample of the vast literature on how to improve statistics instruction, see Becker (1987, 1998), Becker and Greene (2001), Carlson (1999), Gandhi et al. (1991), Hakeem (2001), Hillmer (1996), McAlevey et al. (2001), and Parr and Smith (1998). For discussion of inadequate math preparation prior to entering college, see, Kronholz (2004) and Tomsho (2006). Finally, for a discussion of inadequate math prerequisites, see Green et al. (2007).

[3] We exclude from this study both the students who did not complete the course (who received a W) and those who are taking it for the second or third time.

[4] See, for example, Johnson and Kuennen (2006), Larson (2003) and Rochelle and Dotterweich (2007).

[5] Green et al. (2007) examined the impact of prerequisite math course changes on the DFW rate using a binary probit analysis. They did not examine the letter grade performance of students in the business statistics course.

[6] It excludes 174 transfer/transient students who completed ECON 221 but who had taken their prerequisite math courses elsewhere and 157 students who received a W in ECON 221 because they withdrew from the course before the end of the semester.

[7] Other studies have noted similarly low levels of performance for business statistics students. See, for example, Green et al. (2007), Hakeem (2001), Keeler and Steinhorst (1994), and Lewis et al. (2005).

[8] They are also required to complete two courses in each of accounting, economics, and English and a computer skills course.

[9] The number of credit hours and the academic units for the course are identical.

[10] MATHS 132 was not offered after the Spring 2004 semester.

[11] The credit hours listed for the various math sequences are the *maximum* that students would have to take. They were allowed to enroll in MATHS 131, MATHS 135, MATHS 161 or MATHS 165 without taking all the math prerequisites for these courses if they had sufficient background in algebra and trigonometry as evidenced by their high school record, SAT/ACT score, and/or score on the mathematics placement test, or permission from the chair of the mathematics department.

[12] Transitional period for change in math prerequisites in Fall 2004. See text for explanation.

[13] MATHS 131 was only permitted as the prerequisite math class for business statistics in the 2004/05 academic year. However, comparison of MATHS 131 with the other prerequisite math courses is particularly relevant because, initially, discussion in the college curriculum committee favored simply dropping MATHS 132 and making MATHS 131 the prerequisite math course.

[14] There is a third important difference between the math courses generally taken by pre-business students (MATHS 131/132 and 135) and math and science students (MATHS 161 and 165). The former classes are generally taught by contract faculty, while the latter courses are typically taught by tenure-track faculty.

[15] The error term is assumed to be normally distributed with the usual properties.

[16] In our preliminary research, we examined the impact of the student's math SAT score, gender and participation in varsity athletics which are often used in student performance studies. However, none of these variables were statistically significant and, therefore, we excluded them from our model.

[17] For a discussion of the impact of increased class time on math skills, see Toppo (2007).

[18] For example, see Borde et al. (1998), Brasfield et al. (1992), Butler et al. (1998), Didia and Hasnat (1998), Ely and Hittle (1990), Green et al. (2007), Haley et al. (2007) and Von Allmen (1996).

[19] See, for example, Austin and Gustafson (2006), Garces-Ozanne (2004), Green et al. (2007), and Terry and Galchus (2003).

[20] Kennedy (1998), p. 236.

[21] Another model used in such studies is the ordered logit model. The probit model is used if the cumulative density function for the error term is assumed to follow a normal distribution. The logit model is used if the cumulative density function for the error term is assumed to follow a logistic distribution. The probit and logit models generally produce similar results. See, for example, Greene (2008), p. 832 or Kennedy (1998), p. 247.

[22] Kennedy (1998), p. 236.

[23] For detailed discussion of probit and logit model estimation, see Greene (2008), chapter 23 or Wooldridge (2009), chapter 17.

[24] * denotes significance at the .01 level; ** denotes that the pair are jointly significant at the .05 level (F-test for redundant variables).

[25] Wooldridge (2009), p. 580.

[26] Kennedy (1998), p. 238.

[27] See Kennedy (1998), p. 239; Wooldridge (2009), pp. 580-1.

28 Kennedy (1998), p. 239.

[29] Results from the default model are available from the authors by request.

[30] For references showing how these probabilities are derived from the estimated coefficients and the limit points, see EViews (2007), pp. 226-7 or Greene (2008) pp. 831-832.

[31] These conditional probabilities for the designated math sequence and math course are estimated with the other explanatory variables valued at their mean values.

[32] The increase was even more spectacular for MATHS 165 but, once again, this result is produced by

the size of the negative estimated math grade interaction term for D165M.

[33] The changes in probabilities for a one-unit change in GPA are calculated holding other explanatory variables constant at their mean values. The changes in probabilities for D131 through D165 are calculated by subtracting the probabilities for MATHS 132 from those for MATHS 131 through 165, respectively, in column 2 of Table 6. To see how these marginal probabilities are calculated, see Greene (2008), p. 833-4.

[34] Because the numbers shown in the Table are *changes* in the probabilities of earning the designated statistics grades, they must necessarily sum to zero.

[35] See Yang and Raehsler (2005, p. 8) for very similar results for the marginal impact of GPA on intermediate microeconomics grades.

[36] Here the one-unit change is the change in the appropriate dummy variable from 0 to 1.

[37] For this reason, colleges and universities often impose minimum grade requirements on student performance in specific courses and minimum GPA requirements for entry into many degree programs. For example, the College of Business currently requires students to earn a minimum grade of C in the prerequisite math course, a minimum GPA of 2.25 in the pre-business core courses and a minimum GPA of 2.0 when they attain junior standing in order to enroll in upper-division business courses.

[38] The low probabilities for MATHS 165 students remains puzzling and runs counter to the argument presented in the text. Since this category includes only 5 students, we regard this result as anomalous rather than a disconfirmation of our general analysis.

[39] There is some anecdotal evidence for this conjecture. The requirement that students must earn a minimum grade of C in MATHS 135 was enforced beginning in the Fall 2006 semester, after it had been published in the 2006-8 university catalog and, thus, became official policy. However, pre-business students were informed that this standard would be "unofficially" applied (with a few exceptions) starting in Fall 2005. Table 6 shows that only 9.5 percent of the MATH 135 students earned a grade of D before taking ECON 221, compared to 15.2 and 17 percent of the MATHS 131 and 132 students, respectively. Moreover, about 11 percent of MATHS 161 students earned a D grade before taking ECON 221.

[40] Yang and Raeshler (2005), p. 6 comment that: "As is well known in the literature, SAT is a good predictor for freshmen, and its explanatory power wears off as students become sophomores and so on." ECON 221 students are typically either second-semester sophomores or juniors.

[41] Hoag and Benedict (2007), p. 15.

[42] Ibid.

[43] Our university requires that researchers using student records must obtain explicit written permission from students if they can be identified from the data used. Accordingly, they provided the data we requested in a format that excluded student identities (e.g., names, student IDs, course section numbers, etc.).

[44] Because our data set ended in Summer 2006, we are unable to directly examine the impact of this math

grade requirement on student performance in the business statistics course. However, footnote 38 contains some anecdotal evidence on its effect.

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