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The Availability Heuristic: A Redux

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Abstract

This article reports on a subset of results from a larger study which examined middle and high school students' probabilistic reasoning. Students in grades 5, 7, 9, and 11 at a boys' school (n=173) completed a Probability Inventory, which required students to answer and justify their responses to ten items. Supplemental clinical interviews were conducted with 33 of the students. This article describes students' specific reasoning strategies to a task familiar from the literature (Tversky and Kahneman, 1973). The results call into question the dominance of the availability heuristic among school students and present other frameworks of student reasoning.

1. Background

Suppose a group of three people (Axel, Beatrice, and Claude) want to form a 2-person team. They could create three different teams: Axel and Beatrice, Axel and Claude, or Beatrice and Claude. In other words, there are three different 2-person teams.

Teams Item: Now consider a group of 10 Dutchmen[1] who want to form a 2-person team. Also consider a group of 10 Dutchmen who want to form an 8-person team.

a) There are more 2-person teams
b) There are more 8-person teams

[1] The Dutchman is the mascot of the school at which the study was undertaken.

c) There are the same number of 2-person and 8-person teams

The number of 2-person teams in the above item is equal to the number of 8-person teams. However, on similarly constructed tasks, adults typically indicate that there are more 2-person teams (Shaughnessy, 1981; Tversky and Kahneman, 1973). Their reasoning for this response can be explained by the "availability heuristic," by which one estimates the probability of an event according to the ease with which instances of the event can be conceived (Tversky and Kahneman, 1973). Two-person teams are considered more available than eight-person teams for two reasons. First of all, ten people can be split into five distinct two-person teams at one time, whereas ten people can form only one eight-person team at a time. Secondly, any specific eight-person team has a greater overlap with the other eight-person teams. Median values of college students' estimations of the number of distinct teams of size n out of a total of ten people were found to decrease with larger values of n (Tversky and Kahneman, 1973). Specifically, the median value of the subjects' estimations of the number of 2-person teams was greater than sixty, while the median value of the estimations of the number of 8-person teams was less than twenty.

More recently, Fischbein and Schnarch (1997) posed a question similar to the Teams Item to a convenience sample of undergraduate students and students in 5^{th}, 7^{th}, 9^{th} and 11^{th} grades, and found, surprisingly, that higher proportions of the *older* students responded that there are more two-person teams. In their study, 10% of 5^{th} graders, 20% of 7^{th} graders, 65% of 9^{th} graders, 85% of 11^{th} graders, and 72% of undergraduate education students indicated that there are more two-person teams. Fischbein and Schnarch explained this surprising result in terms of the development of the cognitive schema of combinatorial reasoning. In other words, since older students possess more developed combinatorial reasoning skills, they can better conceptualize the number of combinations of two objects out of ten, which leads them toward the availability heuristic. At the same time, though, 55% of 5^{th} graders and 40% of 7^{th} graders in Fischbein and Schnarch's study did not respond to this particular item, in contrast with the nearly complete response rates of the older students. The goal of this article is to further investigate Fischbein and Schnarch's results by presenting the Teams Item to a new sample of students and by including an analysis of students' justifications of their responses.

2. Related Literature

The availability heuristic is typically cited as the most prevalent solution method to the above combinations task (e.g., Shaughnessy, 1992; Jones, 2005). However, there are other ways to reason about the Teams Item. It is also possible to categorize thinking about this item in terms of two of Tirosh and Stavy's (1999, 2000) intuitive rules, More A More B and Same A Same B, which pervade school student thinking about a variety of comparison tasks in statistics, mathematics, and science. In these comparison tasks, students are presented with two objects or systems that differ in a certain quality, A. They are then asked to compare those objects or systems in terms of a second quality, B. If one

object has more of quality A, students often indicate that it will also have more of quality B, which Tirosh and Stavy (1999, 2000) term *More A More B*.
For example, consider the comparison of two jars of marbles, one containing six black marbles out of a total of eight marbles, and the other containing three black marbles out of a total of four. When school students say that it is more likely to pick a black marble out of the first jar, a typical justification is that there are more black marbles in that jar. In other words, they have compared the two jars in terms of one quality, the number of black marbles. They then use the More A More B intuitive rule to reason that since the first jar has more black marbles, it will also have a greater likelihood to yield a black marble.

The Teams Item is an additional example of a comparative task, in which information is provided about one quality of the two objects, the size of each team. If a student indicates erroneously that there will be more eight-person teams, this could be interpreted as a result of the More A More B scheme. The eight-person teams have more members, so seemingly there ought to be more of them.

If the two objects or systems are equal in terms of one quality A, students often indicate that they will be equal in terms of a second quality B, which Tirosh and Stavy term *Same A Same B*. For example, when comparing the likelihood that a two-child family is comprised of a son and a daughter with the likelihood that a four-child family is comprised of two sons and two daughters, a typical response is that these events are equally likely since the ratios of the number of boys to the number of girls in each family are equal. In other words, since the two families are the same in terms of one quality, the ratio of girls to boys, they should seemingly be the same in terms of a second quality, their likelihoods. In the case of the Teams item, one could reason that since we are interested in forming two-person teams out of a group of ten people, and eight-person teams out of a group of ten people, the constant ten people in each scenario is an implication that there will be the same number of 2-person and 8-person teams.

3. Methods

I explored Fischbein and Schnarch's results by posing the Teams Item to a sample of American middle and high school students. I selected a convenience sample of 173 students in grades 5, 7, 9 and 11 at a boys' school, and included the Teams Item as part of a larger Probability Inventory (Rubel, 2002). The 11th graders had completed a unit focused on probability and combinatorics as part of a precalculus course, the ninth-grade students had worked with geometric probability tasks in their geometry course, and the fifth and seventh graders had limited exposure to counting problems in their mathematics classes that year.

Familiar to the students as a teacher at this school, I visited each of the twelve represented mathematics classes and delivered an informal presentation about their participation this particular study. I gave students an opportunity to ask questions about the study and they could opt out of participation at any time. The students completed the Probability Inventory, a written questionnaire, during their regular mathematics class period. Each item on the Probability Inventory prompted students for an explanation or justification of the given answer. The Task Item, as stated at the beginning of this article, was one of the ten items. I categorized the responses to each item on the Probability Inventory in two phases, first according to students' answers, and then in terms of justification type. The age levels of this sample match those of Fischbein and Schnarch's sample, as does the written format of the task. In contrast with Fischbein and Schnarch's study, in this case, students were also asked to explain their answer. 33 students were selected to participate in clinical interviews using the tasks from the Probability Inventory. Data from these interviews are used in this article to instantiate or further clarify justification categories.

4. Results

In this section, I present the distribution of students' answers to the Teams item, categorize students' justifications to their responses, and explore several implications of these results. While the literature typically explains reasoning on this or similar tasks only in terms of the availability heuristic, here, I explore other forms of student reasoning as well.

Table 1: Distribution of Responses

	Grade 5 (n=36)	Grade 7 (n=45)	Grade 9 (n=50)	Grade 11 (n=42)	Total (n=173)
More 2-person teams	61% (22)	60% (27)	58% (29)	31% (13)	53% (91)
More 8-person teams	8% (3)	16% (7)	16% (8)	36% (15)	19% (33)
Same number of 2-person and 8-person teams (correct answer)	17% (6)	22% (10)	18% (9)	19% (8)	19% (33)
Other or no answer	14% (5)	2% (1)	8% (4)	14% (6)	9% (16)

Table 1 contains the distribution of responses to the Teams item. About 19% of the students at each grade level gave the correct answer. About half of all of the students answered that there are more 2-person teams, and another 19% answered that there are more 8-person teams. While we might expect that the correct response rate would be higher among the older students, the correct response rates remain roughly stable across the grade levels. However, if we categorize students' responses according to their justifications, a different picture emerges, as explained below.

Table 2 further specifies the results by including students' method of justification along with their response type. In this section, I present student reasoning leading to the "more 2-person teams" response, continue with student reasoning for the "more 8-person teams" response, and conclude with student reasoning for the response "same number of teams."

Table 2 : Distribution of Justifications

	Grade 5 (n=36)	Grade 7 (n=45)	Grade 9 (n=50)	Grade 11 (n=42)	Total (n=173)
MORE 2-PERSON TEAMS	61% (22)	60% (27)	58% (29)	31% (13)	53% (91)
Availability	50% (11)	37% (10)	41% (12)	62% (8)	45% (41)
Partition interpretation	(36%) (8)	(52%) (14)	(48%) (14)	(23%) (3)	43% (39)
Other or no justification	14% (3)	11% (3)	10% (3)	15% (2)	12% (11)
MORE 8-PERSON TEAMS: More A More B	8% (3)	16% (7)	16% (8)	36% (15)	19% (33)
SAME NUMBER of 2-PERSON and 8-PERSON TEAMS (correct answer)	17% (6)	22% (10)	18% (9)	19% (8)	19% (33)
Counting	17% (1)	0	0	25% (2)	
Inclusion/exclusion	17% (1)	10% (1)	44% (4)	75% (6)	
Same A Same B	34% (2)	50% (5)	33% (3)	0	
Other justification	34% (2)	40% (4)	22% (2)	0	
Other or no answer	14% (5)	2% (1)	8% (4)	14% (6)	9% (16)

4.1 "More 2-Person Teams"

While slightly more than half of all of the students (91 of 173 students) answered that there are more 2-person teams, their justifications to that response were not uniform and were not limited to the availability heuristic. In fact, a justification as common as the availability heuristic involved an alternate interpretation of the task itself. Students' justifications to the response "more two-person teams" can be categorized as follows:

Availability (41 of 91 students). Students using this justification indicated that since two is a smaller number than eight, there are more possible combinations. For example, as 11th grader indicated, "There are less 8-person teams. It's obvious…2 is a lot less than 8 out of 10. There are a lot more combinations." Seven of these students counted approximately forty-five possible two-person teams, and then said that this must be more than the number of eight-person teams.

Alternate interpretation of the task: partitioning (39 of 91 students). To explain this form of student reasoning, let us consider a new question: Suppose there are ten people in a room. Scenario A requires that these people divide themselves, at one time, into two-person teams. This scenario is a partition interpretation of the Teams item, and is fundamentally different from counting the number of unique ways to choose two people out of ten. The number of ways to complete Scenario A is given by $\frac{10!}{(2!)^5}$, or 113,400. Scenario B, on the other hand, requires that these people divide themselves into eight-person teams. Since one cannot partition ten people strictly into groups of size eight, we are effectively counting the number of ways to choose eight people out of ten, which is forty-five. So, we see, using a partition interpretation, the number of two-person teams is greater than the number of eight-person teams. While none of the students gave the complete argument outlined above, some students wrote that ten people could be split into five two-person teams as opposed to only one eight-person team, indicating a partition interpretation of the task. For example, five 5th graders responded that there are more 2-person teams and justified this answer by writing "5>1." An eleventh grader's response further explains this reasoning: "In 10 people one can create 5 teams made up of two people and in the same group only one 8-person team can be made."

4.2 "More 8-Person Teams": "More A More B."

19% of all students, nearly half of whom were 11th graders, indicated that there are more 8-person teams. Simply put, a 5th grader wrote, "There are more people, so there are more possibilities." A 9th grade student explained, "If you have 2-person teams you can find out pretty easily. It's 9+8+7+6+5+4+3+2+1. It's like the handshake problem. But with eight people, person nine and ten could sub in for every other person. I think there are more eight-person teams. You take any eight people and keep subbing in two people whereas for two-person teams you can only team up with oh so many people." As described earlier in this article, we can interpret this response in terms of the More A

More B intuitive reasoning rule (Tirosh and Stavy, 1999). Fischbein & Schnarch claimed that older students have more developed combinatorial reasoning schema. We agree with this claim but conjecture that the results from the present study, specifically that more of the older students claimed that there are more 8-person teams, is evidence of this claim. In other words, older students have more experience with permutation situations, situations in which More A More B reasoning is correct (ie a greater collection of objects can be permuted in more ways than a smaller collection),

4.3 "Equal Number of Teams"

A small number of students (33) offered the correct response to the Teams Item. Interestingly, the proportion of students who answered the question correctly is approximately 19% at each of the grade levels. This result might seem surprising across such a wide span of age levels. However, if we categorize these responses according to students' justifications, the pattern of stability vanishes and we find that the frequency of correct reasoning was more common among the older students. Twenty-five of the 33 students who provided the correct answer gave justifications that can be categorized according to the following three categories.

Counting (3 of 33 students). One 5th grader and two 11th graders counted the number of combinations of 2-person teams and the number of 8-person teams.

Inclusion/exclusion (12 of 33 students). One 5th grader, one 7th grader, four 9th graders, and six 11th graders reasoned that including two people on a team is equivalent to excluding the other eight people from the team. For example, a 5th grader wrote, "For two-person teams, there will be eight people who aren't. 1-8 | 9-10. You don't know which is the team." Similarly, a 9th grader wrote, "Choosing a 2-person team is the same as choosing eight people to not put on the team."

"Same A Same B" (10 of 33 students). Two 5th grader, five 7th graders, and three 9th graders answered the question correctly but explained their answer using Same A Same B reasoning (Tirosh and Stavy, 1999). In other words, the same number of people in total makes the same number of teams, whatever their size. For example, a 5th grader wrote, "because the same amount of people do the same amount of teams." A 7th grader wrote, "You have the same number of people for both." The Same A Same B leads to a correct answer with the parameters of the Teams Item as stated. However, this reasoning would lead to an incorrect answer if comparing the number of possible 2-person and 3-person teams from the pool of ten people.

5. Conclusions and Implications

Fischbein and Schnarch (1997) utilized the results of a similar task to demonstrate that the presence of the availability misconception is more prevalent among older school students. The results of this study show a decrease in the frequency of the response "there are more 2-person teams" from the 5th, 7th, and 9th graders to the 11th graders, in contrast to Fischbein and Schnarch's results.

The task was difficult for most of the students in this sample, as only 33 of the 173 students answered this question correctly, and even fewer students provided a correct justification to that response. More of the older students used the mathematically precise inclusion/exclusion approach, while more of the younger students used the intuitive Same A Same B approach. About half of the students responded that there are more two-person teams, as the availability heuristic would dictate. However, fewer than half of those students actually justified their answer using availability reasoning. This calls into question the widespread acceptance of the availability heuristic as the dominant solution strategy to this task. Almost as many students interpreted this question as a partition question, in which case there are many more two-person teams. The partition interpretation of the task seems to be a new finding and warrants attention in future research.

This study also has instructional implications that extend from its methodology. Students were asked to answer a question and to justify their answer. This enabled the analysis to include descriptions of student reasoning to mathematically correct as well as incorrect responses. As a result, insights were gained, both into the types of errors students make, as well as the methods students use to arrive at correct answers. Statistics educators can benefit from a better understanding of students' reasoning on this specific task, but more broadly, statistics educators could use this, or a similarly constructed, item and its categorization of responses as a formative assessment tool. The actual process of asking for justifications and then paying attention to students' reasoning leading to correct and incorrect answers has significance for classroom assessment as well as future statistics education research.

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